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Research Article

On Topological Indices of Total Graph and Its Line Graph for Kragujevac Tree Networks

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Kragujevac tree is indicated by K; $K \in Kg_{q=s(2t+1)+1,s}$ with order and size $s(2t+1)+1$ and $s(2t+1)$, respectively. In this paper, we have a look at certain topological features of the total graph and line graph of the total graph of the considered tree, i.e., Kragujevac tree, by computing different topological indices and polynomials.

1. Introduction

The graphs throughout the discussion are simple. In a graph H, $V(H)$ and $E(H)$ denote the sets of nodes and lines accordingly. Consider $d_H(u)$ be the degree of a vertex u. Here, we assume that $A_1, A_2, A_3, \ldots$ are the branches which are shown in Figure 1. A Kragujevac tree [1–3] with central vertex consisting of degree at least 3 is a proper tree having branches $A_1, A_2, A_3, \ldots$.

A group of proper Kragujevac trees whose central vertex consist of degree s and order q is symbolized by $Kg_{q,s}$, and by adding a new vertex of degree 2 to a pendant line, an improper Kragujevac tree is attained [4]. It is symbolized by $Kg_{q,s}$; see [5–7] for more details.

Here, we have the branches $A_{l_1}, A_{l_2}, \ldots, A_{l_m}$ where $l_i \geq 2$ for all $\{i = 1, \ldots, m\}$ connected to the mid vertex. Our concern in this research work is with a specific type of Kragujevac tree when $A_{l_1}, A_{l_2}, \ldots, A_{l_m}$ are identical, i.e., $l_1 = l_2 = \cdots = l_m = p$. Thus, order and size of Kragujevac tree $K \in Kg_{q,s}$ are $q = s(2t+1)+1$ and $s(2t+1)$ accordingly, see Figure 2.

Any function on a graph that does not rely on the labelling of its vertices is referred to as a graph invariant or a topological index. Many invariants have been used in QSPR/QSAR studies with varying degrees of success. Gutman and Trinajstic [8] proposed Zagreb indices three decades before. Balaban et.al. later referred to them as Zagreb group indices [9, 10].

Then, the first Zagreb index and second Zagreb index are formulated as

$$M_1(H) = \sum_{l \in V(H)} (d_l)^2 = \sum_{ml \in E(H)} d_m + d_l, \qquad M_2(H) = \sum_{ml \in E(H)} d_m d_l. \tag{1}$$

In 2011, Fath-Tabar [11] suggested third Zagreb index for some graph H symbolized by $M_3(H)$ and is specifically defined as

$$M_3(H) = \sum_{ml \in E(H)} |d_m - d_l|. \tag{2}$$

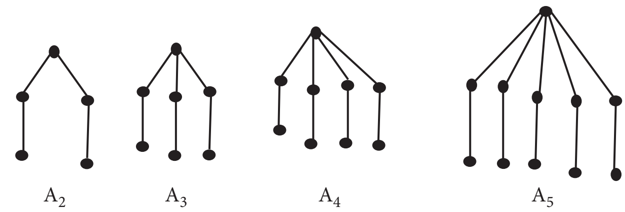

Figure 1: Branches (a) A_2, (b) A_3, (c) A_4, and (d) A_5.

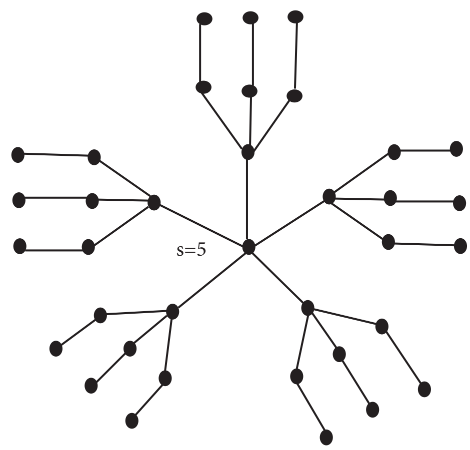

Figure 2: Kragujevac tree K from class $Kg_{36,5}$.

The same author introduced Zagreb polynomials in the same year, naming them first, second, and third Zagreb polynomials, and they are formulated as

$$M_1(H,u) = \sum_{ml\in E(H)} u^{d_m+d_l},$$
$$M_2(H,u) = \sum_{ml\in E(H)} u^{d_m d_l}, \quad (3)$$
$$M_3(H,u) = \sum_{ml\in E(H)} u^{|d_m-d_l|},$$

respectively. Modified Zagreb indices [12] are another variant of Zagreb indices that are associated with degrees of vertices. The modified Zagreb indices first and second are signified by $mM_1(H)$ and $mM_2(H)$, respectively, and are defined as

$$^mM_1(H) = \sum_{ml\in E(H)} \frac{1}{d_m+d_l},$$
$$^mM_2(H) = \sum_{ml\in E(H)} \frac{1}{d_m d_l}. \quad (4)$$

Hyper-Zagreb index, proposed by Shirdel et.al. [13], was formulated as

$$HM(H) = \sum_{ml\in E(H)} (d_m+d_l)^2. \quad (5)$$

In 2013, Ranjini et.al. suggested redefined Zagreb indices [14] and were formulated as

$$\text{redefined first Zagreb index} = \operatorname{Re} M_1(H) = \sum_{ml\in E(H)} \frac{d_m+d_l}{d_m d_l}, \quad (6)$$

$$\text{redefined second Zagreb index} = \operatorname{Re} M_2(H) = \sum_{ml\in E(H)} \frac{d_m d_l}{d_m+d_l}, \quad (7)$$

$$\text{redefined third Zagreb index} = \operatorname{Re} M_3(H) = \sum_{ml\in E(H)} (d_m d_l)(d_m+d_l). \quad (8)$$

Furtula et.al. introduced forgotten index which is marked by $F(H)$ for graph H [15], formulated as

$$F(H)=\sum_{l\in V(H)}(d_l)^3=\sum_{ml\in E(H)}\left[(d_m)^2+(d_l)^2\right]. \quad (9)$$

Parallel to the notion of forgotten topological index is that of forgotten polynomial. Forgotten polynomial of a graph H is given by

$$F(H,u)=\sum_{ml\in E(H)}u^{\left[(d_m)^2+(d_l)^2\right]}. \quad (10)$$

Reciprocal Randić index, reduced second Zagreb index, and reduced reciprocal Randić index were proposed by Gutman et al. [16] in 2014 as follows:

$$\begin{aligned}\mathrm{RR}(H)&=\sum_{ml\in E(H)}\sqrt{(d_m)(d_l)},\\ RM_2(H)&=\sum_{ml\in E(H)}(d_m-1)(d_l-1),\end{aligned} \quad (11)$$

$$\mathrm{RRR}(H)=\sum_{ml\in E(H)}\sqrt{(d_m-1)(d_l-1)}. \quad (12)$$

Many new graph invariants [17–20] known as family of Gourava indices were brought into existence by V. R. Kulli in 2017 and are described as follows:

$$\text{first Gourava index}=\mathrm{G_1O}(H)=\sum_{ml\in E(H)}[d_m+d_l+d_md_l], \quad (13)$$

$$\text{second Gourava index}=\mathrm{G_2O}(H)=\sum_{ml\in E(H)}[(d_m+d_l)(d_md_l)], \quad (14)$$

$$\text{product connectivity Gourava index}=\mathrm{PGO}(H)=\sum_{ml\in E(H)}\frac{1}{\sqrt{(d_m+d_l)(d_md_l)}}, \quad (15)$$

$$\text{sum connectivity Gourava index}=\mathrm{SGO}(H)=\sum_{ml\in E(H)}\frac{m}{\sqrt{(d_m+d_l)+(d_md_l)}}, \quad (16)$$

$$\text{first hyper}-\text{Gourava index}=\mathrm{HGO_1}(H)=\sum_{ml\in E(H)}[(d_m+d_l)+(d_md_l)]^2, \quad (17)$$

$$\text{second hyper}-\text{Gourava index}=\mathrm{HGO_2}(H)=\sum_{ml\in E(H)}[(d_m+d_l)(d_md_l)]^2. \quad (18)$$

M-polynomial [21] is given as

$$M(H;u,v)=\sum_{\delta\le s\le p\le\Delta}x_{sp}(H)u^sv^p, \quad (19)$$

where δ and Δ are respectively minimum and maximum of vertex degrees in graph H and x_{sp} represents the number of edges $ml\in E(H)$ such that $\{d_m,d_l\}=\{s,p\}$. We can deduce various indices from M-polynomial such as

$$\text{first Zagreb index}=M_1(H)=(D_u+D_v)(M(H;u,v))_{u=v=1}, \quad (20)$$

$$\text{second Zagreb index}=M_2(H)=(D_uD_v)(M(H;u,v))_{u=v=1}, \quad (21)$$

$$\text{modified second Zagreb index}={}^mM_2(H)=(S_uS_v)(M(H;u,v))_{u=v=1}, \quad (22)$$

$$\text{Randić index}=R_\alpha(H)=(S_u^\alpha S_v^\alpha)(M(H;u,v))_{u=v=1}=\sum_{ml\in E(H)}(d_md_l)^\alpha, \quad (23)$$

$$\text{inverse Randić index}=\mathrm{RR}(H)=(D_u^\alpha D_v^\alpha)(M(H;u,v))_{u=v=1}=\sum_{ml\in E(H)}\sqrt{d_md_l}. \quad (24)$$

Symmetric division deg index

$$\mathrm{SDD}(H)=\left(D_uS_v+D_vS_u\right)\left(M(H;u,v)\right)_{u=v=1}=\sum_{ml\in E(H)}\left\{\frac{\min\left(d_m,d_l\right)}{\max\left(d_m,d_l\right)}+\frac{\max\left(d_m,d_l\right)}{\min\left(d_m,d_l\right)}\right\}, \tag{25}$$

$$\text{harmonic index}=H(H)=2S_uJ\left(M(H;u,v)\right)_{u=v=1}=\sum_{ml\in E(H)}\frac{2}{d_m+d_l}, \tag{26}$$

$$\text{inverse sum indeg index}=\mathrm{ISI}(H)=S_uJD_uD_v\left(M(H;u,v)\right)_{u=v=1}=\sum_{ml\in E(H)}\frac{d_md_l}{d_m+d_l}, \tag{27}$$

and augmented Zagreb index

$$\begin{aligned}A(H)&=S_u^3Q_{-2}D_u^3D_v^3\left(M(H;u,v)\right)_{u=v=1}\\&=\sum_{ml\in E(H)}\left\{\frac{d_md_l}{d_m+d_l-2}\right\}^3,\end{aligned} \tag{28}$$

where

$$\begin{aligned}D_uM(H;u,v)&=u\frac{\partial\left(M(H;u,v)\right)}{\partial u},\\D_vM(H;u,v)&=v\frac{\partial\left(M(H;u,v)\right)}{\partial v},\\S_uM(H;u,v)&=\int_0^u\frac{M(H;t,v)}{t}\,\mathrm{d}t,\\S_vM(H;u,v)&=\int_0^v\frac{M(H;u,t)}{t}\,\mathrm{d}t,\\JM(H;u,v)&=M(H;u,u),\\Q_\alpha M(H;u,v)&=u^\alpha M(H;u,v).\end{aligned} \tag{29}$$

Since the past three years, people have conducted a lot of research on graphs, various operations, and chemical invariants of graphs. Observe simple charts under different chart operations such as line chart, subdivision, half-full point chart, half-full line chart, and general chart. The topological index of these operations on the graph is studied in depth.

Eliasi et.al. [22] displayed operations in the chart, such as S, R, Q, T, named subdivision, semitotal dot chart, semitotal line chart, and general chart. The subdivision graph represented by $S(H)$ is derived from H by replacing each of its edges or lines with a path of length 2, or in other words, by placing a new vertex (called white vertex) on each line of H [23–25]. In order to maintain the difference between the existing vertices and the newly inserted vertices, the vertices in H are called black vertices. If two white vertices are adjacent to each other in H, it is called correlation. Similarly, if two black vertices are adjacent in H, it is called correlation [26, 27].

In 1981, Bertz [28] invented the first topological index established on an online graph. For these graphical operations, many authors quote [21, 29, 30] to compute various topological indices. The general diagram of the Kragujevac tree is displayed in Figure 3.

Next part of this research paper comprises of two main Sections 2 and 3. In Section 2, we put forward computations for several chemical invariants (through direct calculations 2.2, 3.2 and using relation between polynomials 2.1, 3.1 and chemical invariants 2.3, 3.3) for Kragujevac tree. In Section 2, graphic operation T, total graph is applied at K, whereas in Section 3, same computation is made for $L(T(K))$.

2. Certain Topological Indices and Polynomials of $T(K)$, where $K\in Kg_{q==s(2t+1)+1,s}$

In this section, we compute certain topological indices, polynomials, and several other chemical indices in terms of these polynomials as mentioned in previous section for the total graph of Kragujevac tree. The edge partition is depicted in Table 1.

Theorem 1. *Let H be the total graph of a Kragujevac tree; $K\in Kg_{q=s(2t+1)+1,s}$. Then,*

$$\begin{aligned}F(H,u)&=\sum_{ml\in E(H)}u^{\left(d_m\right)^2+\left(d_l\right)^2}=su^{\left[(2s)^2+(s+t+1)^2\right]}+su^{\left[(s+t+1)^2+(2t+2)^2\right]}\\&\quad+stu^{\left[(s+t+1)^2+(t+3)^2\right]}+\frac{st^2}{2}u^{\left[(t+3)^2+(t+3)^2\right]}-\frac{st}{2}u^{\left[(t+3)^2+(t+3)^2\right]}\end{aligned}$$

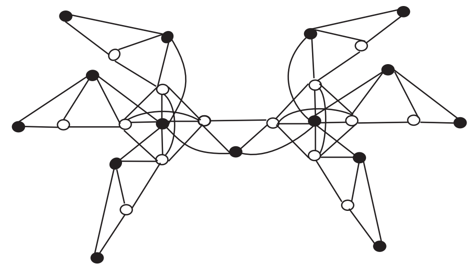

Figure 3: Total graph of Kragujevac tree.

Table 1: Line distribution in $T(K)$, where $K \in Kg_{q=s(2t+1)+1,s}$.

Lines of type	Number of lines
$E_{(2s,s+t+1)}$	s
$E_{(s+t+1,2t+2)}$	s
$E_{(s+t+1,t+3)}$	st
$E_{(t+3,t+3)}$	$st(t-1)/2$
$E_{(t+3,4)}$	st
$E_{(t+3,3)}$	st
$E_{(4,2)}$	st
$E_{(4,3)}$	st
$E_{(3,2)}$	st
$E_{(s+t+1,s+t+1)}$	$s(s-1)/2$
$E_{(t+3,2t+2)}$	st
$E_{(2t+2,2s)}$	s
$E_{(2t+2,4)}$	st

$$+ stu^{\left[(t+3)^2+(4)^2\right]} + stu^{\left[(t+3)^2+(3)^2\right]} + stu^{\left[(4)^2+(2)^2\right]} + stu^{\left[(4)^2+(3)^2\right]}$$
$$+ stu^{\left[(3)^2+(2)^2\right]} + \frac{s^2}{2}u^{\left[(s+t+1)^2+(s+t+1)^2\right]} + stu^{\left[(t+3)^2+(2t+2)^2\right]}$$
$$- \frac{s}{2}u^{\left[(s+t+1)^2+(s+t+1)^2\right]} + su^{\left[(2t+2)^2+(2s)^2\right]} + stu^{\left[(2t+2)^2+(4)^2\right]},$$

$$M_1(H,u) = \sum_{ml\in E(H)} u^{d_m+d_l} = su^{3s+t+1} + su^{s+t+1+2t+2} + stu^{s+t+1+t+3}$$
$$+ \frac{st^2}{2}u^{t+3+t+3} - \frac{st}{2}u^{t+3+t+3} + stu^{t+3+4} + stu^{t+3+3} + stu^{4+2}$$
$$+ stu^{4+3} + stu^{3+2} + \frac{s^2}{2}u^{s+t+1+s+t+1} - \frac{s}{2}u^{s+t+1+s+t+1} + stu^{t+3+2t+2}$$
$$+ su^{2t+2+2s} + stu^{2t+2+4} = s\left[u^{3s+t+1} + u^{s+3t+3} - \frac{u^{2s+2t+2}}{2}\right] + st\left[u^{s+2t+4}\right.$$
$$\left. - \frac{u^{2t+6}}{2} + u^{t+7} + u^{t+6} + u^6 + u^7 + u^5\right] + st^2\left[\frac{u^{2t+6}}{2}\right]$$
$$+ s^2\left[\frac{u^{2s+2t+2}}{2}\right] + stu^{3t+5} + su^{2t+2+2s} + stu^{2t+6},$$

$$M_2(H,u) = \sum_{ml\in E(H)} u^{d_m d_l} = su^{(2s)\times(s+t+1)} + su^{(s+t+1)\times(2t+2)}$$
$$+ stu^{(s+t+1)\times(t+3)} + \frac{st^2}{2}u^{(t+3)\times(t+3)} - \frac{st}{2}u^{(t+3)\times(t+3)}$$

$$+ stu^{(t+3)\times(4)} + stu^{(t+3)\times(3)} + stu^{(3)\times(2)} + su^{(2t+2)\times(2s)}$$
$$+ \frac{s^2}{2}u^{(s+t+1)\times(s+t+1)} - \frac{s}{2}u^{(s+t+1)\times(s+t+1)} + stu^{(t+3)\times(2t+2)}$$
$$+ stu^{(2t+2)\times(4)} + stu^{(4)\times(2)} + stu^{(4)\times(3)} + stu_{(t+3)\times(4)} = s\bigg[u^{2s^2+2st+2s}$$
$$+ u^{2st+4t+2s+2t^2+2} - \frac{u^{s^2+2st+2s+2t+t^2+1}}{2}\bigg] + st\bigg[u^{t^2+st+4t+3s+3}$$
$$+ \frac{u^{t^2+6t+9}}{2} + u^{12} + u^{4t+12} + u^{3t+9} + u^{8} + u^{6}\bigg]$$
$$+ st^2\left[\frac{u^{t^2+6t+9}}{2}\right] + s^2\left[\frac{u^{s^2+2st+2s+2t+t^2+1}}{2}\right] + stu^{2t^2+8t+6} + su^{4st+4s} + stu^{(8t+8)},$$
$$M_3(H,u) = \sum_{ml\in E(H)} u^{|d_m-d_l|} = su^{(2s)-(s+t+1)} + su^{(s+t+1)-(2t+2)}$$
$$+ stu^{(s+t+1)-(t+3)} + \frac{ts^2}{2}u^{(t+3)-(t+3)} - \frac{st}{2}u^{(t+3)-(t+3)} + stu^{(t+3)-(4)}$$
$$+ stu^{(t+3)-(3)} + stu^{(4)-(2)} + stu^{(4)-(3)} + stu^{(3)-(2)} + \frac{s^2}{2}u^{(s+t+1)-(s+t+1)}$$
$$- \frac{s}{2}u^{(s+t+1)-(s+t+1)} + stu^{(t+3)-(2t+2)} + su^{(2t+2)-(2s)} + stu^{(2t+2)-(4)} = s\left[2u^{s-t-1} - \frac{1}{2}\right]$$
$$+ st\left[u^{s-2} - \frac{1}{2} + u^{t-1} + u^{t} + u^{2} + 2u\right] + ts^2\left[\frac{1}{2}\right] + s^2\left[\frac{1}{2}\right] + stu^{1-t} + su^{2t+2-2s} + stu^{2t-2}. \quad (30)$$

Proof. Consider the total graph of Kragujevac tree, denoted by $T(K)$, where $K \in Kg_{q=s(2t+1)+1,s}$. We categorize the lines of $T(K)$ based on the lines of type $E_{(d_m,d_l)}$, where $ml \in E(T(K))$. The line classification is shown in Table 1,

$$F(H,u) = \sum_{ml\in E(H)} u^{(d_m)^2+(d_l)^2} = su^{\left[(2s)^2+(s+t+1)^2\right]} + su^{\left[(s+t+1)^2+(2t+2)^2\right]}$$
$$+ stu^{\left[(s+t+1)^2+(t+3)^2\right]} + \frac{st^2}{2}u^{\left[(t+3)^2+(t+3)^2\right]} - \frac{st}{2}u^{\left[(t+3)^2+(t+3)^2\right]}$$
$$+ stu^{\left[(t+3)^2+(4)^2\right]} + stu^{\left[(t+3)^2+(3)^2\right]} + stu^{\left[(4)^2+(2)^2\right]} + stu^{\left[(4)^2+(3)^2\right]}$$
$$+ stu^{\left[(3)^2+(2)^2\right]} + \frac{s^2}{2}u^{\left[(s+t+1)^2+(s+t+1)^2\right]} + stu^{\left[(t+3)^2+(2t+2)^2\right]}$$
$$- \frac{s}{2}u^{\left[(s+t+1)^2+(s+t+1)^2\right]} + su^{\left[(2t+2)^2+(2s)^2\right]} + stu^{\left[(2t+2)^2+(4)^2\right]},$$
$$M_1(H,u) = \sum_{ml\in E(H)} u^{d_m+d_l} = su^{3s+t+1} + su^{s+t+1+2t+2} + stu^{s+t+1+t+3}$$
$$+ \frac{st^2}{2}u^{t+3+t+3} - \frac{st}{2}u^{t+3+t+3} + stu^{t+3+4} + stu^{t+3+3} + stu^{4+2}$$
$$+ stu^{4+3} + stu^{3+2} + \frac{s^2}{2}u^{s+t+1+s+t+1} - \frac{s}{2}u^{s+t+1+s+t+1} + stu^{t+3+2t+2}$$

$$
\begin{aligned}
&+ su^{2t+2+2s} + stu^{2t+2+4} = s\left[u^{3s+t+1} + u^{s+3t+3} - \frac{u^{2s+2t+2}}{2}\right]\\
&+ st\left[u^{s+2t+4} - \frac{u^{2t+6}}{2} + u^{t+7} + u^{t+6} + u^{6} + u^{7} + u^{5}\right]\\
&+ st^2\left[\frac{u^{2t+6}}{2}\right] + s^2\left[\frac{u^{2s+2t+2}}{2}\right] + stu^{3t+5} + su^{2t+2+2s} + stu^{2t+6},
\end{aligned}
$$

$$
\begin{aligned}
M_2(H,u) = \sum_{ml\in E(H)} u^{d_m d_l} &= su^{(2s)\times(s+t+1)} + su^{(s+t+1)\times(2t+2)}\\
&+ stu^{(s+t+1)\times(t+3)} + \frac{st^2}{2}u^{(t+3)\times(t+3)} - \frac{st}{2}u^{(t+3)\times(t+3)}\\
&+ stu^{(t+3)\times(4)} + stu^{(t+3)\times(3)} + stu^{(3)\times(2)} + su^{(2t+2)\times(2s)}\\
&+ \frac{s^2}{2}u^{(s+t+1)\times(s+t+1)} - \frac{s}{2}u^{(s+t+1)\times(s+t+1)} + stu^{(t+3)\times(2t+2)}\\
&+ stu^{(2t+2)\times(4)} + stu^{(4)\times(2)} + stu^{(4)\times(3)} + stu_{(t+3)\times(4)} = s\left[u^{2s^2+2st+2s}\right.\\
&\left.+ u^{2st+4t+2s+2t^2+2} - \frac{u^{s^2+2st+2s+2t+t^2+1}}{2}\right] + st\left[u^{t^2+st+4t+3s+3}\right.\\
&\left.+ \frac{u^{t^2+6t+9}}{2} + u^{12} + u^{4t+12} + u^{3t+9} + u^{8} + u^{6}\right]\\
&+ st^2\left[\frac{u^{t^2+6t+9}}{2}\right] + s^2\left[\frac{u^{s^2+2st+2s+2t+t^2+1}}{2}\right]\\
&+ stu^{2t^2+8t+6} + su^{4st+4s} + stu^{(8t+8)},
\end{aligned}
$$

$$
\begin{aligned}
M_3(H,u) = \sum_{ml\in E(H)} u^{\left|d_m - d_l\right|} &= su^{(2s)-(s+t+1)} + su^{(s+t+1)-(2t+2)}\\
&+ stu^{(s+t+1)-(t+3)} + \frac{ts^2}{2}u^{(t+3)-(t+3)} - \frac{st}{2}u^{(t+3)-(t+3)}\\
&+ stu^{(t+3)-(4)} + stu^{(t+3)-(3)} + stu^{(4)-(2)}\\
&+ stu^{(4)-(3)} + stu^{(3)-(2)} + \frac{s^2}{2}u^{(s+t+1)-(s+t+1)} - \frac{s}{2}u^{(s+t+1)-(s+t+1)}\\
&+ stu^{(t+3)-(2t+2)} + su^{(2t+2)-(2s)} + stu^{(2t+2)-(4)} = s\left[2u^{s-t-1} - \frac{1}{2}\right]\\
&+ st\left[u^{s-2} - \frac{1}{2} + u^{t-1} + u^{t} + u^{2} + 2u\right] + ts^2\left[\frac{1}{2}\right] + s^2\left[\frac{1}{2}\right]\\
&+ stu^{1-t} + su^{2t+2-2s} + stu^{2t-2}.
\end{aligned} \tag{31}
$$

□

Theorem 2. *Let H be the total graph of Kragujevac tree; $K \in Kg_{q=s(2t+1)+1,s}$, then we have*

$$\begin{aligned}
HM(H) &= 12s^3 + 56st^2 + 8s + 209st + 20s^2t + 10s^2 + 5s^3t + 16st^3 + 6s^2t^2 \\
&\quad + 2st^4 + 2s^4 + 13st^3 + 69st + 58st^2 + 8st^2 + 4s + 4s^3 + 8s^2, \\
\operatorname{Re} M_1(H) &= s\left[\frac{3}{2s+2t+2} + \frac{3}{2st+4t+2s+2t^2+2} - \frac{1}{s^2+2st+2s+2t+t^2+1}\right] \\
&\quad + st\left[\begin{array}{c} \frac{3}{2st+2s+2t^2+4t+2} + \frac{4}{t^2+st+4t+3s+3} - \frac{3}{t^2+6t+9} + \frac{7}{4t+12} \\ + \frac{2}{t+3} + \frac{1}{4} + \frac{7}{12} + \frac{5}{6} - \frac{1}{s^2+2st+2s+2t+t^2+1} \end{array}\right] \\
&\quad + s^2t\left[\frac{1}{t^2+st+4t+3s+3} + \frac{1}{s^2+2st+2s+2t+t^2+1}\right] \\
&\quad + st^2\left[\frac{2}{t^2+st+4t+3s+3} + \frac{3}{t^2+6t+9} - \frac{1}{t^2+6t+9} + \frac{1}{4t+12} + \frac{1}{3t+9}\right] \\
&\quad + st^3\left[\frac{1}{t^2+6t+9}\right] + st\frac{2t+6}{(8t+8)} + t\left[\frac{1}{2t+2s+2}\right] \\
&\quad + s^3\left[\frac{1}{s^2+2st+2s+2t+t^2+1}\right] + st\frac{3t+5}{2t^2+8t+6} + s\frac{2t+2+2s}{(4st+4s)}, \\
\operatorname{Re} M_2(H) &= \frac{2s^3}{3s+t+1} + \frac{2s^2t}{3s+t+1} + \frac{2s^2}{3s+t+1} + \frac{2s^2t}{s+3t+3} + \frac{4st}{s+3t+3} + \frac{2s^2}{s+3t+3} + \frac{2st^2}{s+3t+3} \\
&\quad + \frac{2s}{s+3t+3} + \frac{st^3}{s+2t+4} + \frac{s^2t^2}{s+2t+4} + \frac{4st^2}{s+2t+4} + \frac{3s^2t}{s+2t+4} + \frac{3st}{s+2t+4} + \frac{st^4}{4t+12} \\
&\quad + \frac{3st^3}{2t+6} + \frac{9st^2}{4t+12} - \frac{st^3}{4t+12} - \frac{6st^2}{4t+12} - \frac{9st}{4t+12} + \frac{4st^2}{t+7} + \frac{12st}{t+7} + \frac{3st^2}{t+6} \\
&\quad + \frac{9st}{t+6} + \frac{4st}{3} + \frac{12st}{7} + \frac{6st}{5} + \frac{s^4}{4s+4t+4} + \frac{s^3t}{2s+2t+2} + \frac{s^3}{2s+2t+2} + \frac{s^2t}{2s+2t+2} \\
&\quad + \frac{s^2t^2}{4s+4t+4} + \frac{s^2}{4s+4t+4} - \frac{s^3}{4s+4t+4} - \frac{s^2t}{2s+2t+2} - \frac{s^2}{2s+2t+2} - \frac{st}{2s+2t+2} \\
&\quad - \frac{st^2}{4s+4t+4} - \frac{s}{4s+4t+4} + st\frac{\left(2t^2+8t+6\right)}{t+3+2t+2} + s\frac{4st+4s}{2t+2+2s} + st\frac{8t+8}{2t+6}, \\
\operatorname{Re} M_3(H) &= s^5 + s^4[6] + s^3[8] - s^2[8] + s[5] + 36s^2t + 104st^2 + 24s^2t^2 + 4s^2t^3 + 13s^3t \\
&\quad + 42st^3 + 4s^3t^2 + 2s^3 + 6st^4 + 50st^3 + 122st^2 + 78st + 16s^2t + 8s^2t^2 + 8s^2 + 8s^3, \\
{}^{m}M_1(H) &= s\left[\frac{1}{3s+t+1} + \frac{1}{s+3t+3} - \frac{1}{4s+4t+4}\right] + st\left[\frac{1}{s+2t+4} - \frac{1}{4t+12} + \frac{1}{t+7} + \frac{1}{t+6} + \frac{1}{6} + \frac{1}{7} + \frac{1}{5}\right] \\
&\quad + st^2\left[\frac{1}{4t+12}\right] + s^2\left[\frac{1}{4s+4t+4}\right] + st\frac{1}{3t+5} + s\frac{1}{2t+2+2s} + st\frac{1}{2t+6},
\end{aligned} \tag{32}$$

$$RM_2(H) = \frac{s^4}{2} + \frac{3s^3}{2} + 2s^2 - s + s^3t + 9s^2t + 22st + \frac{39st^2}{2} + \frac{3s^2t^2}{2} + \frac{9st^3}{2} + \frac{st^4}{2},$$

$$\begin{aligned}\mathrm{RRR}(H) = {} & s\left[\sqrt{2s^2 + 2st - s - t} + \sqrt{2st + 2t^2 + s + t} - \frac{\sqrt{s^2 + 2st + t^2}}{2}\right] + st\left[\sqrt{st + 2s + t^2 + 2t}\right. \\ & \left. - \frac{\sqrt{t^2 + 4t + 4}}{2} + \sqrt{3t + 6} + \sqrt{2t + 4} + \sqrt{3} + \sqrt{6} + \sqrt{2}\right] + st^2\left[\frac{\sqrt{t^2 + 4t + 4}}{2}\right] \\ & + s^2\left[\frac{\sqrt{s^2 + 2st + t^2}}{2}\right] + st\sqrt{(t + 2)(2t + 1)} + s\sqrt{(2t + 1)(2s - 1)} + st\sqrt{(2t + 1)(3)},\end{aligned}$$

$$\begin{aligned}\mathrm{PGO}(H) = {} & s\frac{1}{\sqrt{\left[6s^3 + 8st^2 + 8s^2t + 4st + 2s + 2st^2\right]}} + s\frac{1}{\sqrt{\left[2s^2t + 8st^2 + 10st + 24t^2 + 6t^3 + 18t + 2s + 6\right]}} \\ & + st\frac{1}{\sqrt{\left[3st^2 + 2t^3 + 12t^2 + 14st + 16t + 3s^2 + 15s + 18\right]}} + \frac{st^2}{2}\frac{1}{\sqrt{\left[2t^3 + 18t^2 + 58t + 54\right]}} \\ & + s\frac{1}{\sqrt{[(2t + 2 + 2s)(4st + 4s)]}} - \frac{st}{2}\frac{1}{\sqrt{\left[2t^3 + 18t^2 + 58t + 54\right]}} + st\frac{1}{\sqrt{\left[4t^2 + 40t + 84\right]}} \\ & + st\frac{1}{\sqrt{[48]}} + st\frac{1}{\sqrt{[84]}} + \frac{s^2}{2}\frac{1}{\sqrt{\left[2s^3 + 6s^2t + 6st^2 + 12st + 8s^2 + 6s + 4t^2 + 2t^3 + 2 + 6t\right]}} \\ & - \frac{s}{2}\frac{1}{\sqrt{\left[2s^3 + 6s^2p + 6st^2 + 12st + 8s^2 + 6s + 4t^2 + 2t^3 + 2 + 6t\right]}} + st\frac{1}{\sqrt{\left[3t^2 + 27t + 54\right]}} \\ & + st\frac{1}{\sqrt{\left[(3t + 5)\left(2t^2 + 11t + 11\right)\right]}} + s\frac{1}{\sqrt{[(2t + 2 + 2s)(4st + 4s)]}} + st\frac{1}{\sqrt{[(2t + 6)(8t + 8)]}} \\ & + st\frac{1}{\sqrt{\left[(3t + 5)\left(2t^2 + 8t + 6\right)\right]}} + st\frac{1}{\sqrt{[(2t + 6)(8t + 8)]}} + st\frac{1}{\sqrt{[30]}},\end{aligned}$$

$$\begin{aligned}\mathrm{GO}_1(H) = {} & s\left[5s + 2s^2 + 2st + t + 1\right] + s\left[3s + 7t + 2st + 2t^2 + 5\right] + st\left[s + 2t + 4 + st + t^2 + t + 3s + 3t + 1\right] \\ & + st\frac{(t - 1)}{2}\left[t^2 + 8t + 15\right]\Big] + st[5t + 19] + st[4t + 15] + st[14] + st[21] + st[11] \\ & + \frac{s(s - 1)}{2}\left[4s + 4t + 3 + s^2 + t^2 + 2st\right] + st\left[\left(2t^2 + 11t + 11\right)\right] + s[(4st + 2t + 2 + 6s)] + st[(10t + 14)],\end{aligned}$$

$$\begin{aligned}\mathrm{GO}_2(H) = {} & s^5 + s^4[6] + s^3[8] + s[5] + 52s^2t + 226st^2 + 32s^2t^2 + 4s^2t^3 + 29s^3t + 92st^3 \\ & + 4s^3t^2 + 2s^3 + 78st + 8s^2,\end{aligned}$$

$$\begin{aligned}\mathrm{HGO}_1(H) = {} & 37s^3 + 12s^2t + 5s^2 + 24s^3t + st^2 + 4s^2t^2 + 10s^4 + 4s^5 + 6s^4t + 2s^3t^2 + 9s^3 + 62s^2t \\ & + 40s^2t^2 + 12s^3t + 30s^2 + 69st^2 + 28st^3 + 24st^4 + 8s^2t^2 + 35st + 25s + st^5 + 2s^2t^4 + 6st^4 \\ & + 14st^3 + s^3t^3 + 8s^3t^2 + 31s^2t^2 + 16s^3t + 63st^2 + 42s^2t + 49st + \frac{st^6}{2} + 8st^5 + 47st^4 + 120st^3 \\ & + \frac{225st^2}{2} - \frac{st^5}{2} - 8st^4 - 47st^3 - 120st^2 - \frac{225st}{2} + 25st^3 + 190st^2 + 361st + 121st + \frac{s^6}{2} \\ & + 3s^4t^2 + 3s^5t + 4s^5 + 12s^4t + 11s^4 + 27s^2t^4 + 2s^2t^4 + 2s^3t^3 + 12s^3t^2 + 11s^2t^2 + 22s^3t \\ & + 2s^2t^3 + 12s^2t + 6s^3 + \frac{9t^2}{2} - \frac{s^5}{2} - 3s^3t^2 - 3s^4t - 4s^4 - 12s^3t + 11s^2t^3 \\ & - 11t^3 - \frac{5st^4}{2} - 2s^3t^3 - 12s^2t^2 - 11st^2 - 22s^2t - 2st^3 - 12st - 6s^2 - \frac{9t}{2},\end{aligned}$$

$$\begin{aligned}\mathrm{HGO}_2(H) = {} & s\left[6s^3 + 8s^2 + 8s^2t + 4st + 2s + 2st^2\right]^2 + s\left[2s^2t + 2s^2 + 16st + 8st^2 + 18t^2 + 6t^3\right. \\ & \left. + 18t + 8s + 6\right]^2 + st\left[3st^2 + 2t^3 + 12t^2 + 14st + s^2t + 15s + 12 + 12t\right]^2 + \frac{st^2}{2}\left[2t^3\right.\end{aligned}$$

$$+18t^2+54t+54\big]^2-\frac{st}{2}\big[2t^3+18t^2+54t+54\big]^2+st\big[4t^2+40t+84\big]^2+st\big[3t^2+27t+54\big]^2$$
$$+st[48]^2+st[84]^2+st[30]^2+\frac{s^2}{2}\big[2s^3+6s^2t+6st^2+12st+6s^2+6s+6t^2+2t^3+2+6t\big]^2$$
$$-\frac{s}{2}\big[2t^3+6s^2t+6st^2+12st+6s^2+6s+6t^2+2t^3+2+6t\big]^2, \tag{33}$$

$$\begin{aligned}\mathrm{SGO}(H)&=s\frac{1}{\sqrt{[3s+t+1+2s^2+2st+2s]}}+s\frac{1}{\sqrt{[s+3t+3+2s+4t+2t^2+2]}}\\&+\frac{st^2}{2}\frac{1}{\sqrt{[t^2+6t+9+2t+6]}}-\frac{st}{2\sqrt{t^2+6t+9+2t+6}}+st\frac{1}{\sqrt{[5t+12+7]}}\\&+st\frac{1}{\sqrt{[4t+15]}}+st\frac{1}{\sqrt{[14]}}+st\frac{1}{\sqrt{[19]}}+st\frac{1}{\sqrt{[11]}}+st\frac{1}{\sqrt{[(10t+14)]}}\\&+\frac{s^2}{2}\frac{1}{\sqrt{[s^2+2st+2s+2t+t^2+1+2s+2t+2]}}-\frac{s}{2}\frac{1}{\sqrt{[s^2+2st+2s+2t+t^2+1+2s+2t+2]}}\\&+st\frac{1}{\sqrt{[(2t^2+11t+11)]}}+s\frac{1}{\sqrt{[(4st+2t+2+6s)]}}+st\frac{1}{\sqrt{[t^2+st+4t+3s+3+s+4]}},\end{aligned}\tag{34}$$

$$\begin{aligned}F(H)&=4s^3+s^3+st^2+s+2st+2s^2t+2s^2+s^3+st^2+s+2st+2s^2t+2s^2+4st^2+4s+8st\\&+s^3t+st^3+st+2st^2+2s^2t+2s^2t^2+st^3+9st+6st^2+\frac{st^4}{2}+\frac{9st^2}{2}+3st^3+\frac{st^4}{2}\\&+\frac{9st^2}{2}+3st^3-\frac{st^3}{2}-3st^2-\frac{9st}{2}-3st^2-\frac{st^3}{2}-\frac{9st}{2}+st^3+9st+6st^2+16st+st^3\\&+9st+6st^2+9st+20st+28st+13st+\frac{s^4}{2}+s\frac{^2t^2}{2}+\frac{s^2}{2}+s^2t+s^3+s^3t-\frac{s^3}{2}-\frac{st^2}{2}\\&-\frac{s}{2}-st-s^2-\frac{s^3}{2}-\frac{st^2}{2}-2st-\frac{st}{2}-s^2-s^2t+st\big[5t^2+14t+13\big]+s\big[4t^2+4+8t+4s^2\big]\\&+st\big[4t^2+20+8t\big].\end{aligned}$$

Proof. For line classification shown in Table 1, use of formulas (5)–(9), (11)–(18), and (22) will give desired above expressions. □

Theorem 3. *Let H be the total graph of a Kragujevac tree; $K\in Kg_{q=s(2t+1)+1,s}$, then M-polynomial and certain topological indices deducted from M-polynomial are*

$$\begin{aligned}M(H;u,v)&=su^{2s}v^{s+t+1}+su^{s+t+1}v^{2t+2}+stu^{s+t+1}v^{t+3}+\frac{st^2}{2}u^{t+3}v^{t+3}-\frac{st}{2}u^{t+3}v^{t+3}\\&+stu^{t+3}v^4+stu^{t+3}v^3+stu^4v^2+stu^4v^3+stu^3v^2+\frac{s^2}{2}u^{s+t+1}v^{s+t+1}\\&-\frac{s}{2}u^{s+t+1}v^{s+t+1}+stu^{t+3}v^{2t+2}+su^{2t+2}v^{2s}+stu^{2t+2}v^4,\end{aligned}$$

$$M_1(H)=6s^2+\frac{95}{2}st+5s+2s^2t+11st^2+st^3+s^3,$$

$$M_2(H)=\frac{5s^3}{2}+7s^2t+\frac{7s^2}{2}+14st^2+\frac{97st}{2}+\frac{3}{2}s+\frac{3s^2t^2}{2}+\frac{7st^3}{2}$$

$$
\begin{aligned}
&+\frac{st^4}{2}+\frac{s^4}{2}+s^3+16st^2+14st+4s^2+2st^3+4s^2t,\\
{}^{m}M_2(H) &= \frac{1}{2s+2t+2}+s\left[\frac{1}{2t^2+2st+4t+2s+2}-\frac{1}{2(s+t+1)^2}\right]+st\left[\frac{1}{t^2+st+4t+3s+3}\right.\\
&\quad\left.-\frac{1}{2t^2+2t+18}+\frac{1}{4t+12}+st\frac{1}{(2t+2)(4)}+\frac{st}{8}+\frac{1}{3t+9}+\frac{1}{8}+\frac{1}{12}+\frac{1}{6}\right]\\
&\quad+st^2\left[\frac{1}{2t^2+12t+18}\right]+s^2\left[\frac{1}{2(s+t+1)^2}\right]+st\frac{1}{(2t+2)(t+3)}+s\frac{1}{(2t+2)(2s)},\\
R_\alpha(H) &= \frac{s}{(2s)^\alpha(s+t+1)^\alpha}+\frac{s}{(2t+2)^\alpha(s+t+1)^\alpha}+\frac{st}{(t+3)^\alpha(s+t+1)^\alpha}+\frac{(st)^2}{2(t+3)^\alpha(t+3)^\alpha}\\
&\quad-\frac{st}{2(t+3)^\alpha(t+3)^\alpha}+\frac{st}{(4)^\alpha(t+3)^\alpha}+\frac{st}{(3)^\alpha(t+3)^\alpha}+\frac{st}{(3)^\alpha(4)^\alpha}+\frac{st}{(2)^\alpha(3)^\alpha}\\
&\quad+\frac{st}{(4)^\alpha(2)^\alpha}+\frac{s^2}{2(s+t+1)^\alpha(s+t+1)^\alpha}-\frac{s}{2(s+t+1)^\alpha(s+t+1)^\alpha}+st\frac{1}{(2t+2)^\alpha(t+3)^\alpha}\\
&\quad+s\frac{1}{(2s)^\alpha(2t+2)^\alpha}+st\frac{1}{(4)^\alpha(2t+2)^\alpha},
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
\mathrm{RR}_\alpha(H) &= [s(2s)^\alpha(s+t+1)^\alpha+s(s+t+1)^\alpha(2t+2)^\alpha+st(s+t+1)^\alpha(t+3)^\alpha\\
&\quad-\frac{s}{2}(s+t+1)^\alpha(s+t+1)^\alpha+\frac{st^2}{2}(t+3)^\alpha(t+3)^\alpha-\frac{st}{2}(t+3)^\alpha(t+3)^\alpha+st(t+3)^\alpha(4)^\alpha\\
&\quad+st(t+3)^\alpha(3)^\alpha+st(4)^\alpha(2)^\alpha+st(4)^\alpha(3)^\alpha+st(3)^\alpha(2)^\alpha+\frac{s^2}{2}(s+t+1)^\alpha(s+t+1)^\alpha\\
&\quad+st(2t+2)^\alpha(t+3)^\alpha+s(2s)^\alpha(2t+2)^\alpha+st(4)^\alpha(2t+2)^\alpha,\\
\mathrm{SDD}(H) &= s^2\left[\frac{2}{s+t+1}+\frac{1}{2t+2}+1\right]+st\left[\frac{1}{2t+2}+\frac{2}{t+3}+\frac{2}{s+t+1}+\frac{3}{2(t+3)}+\frac{29}{4}\right]\\
&\quad+s^2t\left[\frac{1}{t+3}\right]+\frac{1}{2t+2}+st^2\left[\frac{1}{t+3}+\frac{1}{s+t+1}+\frac{1}{t+3}+\frac{13}{12}\right]+st^3\left[\frac{1}{2(t+3)}\right]\\
&\quad+\frac{1}{2}+\frac{t}{2}+s\left[\frac{1}{s+t+1}+\frac{1}{2}\right]+2st,\\
H(H) &= s\left[\frac{2}{3s+t+1}+\frac{2}{s+3t+4}-\frac{1}{2s+2t+2}\right]+st\left[\frac{2}{s+2t+4}-\frac{1}{2t+6}+\frac{2}{t+7}+\frac{2}{t+6}\right.\\
&\quad\left.+\frac{1}{3}+\frac{2}{7}+\frac{2}{5}\right]+s^2\left[\frac{1}{2s+2t+2}\right]+st^2\left[\frac{1}{2t+6}\right]+st\frac{2}{3t+5}+s\frac{2}{2t+2+2s}+st\frac{2}{2t+6},\\
\mathrm{ISI}(H) &= s(2s)(s+t+1)\frac{1}{3s+t+1}+s(s+t+1)(2t+2)\frac{1}{s+3t+3}+st(s+t+1)(t+3)\frac{1}{s+2t+4}\\
&\quad+\frac{st^2}{2}(t+3)(t+3)\frac{1}{2t+6}+st(4)(3)\frac{1}{7}+st(3)(2)\frac{1}{5}-\frac{st}{2}(t+3)(t+3)\frac{1}{2t+6}\\
&\quad+st(t+3)(4)\frac{1}{t+7}+st(t+3)(3)\frac{1}{t+6}+st(4)(2)\frac{1}{6}+\frac{s^2}{2}(s+t+1)(s+t+1)\frac{1}{2s+2t+2}\\
&\quad-\frac{s}{2}(s+t+1)(s+t+1)\frac{1}{2s+2t+2}+st(t+3)\frac{2t+2}{3t+5}+s(2t+2)\frac{2s}{2t+2s+2}\\
&\quad+st(2t+2)\frac{4}{2t+6},\\
A(H) &= s(2s)^3(s+t+1)^3\frac{1}{(3s+t-1)^3}+s(s+t+1)^3(2t+2)^3\frac{1}{(s+3t+1)^3}
\end{aligned}
$$

$$+ st(s+t+1)^3(t+3)^3\frac{1}{(s+2t+2)^3} + \frac{st^2}{2}(t+3)^3(t+3)^3\frac{1}{(2t+4)^3} - \frac{st}{2}(t+3)^3(t+3)^3\frac{1}{(2t+4)^3}$$
$$+ st(t+3)^3(4)^3\frac{1}{(t+5)^3} + st(t+3)^3(3)^3\frac{1}{(t+4)^3} + st(4)^3(2)^3\frac{1}{(4)^3} + st(4)^3(3)^3\frac{1}{(5)^3}$$
$$+ st(3)^3(2)^3\frac{1}{(3)^3} + \frac{s^2}{2}(s+t+1)^3(s+t+1)^3\frac{1}{(2s+2t)^3} - \frac{s}{2}(s+t+1)^3(s+t+1)^3\frac{1}{(2s+2t)^3}$$
$$+ st(2t+2)^3(2t+3)^3\frac{1}{(3t+3)^3} + s(2s)^3(2t+2)^3\frac{1}{(2t+2s)^3} + st(4)^3(2t+2)^3\frac{1}{(2t+4)^3}. \quad (36)$$

Proof. For line distribution shown in Table 1, use of formulas (19)–(28) will give above desired expressions. □

3. Certain Topological Indices and Polynomials of $L(T(K))$, where $K \in Kg_{q==s(2t+1)+1,s}$

In this section, we calculate specific topological indices and polynomials and many other chemical indices in terms of these polynomials, as presented in the first section of the Kragujevac tree total line graph. Figures 4–7 represents certain topological indices and polynomials.

Theorem 4. *Let H be the line graph of total of Kragujevac tree; $K \in Kg_{q=s(2t+1)+1,s}$. Then,*

$$F(H,u) = \sum_{ml\in E(H)} u^{(d_m)^2+(d_l)^2}$$
$$+ stu^{(t+5)^2+(3t+3)^2} + st(s-1)u^{(t+5)^2+(2t+4)^2} + st(s-1)u^{(t+4)^2+(2t+4)^2} + stu^{(4)^2+(5)^2}$$
$$+ stu^{(t+4)^2+(3t+3)^2} + stu^{(t+4)^2+(2t+s+2)^2} + stu^{(t+4)^2+(5)^2} + stu^{(t+4)^2+(3)^2}$$
$$+ stu^{(t+5)^2+(5)^2} + stu^{(5)^2+(3)^2} + stu^{(2t+s+3)^2+(3t+3)^2} + st(t-1)u^{(3t+3)^2+(2t+4)^2}$$
$$+ st(t-1)u^{(2t+s+2)^2+(2t+4)^2} + \frac{st(t-1)(t-2)}{2}u^{(2t+4)^2+(2t+4)^2} + stu^{(4)^2+(3)^2}$$
$$+ stu^{(4)^2+(t+5)^2} + stu^{(2t+4)^2+(4)^2} + stu^{(2t+4)^2+(5)^2} + stu^{(2t+s+2)^2+(s+3t+1)^2} + \frac{st(t-1)}{2}$$
$$\cdot u^{(2t+s+2)^2+(2t+s+2)^2} + stu^{(3s+t-1)^2+(2t+s+2)^2} + su^{(3s+t-1)^2+(s+3t+1)^2} + st(s-1)$$
$$\cdot u^{(2t+s+2)^2+(2s+2t)^2} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)^2+(2s+2t)^2} + s(s-1)u^{(3s+t-1)^2+(2s+2t)^2}$$
$$+ s(s-1)u^{(s+3t+1)^2+(2s+2t)^2} + s(s-1)u^{(2s+2t)^2+(2s+2t)^2}$$
$$+ \frac{s(s-1)}{2}u^{(3s+t-1)^2+(3s+t-1)^2} + s(s-1)u^{(2s+2t)^2+(3s+t-1)^2}$$
$$+ (s-1)u^{(2s+2t)^2+(2s+2t)^2} + stu^{(2t+4)^2+(s+3t+1)^2} + stu^{(2t+4)^2+(2s+2t)^2}$$
$$+ \frac{st(t-1)}{2}u^{(3t+3)^2+(3t+3)^2} + stu^{(3t+3)^2+(s+3t+1)^2}$$
$$+ stu^{(3t+3)^2+(2s+2t)^2} + su^{(s+3t+1)^2+(2s+2t)^2},$$

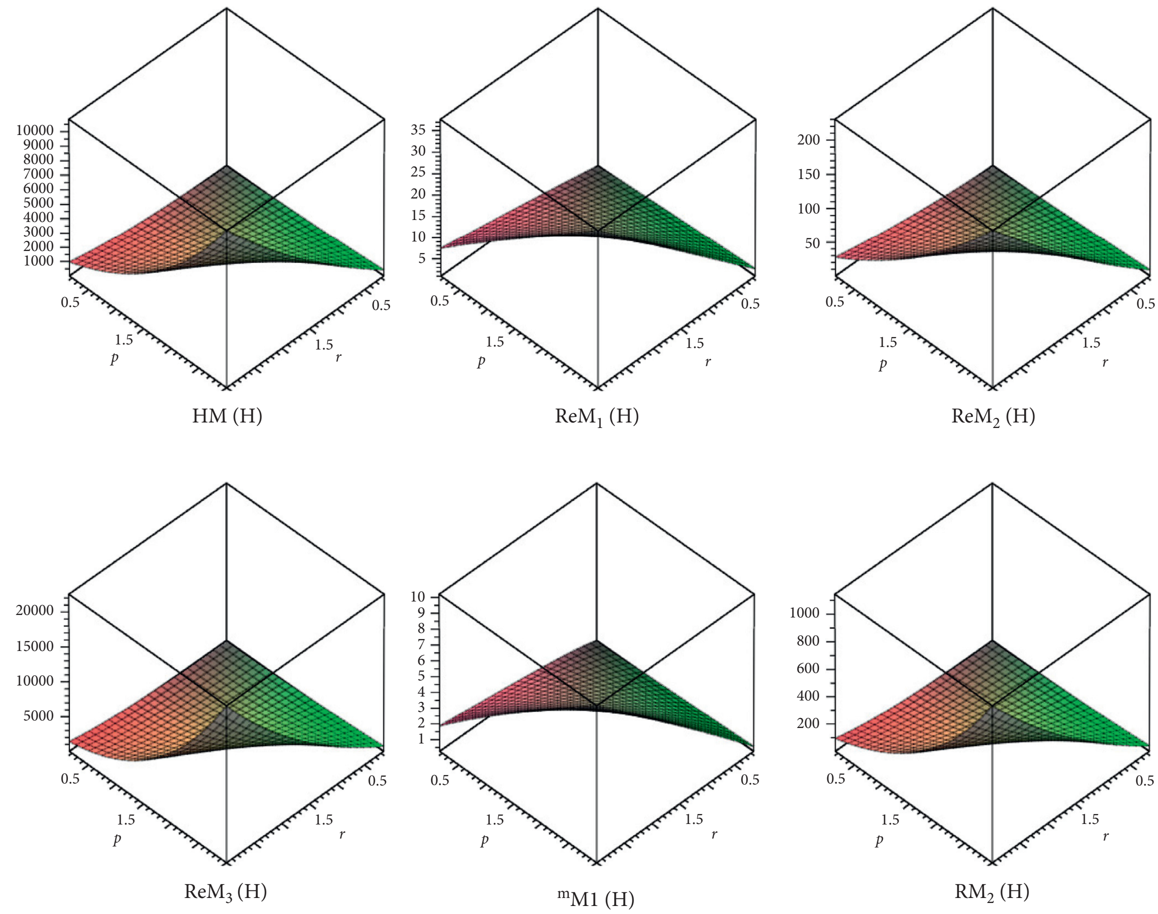

Figure 4: The pictorial representation of (a) $HM(H)$, (b) Re $M_1(H)$, (c) Re $M_2(H)$, (d) Re $M_3(H)$, (e) ${}^{m}M_1(H)$, and (f) $RM_2(H)$ in space.

$$\begin{aligned}
M_1(H,u) = {} & \sum_{ml\in E(H)} u^{d_m+d_l} \\
& + stu^{(t+5)+(3t+3)} + st(s-1)u^{(t+5)+(2t+4)} + st(s-1)u^{(t+4)+(2t+4)} + stu^{(t+4)+(3t+3)} \\
& + stu^{(t+4)+(2t+s+2)} + stu^{(t+4)+(5)} + stu^{(t+4)+(3)} + stu^{(t+5)+(5)} + stu^{(5)+(3)} \\
& + stu^{(2t+s+3)+(3t+3)} + st(t-1)u^{(3t+3)+(2t+4)} + st(t-1)u^{(2t+s+2)+(2t+4)} \\
& + su^{(s+3t+1)+(2s+2t)} + \frac{st(t-1)(t-2)}{2}u^{(2t+4)+(2t+4)} + stu^{(4)+(3)} + stu^{(4)+(5)} \\
& + stu^{(4)+(t+5)} + stu^{(2t+4)+(4)} + stu^{(2t+4)+(5)} + stu^{(2t+s+2)+(s+3t+1)} + \frac{st(t-1)}{2} \\
& \cdot u^{(2t+s+2)+(2t+s+2)} + stu^{(3s+t-1)+(2t+s+2)} + su^{(3s+t-1)+(s+3t+1)} + st(s-1) \\
& \cdot u^{(2t+s+2)+(2s+2t)} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)+(2s+2t)} + s(s-1)u^{(3s+t-1)+(2s+2t)} \\
& + s(s-1)u^{(s+3t+1)+(2s+2t)} + s(s-1)u^{(2s+2t)+(2s+2t)} + \frac{s(s-1)}{2}u^{(3s+t-1)+(3s+t-1)} \\
& + s(s-1)u^{(2s+2t)+(3s+t-1)} + (s-1)u^{(2s+2t)+(2s+2t)} + stu^{(2t+4)+(s+3t+1)} + st \\
& \cdot u^{(2t+4)+(2s+2t)} + \frac{st(t-1)}{2}u^{(3t+3)+(3t+3)} + stu^{(3t+3)+(s+3t+1)} + stu^{(3t+3)+(2s+2t)},
\end{aligned} \tag{37}$$

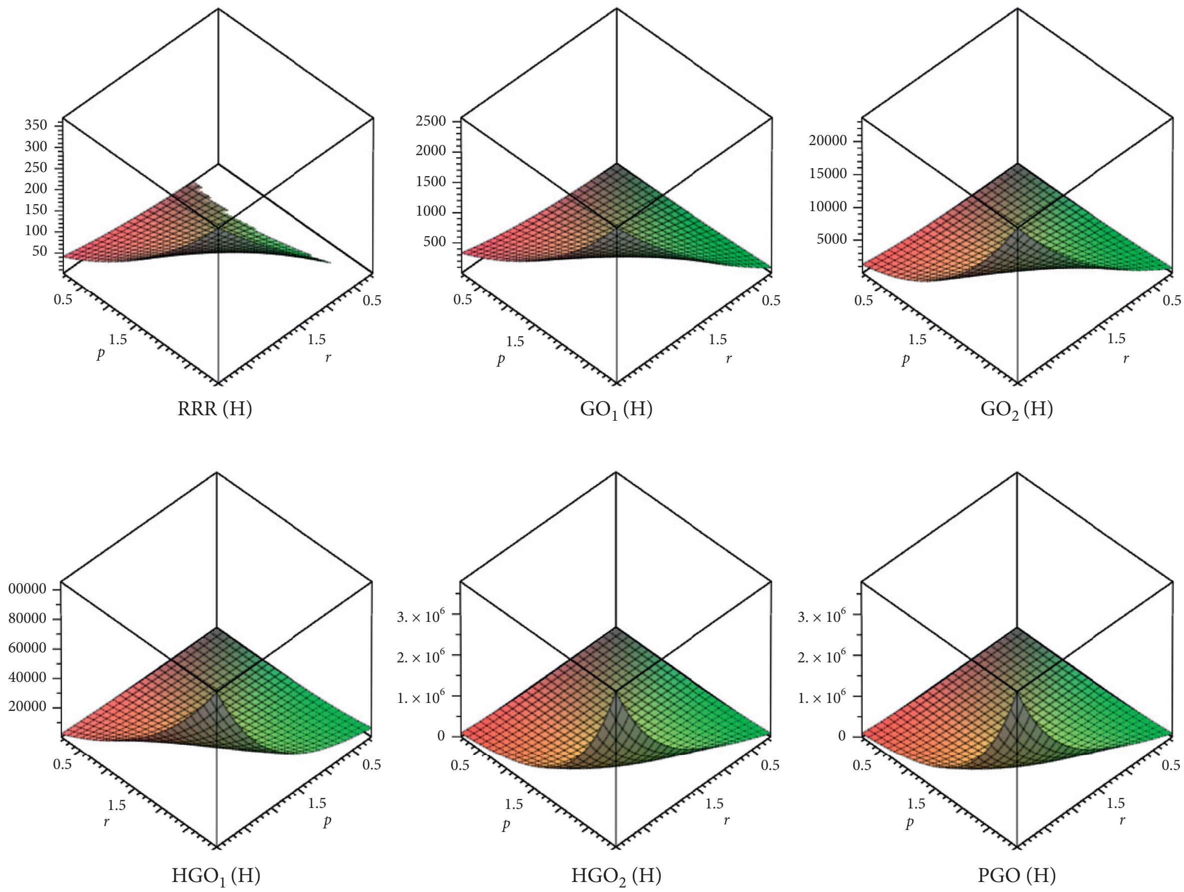

FIGURE 5: The pictorial characterization of (a) RRR (H), (b) $GO_1(H)$, (c) $GO_2(H)$, (d) $HGO_1(H)$, (e) $HGO_2(H)$, and (f) PGO (H) in space.

$$\begin{aligned}
M_2(H,u) = {} & \sum_{ml\in E(H)} u^{d_m d_l} \\
& + stu^{(t+5)\times(3t+3)} + st(s-1)u^{(t+5)\times(2t+4)} + st(s-1)u^{(t+4)\times(2t+4)} \\
& + stu^{(t+4)\times(3t+3)} + stu^{(t+4)\times(2t+s+2)} + stu^{(t+4)\times(5)} + stu^{(t+4)\times(3)} \\
& + stu^{(t+5)\times(5)} + stu^{(5)\times(3)} + stu^{(2t+s+3)\times(3t+3)} + st(t-1)u^{(3t+3)\times(2t+4)} \\
& + st(t-1)u^{(2t+s+2)\times(2t+4)} + \frac{st(t-1)(t-2)}{2}u^{(2t+4)\times(2t+4)} + stu^{(4)\times(3)} \\
& + stu^{(4)\times(5)} + stu^{(4)\times(t+5)} + stu^{(2t+4)\times(4)} + stu^{(2t+4)\times(5)} + stu^{(2t+s+2)\times(s+3t+1)} \\
& + \frac{st(t-1)}{2}u^{(2t+s+2)\times(2t+s+2)} + stu^{(3s+t-1)\times(2t+s+2)} + su^{(3s+t-1)\times(s+3t+1)} \\
& + st(s-1)u^{(2t+s+2)\times(2s+2t)} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)\times(2s+2t)} + s(s-1) \\
& \cdot u^{(3s+t-1)\times(2s+2t)} + s(s-1)u^{(s+3t+1)\times(2s+2t)} + s(s-1)u^{(2s+2t)\times(2s+2t)} + \frac{s(s-1)}{2} \\
& \cdot u^{(3s+t-1)\times(3s+t-1)} + s(s-1)u^{(2s+2t)\times(3s+t-1)} + (s-1)u^{(2s+2t)\times(2s+2t)} \\
& + stu^{(2t+4)\times(s+3t+1)} + stu^{(2t+4)\times(2s+2t)} + \frac{st(t-1)}{2}u^{(3t+3)\times(3t+3)} \\
& + stu^{(3t+3)\times(s+3t+1)} + stu^{(3t+3)\times(2s+2t)} + su^{(s+3t+1)\times(2s+2t)},
\end{aligned}$$

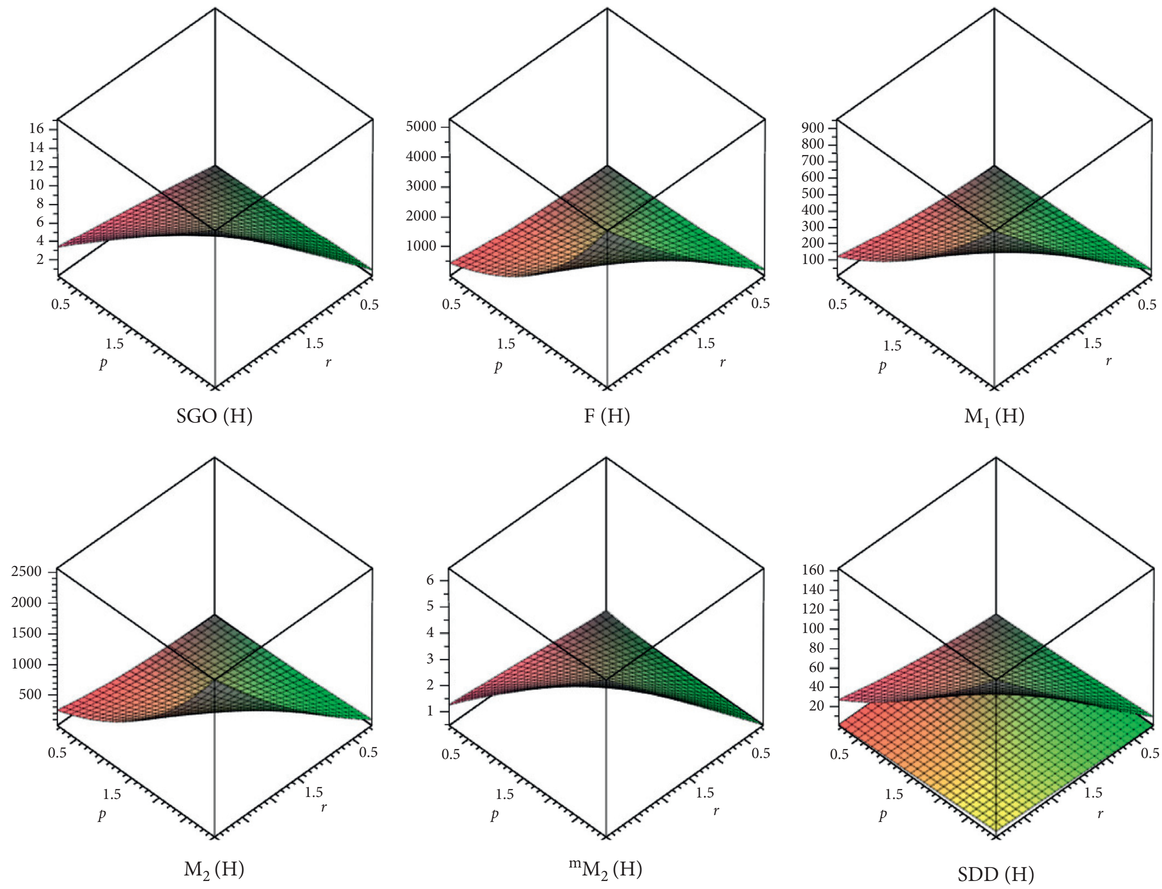

Figure 6: The pictorial representation of (a) SGO(H), (b) $F(H)$, (c) $M_1(H)$, (d) $M_2(H)$, (e) ${}^{m}M_2$, and (f) SDD(H) in space.

$$\begin{aligned}
M_3(H,u) ={}& \sum_{ml\in E(H)} u^{|d_m-d_l|} \\
&+ stu^{(t+5)-(3t+3)} + st(s-1)u^{(t+5)-(2t+4)} + st(s-1)u^{(t+4)-(2t+4)} + stu^{(t+4)-(3t+3)} \\
&+ stu^{(t+4)-(2t+s+2)} + stu^{(t+4)-(5)} + stu^{(t+4)-(3)} + stu^{(t+5)-(5)} + stu^{(5)-(3)} \\
&+ stu^{(2t+s+3)-(3t+3)} + st(t-1)u^{(3t+3)-(2t+4)} + st(t-1)u^{(2t+s+2)-(2t+4)} + \frac{st(t-1)(t-2)}{2} \\
&\cdot u^{(2t+4)-(2t+4)} + stu^{(4)-(3)} + stu^{(4)-(5)} + stu^{(4)-(t+5)} + stu^{(2t+4)-(4)} + stu^{(2t+4)-(5)} \\
&+ stu^{(2t+s+2)-(s+3t+1)} + \frac{st(t-1)}{2}u^{(2t+s+2)-(2t+s+2)} + stu^{(3s+t-1)-(2t+s+2)} \\
&+ su^{(3s+t-1)-(s+3t+1)} + st(s-1)u^{(2t+s+2)-(2s+2t)} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)-(2s+2t)} \\
&+ s(s-1)u^{(3s+t-1)-(2s+2t)} + s(s-1)u^{(s+3t+1)-(2s+2t)} + s(s-1)u^{(2s+2t)-(2s+2t)} \\
&+ \frac{s(s-1)}{2}u^{(3s+t-1)-(3s+t-1)} + s(s-1)u^{(2s+2t)-(3s+t-1)} + (s-1)u^{(2s+2t)-(2s+2t)} \\
&+ stu^{(2s+2t)-(s+3t+1)} + stu^{(2t+4)-(2s+2t)} + \frac{st(t-1)}{2}u^{(3t+3)-(3t+3)} + stu^{(3t+3)-(s+3t+1)} \\
&+ stu^{(3t+3)-(2s+2t)} + su^{(s+3t+1)-(2s+2t)}.
\end{aligned} \tag{38}$$

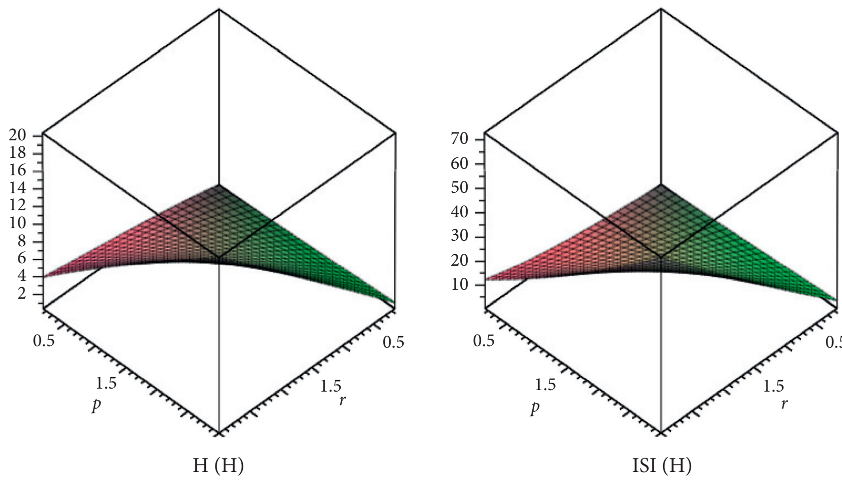

Figure 7: The pictorial characterization of (a) $H(H)$ and (b) $\mathrm{ISI}(H)$ in space.

Proof. Consider the line graph of total graph of Kragujevac tree, denoted by $L(T(K))$. We categorize the lines of $L(T(K))$ based on the lines of type $E_{(d_m,d_l)}$, where $ml \in E(L(T(K)))$. The line distribution is shown in Table 2.

$$
\begin{aligned}
F(H,u) &= \sum_{ml\in E(H)} u^{(d_m)^2+(d_l)^2} \\
&+ stu^{(t+5)^2+(3t+3)^2} + st(s-1)u^{(t+5)^2+(2t+4)^2} + st(s-1)u^{(t+4)^2+(2t+4)^2} + stu^{(4)^2+(5)^2} \\
&+ stu^{(2t+4)^2+(3t+3)^2} + stu^{(t+4)^2+(2t+s+2)^2} + stu^{(t+4)^2+(5)^2} + stu^{(t+4)^2+(3)^2} \\
&+ stu^{(t+5)^2+(5)^2} + stu^{(5)^2+(3)^2} + stu^{(2t+s+3)^2+(3t+3)^2} + st(t-1)u^{(3t+3)^2+(2t+4)^2} \\
&+ st(t-1)u^{(2t+s+2)^2+(2t+4)^2} + \frac{st(t-1)(t-2)}{2}u^{(2t+4)^2+(2t+4)^2} + stu^{(4)^2+(3)^2} \\
&+ stu^{(4)^2+(t+5)^2} + stu^{(2t+4)^2+(4)^2} + stu^{(2t+4)^2+(5)^2} + stu^{(2t+s+2)^2+(s+3t+1)^2} \\
&+ \frac{st(t-1)}{2}u^{(2t+s+2)^2+(2t+s+2)^2} + stu^{(3s+t-1)^2+(2t+s+2)^2} + su^{(3s+t-1)^2+(s+3t+1)^2} \\
&+ st(s-1)u^{(2t+s+2)^2+(2s+2t)^2} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)^2+(2s+2t)^2} + s(s-1)u^{(3s+t-1)^2+(2s+2t)^2} \\
&+ s(s-1)u^{(s+3t+1)^2+(2s+2t)^2} + s(s-1)u^{(2s+2t)^2+(2s+2t)^2} \\
&+ \frac{s(s-1)}{2}u^{(3s+t-1)^2+(3s+t-1)^2} + s(s-1)u^{(2s+2t)^2+(3s+t-1)^2} \\
&+ (s-1)u^{(2s+2t)^2+(2s+2t)^2} + stu^{(2t+4)^2+(s+3t+1)^2} + stu^{(2t+4)^2+(2s+2t)^2} \\
&+ \frac{st(t-1)}{2}u^{(3t+3)^2+(3t+3)^2} + stu^{(3t+3)^2+(s+3t+1)^2} \\
&+ stu^{(3t+3)^2+(2s+2t)^2} + su^{(s+3t+1)^2+(2s+2t)^2},
\end{aligned}
$$

Table 2: Line classification of $L(T(K))$.

Lines of type	Number of lines
$E_{(t+5,t+4)}$	st
$E_{(2t+4,t+5)}$	st
$E_{(t+5,2t+s+2)}$	st
$E_{(t+5,3t+3)}$	st
$E_{(t+5,2t+4)}$	$st(s-1)$
$E_{(t+4,2t+4)}$	$st(s-1)$
$E_{(t+4,3t+3)}$	st
$E_{(t+4,2t+s+2)}$	st
$E_{(t+4,5)}$	st
$E_{(t+4,3)}$	st
$E_{(t+5,5)}$	st
$E_{(5,3)}$	st
$E_{(2t+s+2,3t+3)}$	st
$E_{(3t+3,2t+4)}$	$st(s-1)$
$E_{(2t+s+2,2t+4)}$	$st(s-1)$
$E_{(2t+4,2t+4)}$	$(st(t-1)(t-2))/2$
$E_{(4,3)}$	st
$E_{(4,5)}$	st
$E_{(4,t+5)}$	st
$E_{(2t+4,4)}$	st
$E_{(2t+4,5)}$	st
$E_{(2t+s+2,2t+s+2)}$	$st(t-1)/2$
$E_{(2t+s+2,s+3t+1)}$	st
$E_{(3s+t-1,2t+s+2)}$	st
$E_{(3s+t-1,s+3t+1)}$	s
$E_{(2t+s+2,2s+2t)}$	$st(s-1)$
$E_{(2s+2t,2s+2t)}$	$(s(s-1)(s-2))/2$
$E_{(3s+t-1,2s+2t)}$	$s(s-1)$
$E_{(s+3t+1,2s+2t)}$	$s(s-1)$
$E_{(2s+2t,2s+2t)}$	$s(s-1)$
$E_{(3s+t-1,3s+t-1)}$	$s(s-1)/2$
$E_{(2s+2t,3s+t-1)}$	$s(s-1)$
$E_{(2s+2t,2s+2t)}$	$s-1$
$E_{(2t+4,s+3t+1)}$	st
$E_{(2t+4,2s+2t)}$	st
$E_{(3t+3,3t+3)}$	$st(t-1)/2$
$E_{(3t+3,s+3t+1)}$	st
$E_{(3t+3,2s+2t)}$	st
$E_{(s+3t+1,2s+2t)}$	s

$$
\begin{aligned}
M_1(H,u) = & \sum_{ml\in E(H)} u^{d_m+d_l} \\
& + stu^{(t+5)+(3t+3)} + st(s-1)u^{(t+5)+(2t+4)} + st(s-1)u^{(t+4)+(2t+4)} + stu^{(t+4)+(3t+3)} \\
& + stu^{(t+4)+(2t+s+2)} + stu^{(t+4)+(5)} + stu^{(t+4)+(3)} + stu^{(t+5)+(5)} + stu^{(5)+(3)} \\
& + stu^{(2t+s+3)+(3t+3)} + st(t-1)u^{(3t+3)+(2t+4)} + st(t-1)u^{(2t+s+2)+(2t+4)} \\
& + su^{(s+3t+1)+(2s+2t)} + \frac{st(t-1)(t-2)}{2}u^{(2t+4)+(2t+4)} + stu^{(4)+(3)} + stu^{(4)+(5)} \\
& + stu^{(4)+(t+5)} + stu^{(2t+4)+(4)} + stu^{(2t+4)+(5)} + stu^{(2t+s+2)+(s+3t+1)} + \frac{st(t-1)}{2}u^{(2t+s+2)+(2t+s+2)} \\
& + stu^{(3s+t-1)+(2t+s+2)} + su^{(3s+t-1)+(s+3t+1)} + st(s-1)u^{(2t+s+2)+(2s+2t)} \\
& + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)+(2s+2t)} + s(s-1)u^{(3s+t-1)+(2s+2t)}
\end{aligned}
$$

$$+ s(s-1)u^{(s+3t+1)+(2s+2t)} + s(s-1)u^{(2s+2t)+(2s+2t)} + \frac{s(s-1)}{2}u^{(3s+t-1)+(3s+t-1)}$$
$$+ s(s-1)u^{(2s+2t)+(3s+t-1)} + (s-1)u^{(2s+2t)+(2s+2t)} + stu^{(2t+4)+(s+3t+1)}$$
$$+ stu^{(2t+4)+(2s+2t)} + \frac{st(t-1)}{2}u^{(3t+3)+(3t+3)} + stu^{(3t+3)+(s+3t+1)} + stu^{(3t+3)+(2s+2t)}, \quad (39)$$

$$M_2(H,u) = \sum_{ml\in E(H)} u^{d_m d_l}$$
$$+ stu^{(t+5)\times(3t+3)} + st(s-1)u^{(t+5)\times(2t+4)} + st(s-1)u^{(t+4)\times(2t+4)}$$
$$+ stu^{(t+4)\times(3t+3)} + stu^{(t+4)\times(2t+s+2)} + stu^{(t+4)\times(5)} + stu^{(t+4)\times(3)}$$
$$+ stu^{(t+5)\times(5)} + stu^{(5)\times(3)} + stu^{(2t+s+3)\times(3t+3)} + st(t-1)u^{(3t+3)\times(2t+4)}$$
$$+ st(t-1)u^{(2t+s+2)\times(2t+4)} + \frac{st(t-1)(t-2)}{2}u^{(2t+4)\times(2t+4)} + stu^{(4)\times(3)}$$
$$+ stu^{(4)\times(5)} + stu^{(4)\times(t+5)} + stu^{(2t+4)\times(4)} + stu^{(2t+4)\times(5)} + stu^{(2t+s+2)\times(s+3t+1)}$$
$$+ \frac{st(t-1)}{2}u^{(2t+s+2)\times(2t+s+2)} + stu^{(3s+t-1)\times(2t+s+2)} + su^{(3s+t-1)\times(s+3t+1)}$$
$$+ st(s-1)u^{(2t+s+2)\times(2s+2t)} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)\times(2s+2t)}$$
$$+ s(s-1)u^{(3s+t-1)\times(2s+2t)} + s(s-1)u^{(s+3t+1)\times(2s+2t)} + s(s-1)u^{(2s+2t)\times(2s+2t)}$$
$$+ \frac{s(s-1)}{2}u^{(3s+t-1)\times(3s+t-1)} + s(s-1)u^{(2s+2t)\times(3s+t-1)} + (s-1)u^{(2s+2t)\times(2s+2t)}$$
$$+ stu^{(2t+4)\times(s+3t+1)} + stu^{(2t+4)\times(2s+2t)} + \frac{st(t-1)}{2}u^{(3t+3)\times(3t+3)}$$
$$+ stu^{(3t+3)\times(s+3t+1)} + stu^{(3t+3)\times(2s+2t)} + su^{(s+3t+1)\times(2s+2t)}, \quad (40)$$

$$M_3(H,u) = \sum_{ml\in E(H)} u^{|d_m - d_l|}$$
$$+ stu^{(t+5)-(3t+3)} + st(s-1)u^{(t+5)-(2t+4)} + st(s-1)u^{(t+4)-(2t+4)} + stu^{(t+4)-(3t+3)}$$
$$+ stu^{(t+4)-(2t+s+2)} + stu^{(t+4)-(5)} + stu^{(t+4)-(3)} + stu^{(t+5)-(5)} + stu^{(5)-(3)}$$
$$+ stu^{(2t+s+3)-(3t+3)} + st(t-1)u^{(3t+3)-(2t+4)} + st(t-1)u^{(2t+s+2)-(2t+4)} + \frac{st(t-1)(t-2)}{2}$$
$$\cdot u^{(2t+4)-(2t+4)} + stu^{(4)-(3)} + stu^{(4)-(5)} + stu^{(4)-(t+5)} + stu^{(2t+4)-(4)} + stu^{(2t+4)-(5)}$$
$$+ stu^{(2t+s+2)-(s+3t+1)} + \frac{st(t-1)}{2}u^{(2t+s+2)-(2t+s+2)} + stu^{(3s+t-1)-(2t+s+2)}$$
$$+ su^{(3s+t-1)-(s+3t+1)} + st(s-1)u^{(2t+s+2)-(2s+2t)} + \frac{s(s-1)(s-2)}{2}u^{(2s+2t)-(2s+2t)}$$
$$+ s(s-1)u^{(3s+t-1)-(2s+2t)} + s(s-1)u^{(s+3t+1)-(2s+2t)} + s(s-1)u^{(2s+2t)-(2s+2t)}$$
$$+ \frac{s(s-1)}{2}u^{(3s+t-1)-(3s+t-1)} + s(s-1)u^{(2s+2t)-(3s+t-1)} + (s-1)u^{(2s+2t)-(2s+2t)}$$
$$+ stu^{(2t+4)-(s+3t+1)} + stu^{(2t+4)-(2s+2t)} + \frac{st(t-1)}{2}u^{(3t+3)-(3t+3)} + stu^{(3t+3)-(s+3t+1)}$$
$$+ stu^{(3t+3)-(2s+2t)} + su^{(s+3t+1)-(2s+2t)}.$$

□

Theorem 5. *Let H be the line graph of total of Kragujevac tree; $K \in Kg_{q=s(2t+1)+1,s}$, then we have*

$$\begin{aligned}
HM(H) &= 74st^4 + 248st^3 + 522st^2 + 830st + 173s^2t + 293s^2t^2 + 50s^2t^3 \\
&\quad + 36s^3t^2 + 8st^5 - 8s + 133s^3t - 115s^3 + 26s^2, \\
\operatorname{Re} M_1(H) &= st^2\Bigg[\frac{2}{t^2+9t+20} + \frac{3}{2t^2+14t+20} + \frac{3}{2t^2+st+12t+5t+14} + \frac{4}{2t^2+st+12t+5s+14} \\
&\quad - \frac{3}{2t^2+14t+20} + \frac{3}{2t^2+12t+16} + \frac{4}{3t^2+15t+12} + \frac{1}{5t+25} + \frac{5}{6t^2+12t+3rt+3s+6} \\
&\quad + \frac{7}{6t^2+18t+12} - \frac{5}{6t^2+18t+12} + \frac{3}{2t^2+6t+st+2s+4} - \frac{2}{2t^2+6t+st+2t+4} \\
&\quad - \frac{1}{4t^2+12t+2st+4s+8} + \frac{1}{4t+20} + \frac{1}{4t+8} + \frac{1}{5t+10} + \frac{5}{6t^2+6t+s^2+16st+3s} \\
&\quad + \frac{2}{4t^2+4st+8t+s^2+8s+4} - \frac{2}{4t^2+4st+8t+s^2+8s+4} + \frac{3}{7st+3s^2+5s+2t^2-2} \\
&\quad - \frac{2}{3st+2t^2+s^2+2s+2t} + \frac{5}{2rt+6t^2+14t+4s+4} + \frac{1}{t^2+st+2t+2s} + \frac{1}{3t^2+6t+3} \\
&\quad - \frac{1}{3t^2+6t+3} + \frac{1}{3t^2+4t+s+st+4} + \frac{5}{6t^2+6st+6t+6s}\Bigg] + st\Bigg[\frac{9}{t^2+9t+20} \\
&\quad + \frac{9}{2t^2+14t+2t} + \frac{7}{2t^2+st+12t+5s+4} + \frac{8}{2t^2+st+12t+5s+4} - \frac{9}{2t^2+14t+20} \\
&\quad - \frac{4}{t^2+6t+8} + \frac{7}{3t^2+15t+12} + \frac{2}{t+5} + \frac{8}{15} + \frac{5}{6t^2+12t+3st+3t+6} \\
&\quad - \frac{7}{6t^2+18t+12} - \frac{3}{2t^2+6t+st+2s+4} + \frac{7}{12} + \frac{9}{20} + \frac{9}{4t+20} + \frac{1}{t+2} \\
&\quad + \frac{9}{10t+20} + \frac{3}{6st+6t^2+8t+s^2+3s} - \frac{2}{4t^2+4st+8t+s^2+8s+4} \\
&\quad + \frac{1}{7st+3s^2+5s+2t^2-2} + \frac{4}{7st+3s^2+5s+2t^2-2} - \frac{1}{3st+2t^2+s^2+2t+2s} \\
&\quad + \frac{1}{s^2+t^2+2st} - \frac{3}{6s^2+2t^2+8st-2s-2t} - \frac{5}{2s^2+8st+2s+2t+6t^2} \\
&\quad - \frac{1}{s^2+t^2+2st} - \frac{1}{9s^2+t^2+6st+1-2t-6s} - \frac{3}{6s^2+2t^2+8st-2t-2s} \\
&\quad + \frac{1}{s^2+t^2+2st} + \frac{5}{6t^2+2st+14t+4s+4} + \frac{1}{s^2+st+2t+2s} \\
&\quad - \frac{1}{3t^2+6t+3} + \frac{4}{9t^2+12t+3t+3st+3} + \frac{1}{2t^2+2st+2t+2s} + \frac{5}{6t^2+2s^2+8st+2t+2s}\Bigg] \\
&\quad + s^2t\Bigg[\frac{1}{2t^2+st+12t+5s+14} + \frac{9}{2t^2+14t+20} + \frac{4}{t^2+6t+8} + \frac{1}{6t^2+12t+3st+3t+6}
\end{aligned} \tag{41}$$

$$-\frac{1}{4t^2+12t+2st+4s+8}+\frac{2}{6st+6t^2+8t+s^2+3s}-\frac{1}{4t^2+4st+8t+s^2+8s+4}$$
$$+\frac{4}{7st+3s^2 5s+2t^2-2}+\frac{1}{3st+2t^2+s^2+2s+2t}-\frac{3}{6st+4t^2+2s^2+4s+4t}$$
$$-\frac{3}{2s^2+2t^2+4st}+\frac{3}{6s^2+2t^2+8st+2s-2t}+\frac{5}{2s^2+8st+2s+2t+6t^2}$$
$$+\frac{1}{t^2+s^2+2st}+\frac{1}{9s^2+t^2+1+6st-2t-6s}+\frac{3}{8st+6s^2+2t^2-2s-2t}$$
$$+\frac{1}{2t^2+2st+4t+4s}+\frac{1}{9t^2+12t+3s+3st+3}+\frac{1}{2st+6t^2+14t+4s+4}$$
$$+\left.\frac{1}{3t^2+3st+3t+3s}\right]+s^2t^2\left[\frac{3}{2t^2+14t+20}+\frac{3}{2t^2+12t+16}\right.$$
$$\left.+\frac{1}{4t^2+12t+2st+4s+8}+\frac{1}{4t^2+4st+8t+s^2+8s+4}+\frac{2}{3st+2t^2+s^2+2s+2t}\right]$$
$$+st^3\left[\frac{5}{6t^2+18t+12}+\frac{4}{4t^2+12t+2st+4s+8}+\frac{2}{4t^2+4st+8t+s^2+8s+4}\right.$$
$$\left.+\frac{1}{3t^2+6t+3}\right]+s^2\left[\frac{4}{7st+3s^2+5s+2t^2-2}+\frac{1}{s^2+t^2+2st}-\frac{5}{6s^2+2t^2+8st+2s-2t}\right.$$
$$-\frac{1}{6s^2+2t^2+8st-2s-2t}+\frac{1}{2s^2+8st+2s+2t+6t^2} \tag{42}$$

$$-\frac{1}{t^2+r^2+2st}-\frac{1}{9s^2+t^2+1+6st-2t-6s}-\frac{3}{9s^2+t^2+1+6st-2t-6s}$$
$$-\frac{1}{8st+6s^2+2t^2-2t-2s}-\frac{5}{8st+6s^2+2t^2-2t-2s}+\frac{1}{t^2+s^2+2st}$$
$$+\left.\frac{3}{6t^2+2s^2+8st+2t+2s}\right]+s^3t\left[\frac{3}{6st+4t^2+2s^2+4s+4t}+\frac{1}{2s^2+2t^2+4st}\right]$$
$$+s^4\left[\frac{1}{2s^2+2t^2+4st}\right]+s^3\left[\frac{-3}{2s^2+2t^2+4st}+\frac{5}{6s^2+2t^2+8s-2s-2t}\right.$$
$$+\frac{3}{2s^2+8st+2s+2t+6t^2}+\frac{1}{t^2+s^2+2st}+\frac{3}{9s^2+t^2+6st-2t-6s}$$
$$+\left.\frac{5}{8st+6s^2+2t^2-2t-2s}\right]+s\left[\frac{1}{6s^2+2t^2+8st-2s-2t}-\frac{1}{2s^2+8st+2s+2t+6t^2}\right.$$
$$-\frac{1}{9s^2+t^2+1+6st-2t-6s}+\frac{1}{8st+6s^2+2t^2-2t-2s}-\frac{1}{s^2+t^2+2st}$$
$$+\left.\frac{1}{6t^2+2s^2 8st+2s+2t}\right]+s\left[\frac{-1}{t^2+s^2+2st}\right],$$

$$\operatorname{Re} M_2(H)=st^3\left[\frac{1}{2t+9}+\frac{2}{3t+9}+\frac{2}{3t+s+7}+\frac{3}{4t+8}-\frac{2}{3t+8}+\frac{3}{4t+7}+\frac{2}{3t+s+6}\right.$$
$$+\frac{6}{5t+s+5}+\frac{18}{5t+7}-\frac{6}{5t+7}+\frac{12}{4t+s+6}+\frac{2}{t+2}+\frac{1}{2t+4}-\frac{4}{t+2}+\frac{6}{5t+2s+3}$$
$$+\frac{2}{2t+s+2}-\frac{1}{2t+s+2}+\frac{2}{4s+3t+1}-\frac{4}{3s+4t+2}+\frac{6}{5t+s+5}+\frac{2}{2t+s+2}+\frac{3}{2t+2}$$
$$-\left.\frac{3}{4t+4}+\frac{9}{6t+s+4}+\frac{6}{5t+2s+3}\right]+st^2\left[\frac{9}{2t+9}+\frac{14}{3t+9}+\frac{2}{3t+s+7}+\frac{12}{2t+s+7}\right.$$
$$+\frac{9}{2t+4}-\frac{12}{3t+8}+\frac{15}{4t+7}+\frac{10}{3t+s+6}+\frac{5}{t+9}+\frac{3}{t+7}+\frac{5}{t+10}+\frac{12}{5t+s+5}$$
$$+\frac{12}{5t+7}-\frac{18}{5t+7}+\frac{8}{4t+s+6}-\frac{2}{2t^2+6t+st+2s+4}+\frac{2}{t+2}-\frac{4}{t+2}+\frac{4}{t+9}$$

$$
\begin{aligned}
&+\frac{4}{t+4}+\frac{10}{2t+9}+\frac{8}{5t+2s+3}+\frac{1}{2t+s+2}-\frac{2}{2t+s+2}+\frac{7}{4s+4t}-\frac{4}{3s+t+2}\\
&-\frac{1}{s+t}-\frac{2}{5s+3t-1}-\frac{6}{3s+5t+1}-\frac{1}{s+t}-\frac{1}{12s+4t-4}-\frac{2}{5s+3t-1}+\frac{1}{s+t}\\
&+\frac{14}{5t+s+5}+\frac{4}{2t+s+2}+\frac{3}{4t+4}-\frac{3}{2t+2}+\frac{12}{6t+s+4}+\frac{6}{5t+2s+3}+\frac{6}{3s+5t+1}\Bigg]\\
&+st\Bigg[\frac{20}{2t+9}+\frac{20}{3t+9}+\frac{14}{2t+s+7}+\frac{15}{4t+8}-\frac{16}{3t+8}+\frac{12}{4t+7}+\frac{8}{3t+s+6}\\
&+\frac{20}{t+9}+\frac{12}{t+7}+\frac{25}{t+10}+\frac{15}{8}+\frac{6}{5t+s+5}-\frac{12}{5t+7}-\frac{3}{2t^2+6t+st+2s+4}\\
&+\frac{2}{t+2}+\frac{12}{7}+\frac{20}{9}+\frac{20}{t+9}+\frac{8}{t+4}+\frac{10}{2t+9}-\frac{1}{2t+s+2}-\frac{2}{4s+3t+1}+\frac{4}{5t+s+5}\\
&-\frac{1}{2s+2t}+\frac{2}{5s+3p-1}-\frac{2}{3s+5p+1}-\frac{3}{6s+2t-2}+\frac{2}{5s+3t-1}-\frac{2}{s+t}\\
&-\frac{3}{2t+4}+\frac{3}{6t+s+4}+\frac{2}{3s+5t+1}\Bigg]+s^2t^2\Bigg[\frac{1}{3t+s+7}+\frac{4}{t+3}+\frac{1}{3t+s+6}\\
&+\frac{3}{5t+s+5}+\frac{4}{4t+s+6}+\frac{6}{3s+5t+1}+\frac{1}{s+t}+\frac{1}{12s+4t+4}+\frac{2}{5s+3t-1}\\
&+\frac{2}{5t+s+5}+\frac{2}{2t+s+2}+\frac{6}{5t+2s+3}+\frac{3}{6t+s+4}\Bigg]+s^2t\Bigg[\frac{5}{3t+r+7}\\
&+\frac{16}{3t+9}+\frac{4}{3t+s+6}+\frac{3}{5t+s+5}-\frac{1}{4t^2+12t+2st+4s+8}+\frac{3}{6t+s+4}\\
&+\frac{3}{5t+2s+3}-\frac{2}{2t+s+2}+\frac{5}{4s+3t+1}+\frac{5}{2s+2t}-\frac{4}{3s+4t+2}+\frac{2}{s+t}\\
&-\frac{2}{5s+3t-1}-\frac{8}{5s+3t-1}+\frac{2}{3s+5t+1}-\frac{8}{3s+5t+1}-\frac{2}{s+t}-\frac{1}{6s+2t-2}\\
&+\frac{3}{6s+2t-2}-\frac{8}{5s+3t-1}-\frac{2}{5s+3t-1}+\frac{2}{s+t}+\frac{4}{5t+s+5}+\frac{4}{2t+s+2}\\
&+\frac{6}{5t+2s+3}+\frac{8}{3s+5t+1}\Bigg]+s^2t^3\Bigg[\frac{2}{3t+9}+\frac{2}{4t+s+6}+\frac{1}{2t+s+2}+\frac{4}{3s+4t+2}\Bigg]
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
&+st^4\Bigg[\frac{6}{5t+7}+\frac{4}{4t+s+6}+\frac{2}{t+2}+\frac{1}{4t+4}-\frac{1}{t+2}+\frac{1}{2t+s+2}\Bigg]+st^5\Bigg[\frac{1}{2t+4}\Bigg]\\
&+s^3t\Bigg[\frac{1}{5t+2s+3}-\frac{1}{8t+4s+8}+\frac{3}{4s+3t+1}+\frac{4}{3s+4t+2}-\frac{2}{3s+4t+2}-\frac{3}{s+t}\\
&+\frac{8}{5s+3t-1}+\frac{8}{3s+5t+1}+\frac{2}{s+t}+\frac{3}{6s+2t-2}+\frac{8}{5s+3t-1}\Bigg]+s^3t^2\Bigg[\frac{1}{8t+4r+8}\\
&+\frac{6}{3s+4t+2}+\frac{1}{2s+2t}\Bigg]+s^3\Bigg[\frac{3}{4s+4t}+\frac{1}{s+t}-\frac{2}{5s+3t-1}-\frac{6}{5s+3t-1}\\
&+\frac{2}{3s+5t+1}-\frac{2}{3s+5t+1}-\frac{1}{s+t}-\frac{3}{6s+2t-2}-\frac{9}{12s+4tt-4}-\frac{2}{5s+3t-1}\\
&-\frac{6}{5s+3t-1}+\frac{1}{s+t}+\frac{2}{3s+5t+1}\Bigg]+s^2\Bigg[\frac{1}{2s+2t}+\frac{2}{5s+3t-1}-\frac{2}{3s+5t+1}\\
&+\frac{1}{12s+4t-4}-\frac{3}{6s+2t-2}+\frac{2}{5s+3t-1}+\frac{2}{3s+5t+1}\Bigg]+s\Bigg[\frac{-1}{4s+4t}-\frac{1}{12s+4t-4}\Bigg]\\
&+s^4t\Bigg[\frac{2}{3s+4t+2}+\frac{1}{s+t}\Bigg]+s^5\Bigg[\frac{1}{2s+2t}\Bigg]+s^4\Bigg[\frac{-3}{2s+2t}+\frac{6}{5s+3t-1}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{2}{3s+5t+1}+\frac{1}{s+t}+\frac{9}{12s+4t-4}+\frac{6}{5s+3t-1}\Bigg]+t^2\left[\frac{-1}{s+t}\right],\\
\operatorname{Re}M_3(H) &= st\left[143t^3+406t^2+1000t+1745+141st^2+48s^2+304st+128s+50s^2t+7s^3\right]\\
&+s^2t\left[28t^3+136t^2+338t+308+6s^3+26s^2t+36st^2+16s^2+40st+8s\right]+st^2\\
&\cdot\left[65t^3+213t^2+203t+39+8s^2t+52s+14s^2+24st^2+72st+2s^3\right]+st^3\left[8t^3+48t^2\right.\\
&\left.+96t+64\right]+s\left[-59s^3+15t^3+9s^2t+67s^2\right]+s\left[29st^2+11t^2-17s-11t+66st\right]\\
&+s^3\left[8t^3+8s^2+24st^2+24s^2t\right]+s^2\left[5t^3+85s^3+105st^2\right.\\
&\left.+153s^2t-51s^2-3t^2-4st+9t+15s-2\right],\\
{}^{m}M_1(H) &= st\left[\frac{1}{2t+9}+\frac{1}{3t+9}+\frac{1}{3t+s+7}+\frac{1}{4t+8}+\frac{1}{3t+9}-\frac{1}{3t+8}+\frac{1}{4t+7}\right.\\
&+\frac{1}{3t+s+6}+\frac{1}{t+9}+\frac{1}{t+7}+\frac{1}{t+10}+\frac{1}{8}+\frac{1}{5t+s+5}-\frac{1}{5t+7}-\frac{1}{4t+s+6}\\
&+\frac{1}{2(4t+8)}+\frac{1}{7}+\frac{1}{9}+\frac{1}{t+9}+\frac{1}{2t+8}+\frac{1}{2t+9}+\frac{1}{5s+2t+3}-\frac{1}{4t+2s+4}\\
&+\frac{1}{4s+3t+1}-\frac{1}{3s+4t+2}+\frac{1}{5t+s+5}+\frac{1}{4t+2s+4}-\frac{1}{6t+6}+\frac{1}{6t+s+4}\\
&\left.+\frac{1}{5t+2s+3}\right]+s^2t\left[\frac{1}{3t+9}+\frac{1}{3t+8}+\frac{1}{5t+7}+\frac{1}{3s+4t+2}\right]+st^2\left[\frac{1}{4t+s+6}\right.\\
&\left.-\frac{1}{4t+8}+\frac{1}{2(4t+2s+4)}+\frac{1}{6t+6}\right]+st^3\left[\frac{1}{4s+4t}+\frac{1}{4s+4t}-\frac{1}{5s+3t-1}\right.\\
&\left.-\frac{1}{3s+5t+1}-\frac{1}{4s+4t}-\frac{1}{2(6s+2t+2)}-\frac{1}{5s+3t-1}+\frac{1}{4s+4t}+\frac{1}{3s+5t+1}\right]\\
&+s^2\left[\frac{-3}{2(4s+4t)}+\frac{1}{5s+3t+1}+\frac{1}{3s+5t+1}+\frac{1}{4s+4t}+\frac{1}{2(6s+2t-2)}+\frac{1}{5t+3t-1}\right]\\
&+s^3\left[\frac{1}{4s+4t}\right]-\frac{1}{4s+4t},\\
RM_2(H) &= st\left[\frac{-51}{2}t^2+53t+102+6st+11s+s^2\right]+s^2t\left[8t^2+20+2s^2+8st+s\right]+st^2\left[\frac{25}{2}t^2\right.\\
&\left.+17t-\frac{11}{2}+6st+4s+\frac{s^2}{2}\right]+\frac{st^3}{2}\left[4t^2+12t+9\right]+s\left[\frac{-19s^2}{2}-\frac{11t^2}{2}\right.\\
&\left.-5st+30s+21t-12\right]+s^3\left[2s^2+2t^2+4st-2s-2t+\frac{1}{2}\right]+s^2\left[\frac{33s^2}{2}+\frac{17t^2}{2}\right.\\
&\left.+17st-19s-13t+\frac{11}{2}\right],\\
\operatorname{RRR}(H) &= st\left[\sqrt{t^2+12+7t}+\sqrt{2t^2+11t+12}+\sqrt{3t^2+14t+8}+\sqrt{2t^2+9t+4s+st+4}\right.\\
&-\sqrt{2t^2+11t+12}-\sqrt{2t^2+7t+3s+st+3}+\sqrt{2t^2+9t+9}+\sqrt{3t^2+11t+6}\\
&+\sqrt{4t+12}+\sqrt{2t+6}+\sqrt{4t+16}+\sqrt{8}+\sqrt{6t^2+7t+2s+3st+2}-\sqrt{6t^2+13t+6}\\
&-\sqrt{4t^2+8t+3s+4st+1}+\frac{1\sqrt{4t^2+12t+(9/2)}}{2}+\sqrt{6}+\sqrt{12}
\end{aligned}
\tag{44}
$$

$$+\sqrt{3t+12}+\sqrt{6t+9}+\sqrt{8t+12}+\sqrt{6t^2+5st+s^2+3t+s},$$

$$\mathrm{GO}_1(H)=st\left[\frac{111t^2}{2}+184t+\frac{603}{2}+55s+23st+2s^2\right]+s^2t\left[6t^2+40t+55+6st+3s^2+4s\right]+st^2 \cdot\left[\frac{29t^2}{2}+51t+\frac{81}{2}+5s+2st\right]+s\left[\frac{5t^2}{2}-\frac{19s^2}{2}-4s-2t-st+\frac{3}{2}\right]+s^3\left[2s+2t+2s^2 +2t^2+4st\right]+s^2\left[\frac{17t^2}{2}+\frac{33s^2}{2}+9s+7t+23st-\frac{1}{2}\right]-\left[4t+4s+14t^2+4s^2+8st\right],$$

$$\mathrm{GO}_2(H)=st\left[143t^3+406t^2+1000t+1745+141st^2+48s^2+304st+128s+50s^2t+7s^3\right]+s^2t \cdot\left[28t^3+136t^2+338t+308+6s^3+26s^2t+36st^2+16s^2+40st+8s\right]+st^2\left[65t^3+213t^2 +203t+39+8s^2t+52s+14s^2+24st^2+72st+2s^3\right]+st^3\left[8t^3+48t^2+96t+64\right] +s\left[-59s^3+15t^3+9s^2t+67s^2+29st^2+11t^2-17s-11t+66st\right]+s^3\left[8t^3+8s^2 +24st^2+24s^2p\right]+s^2\left[5t^3+85s^3+105st^2+153s^2t-51-s^2-3t^2-4st+9t+15s-2\right], \quad (45)$$

$$\mathrm{HGO}_1(H)=st\left[\frac{783t^4}{2}+1851t^2+1114t^3+1148st^2+320s^2t+9596t+126s^2t^2+354st^3 +441s^2+10s^4+1855st+\frac{13611}{1}+654s+12s^3t+59s^3\right]+s^2t\left[24t^4 +734t^2+192t^3+4s^4+28s^3+16s^2t^2+56st^2+32s^2t+57s^2+112st +1738t+28s+1421\right]+st^2\left[\frac{185t^4}{2}+629t^3+1548t^2+16st^3+104st^2+20s^2t +4s^2t^2+25s^2+216st+140s+1622t+\frac{1339}{2}\right]+s^3\left[8t^4+8s^4+24s^2t^2+8s^2 +8t^2+48s^3t+16st+48st^3+16s^3+24t^3+48s^2t+56st^2\right]+s^2\left[36t^4+109s^4 +68t^3+40s^3+224s^3t+118st^3+267s^2t^2+110s^2t+191st^2-s^2+46t^2+138st +10t+4s+\frac{7}{2}\right]+s\left[-108s^4+t^4-212s^3t-10st^3-109s^2t^2-20s^3+4t^3+48s^2 +\left[3t^2+126s^2t+st^2+10t-6s+23st-\frac{3}{2}-\left[16t^4+14s^4+48s^2t^2+16s^2+16t^2 +96s^3t+72st+96st^3+32s^3+48t^3+96s^2t+112st^2\right],$$

$$\mathrm{HGO}_2(H)=st\left[\left(2t^3+27t^2+121t+180\right)^2+\left(6t^3+60t^2+180t+180\right)^2+\left(6t^3+5st^2+5s^2+50t^2 +34st+126t+98+49t+s^2t\right)^2+\left(12t^3+96t^2+204t+120\right)^2-\left(6t^3+60t^2+18t+180\right)^2- \cdot\left(6t^3+52t^2+144t+128\right)^2+\left(12t^3+81t^2+153t+84\right)^2+\left(5t^2+65t+180\right)^2+\left(20t^2+130t +180\right)^2+\left(5t^2+25t+250\right)^2+\left(6t^3-5st^2+s^2t+48+42t^2+28st+84t+4s^2+32s\right)^2 +\left(3t^2+33t+84\right)^2+14400+\left(30t^3+90t^2+3s^2+3s^2t+21st^2+30+42st+40t+21s\right)^2 -\left(30t^3+132t^2+185t+84\right)^2-\left(16t^3+2s^2t+32s+72t^2+4s^2+48+104t+12st^2+40st\right)^2 +\frac{1}{2}\left(16t^3+96t^2+192t+128\right)^2+7056+32400+\left(4t^250t+180\right)^2+\left(16t^2+96t+128\right)^2,$$

$$\mathrm{PGO}(H)=st\left[\frac{1}{\sqrt{2t^3+27t^2+121t+180}}+\frac{1}{\sqrt{6t^3+60t^2+186t+180}}\right.$$

$$+\frac{1}{\sqrt{12t^3+96t^2+204s+120}}-\frac{1}{\sqrt{6t^3+169t^2+186t+180}}+\frac{1}{\sqrt{5t^2+65t+180}}$$
$$-\frac{1}{\sqrt{6t^3+52t^2+144t+128}}+\frac{1}{\sqrt{6t^3+5st^2+5s^2+50t^2+34st+126t+98+49s+s^2t}}$$
$$+\frac{1}{\sqrt{12t^3+81t^2+153t+84}}+\frac{1}{\sqrt{6t^3-5st^2+s^2t+48+42t^2+28st+84t+4s^2+32s}}$$
$$+\frac{1}{\sqrt{5t^2+75t+250}}+\frac{1}{\sqrt{120}}+\frac{1}{\sqrt{30t^3+90t^2+3s^2+3s^2t+21st^2+30+42st+90t+121s}} \quad (46)$$

$$-\frac{1}{\sqrt{30t^3+132t^2+186t+84}}+\frac{1}{\sqrt{84}}$$
$$-\frac{1}{\sqrt{16t^3+2s^2t+32s+72s^2+4s^2+48+104t+112st^2+40st}}$$
$$+\frac{1}{2\sqrt{16t^3+96t^2+192t+128}}+\frac{1}{\sqrt{180}}+\frac{1}{\sqrt{4t^2+50t+180}}+\frac{1}{\sqrt{16t^2+96t+128}}$$
$$+\frac{1}{\sqrt{30t^3+2s^3+9s+34t+9s^2+58t^2+17s^2t+49st+42st^2}}+\frac{1}{\sqrt{3t^2+33t+84}}$$
$$-\frac{1}{2\sqrt{16t^3+2s^3+24st^2+16+48t^2+12s^2t+64st+48t+20s^2+40s}}$$
$$+\frac{1}{\sqrt{12s^3+6t^3+37s^2t-6t+23s^2+29st^2-3s+22st+2t^2-2}}$$
$$-\frac{1}{\sqrt{6s^3+16t^3+26s^2t+36st^2+16s^2+40st+24t^2+8s+8t}}$$
$$+\frac{1}{\sqrt{3t^3+44rt+16st^2+2s^2t+4s^2+20+100t^2+90t+24s}}$$
$$+\frac{1}{\sqrt{16t^3+24st^2+48t^2+64st+8s^2t+16s^2+32t+32s}}$$
$$-\frac{1}{2\sqrt{54t^3+162t^2+16t+54}}+\frac{1}{\sqrt{64t^3+3s^2+108t^2+42st+27st^2+12+66t+3s^2t+15s}}$$
$$+\frac{1}{\sqrt{30t^3+12s^2t+12s^2+18t+18s+48t^2+142st^2+60st}}\Bigg]+s^2t\Bigg[\frac{1}{\sqrt{6t^3+60t^2+186t+180}}$$
$$+\frac{1}{\sqrt{6t^3+52t^2+144t+128}}+\frac{1}{\sqrt{6t^3+16t^3+26s^2t+136st^2+16s^2+40st+24t^2+8s+8t}}\Bigg]$$
$$+st^2\Bigg[\frac{1}{\sqrt{30t^3+132t^2+186t+84}}-\frac{1}{\sqrt{16t^3+96t^2+192t+128}}$$
$$+\frac{1}{\sqrt{16t^3+2s^2t+32s+12t^2+4s^2+48+104s+112st^2+40st}}$$
$$+\frac{1}{2\sqrt{16t^3+2s^3+24st^2+16+48t^2+12s^2t+64st+48t+20s^2+40s}}$$
$$+\frac{1}{2\sqrt{54t^3+162t^2+162t+54}}\Bigg]+st^3\Bigg[\frac{1}{2\sqrt{16t^3+9s^2+182t+128}}\Bigg]$$
$$+s\Bigg[\frac{1}{\sqrt{12s^3+12t^3+52s^2t+8s^2-8t^2+52st^2-4s-4t}}+\frac{1}{\sqrt{16t^3+16s^3+48st^2+48s^2t}} \quad (47)$$

$$
\begin{aligned}
&-\frac{1}{\sqrt{30s^3+6t^3+2t+2s+34st^2+58s^2t-16s^2-24st-8t^2}}\\
&-\frac{1}{\sqrt{6s^3+30t^3+16t^2+34s^2t+2t+2s+8t^2+24st+52}}\\
&-\frac{1}{2\sqrt{54s^3+2t^3+18st^2+18s+54s^2t-36st-54s^2+6t-6t^2-2}}\\
&-\frac{1}{\sqrt{30s^3+6t^3+2t+2s+34st^2+58s^2t-16s^2-24st-8t^2}}\\
&+\frac{1}{\sqrt{16t^3+16s^3+48st^2+48s^2t}}+\frac{1}{\sqrt{58st^2+3t^3+6s^3+34st^2t+2s+2t+24st+8s^2+16t^2}}\Bigg]\\
&+s^3\left[\frac{1}{2\sqrt{16t^3+16s^3+48st^2+48s^2t}}\right]+s^2\Bigg[\frac{1}{\sqrt{30s^3+6t^3+2t+2s+34st^2+58s^2t-16s^2-24st-8t^2}}\\
&-\frac{3}{2\sqrt{16t^3+16s^3+48st^2+48s^2t}}+\frac{1}{\sqrt{6s^3+30t^3+16t^2+34s^2t+2t+2s+8t^2+24st+52st^2}}\\
&+\frac{1}{2\sqrt{54s^3+2t^3+18st^2+18s+54s^2t-36st-54s^2+6t-6t^2-2}}\\
&+\frac{1}{\sqrt{30s^3+6t^3+2t+2s+34st^2+58s^2t-16s^2-24st-8t^2}}\\
&+\frac{1}{\sqrt{16t^3+16s^3+48st^2+48s^2t}}\Bigg]-\left[\frac{1}{\sqrt{16t^3+16s^3+48st^2+48s^2t}}\right],
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
\mathrm{SGO}(H)={}&st\Bigg[\frac{1}{\sqrt{t^2+11t+29}}+\frac{1}{\sqrt{2t^2+17t+29}}+\frac{1}{\sqrt{2t^2+15t+6s+st+21}}+\frac{1}{\sqrt{3t^2+22t+23}}\\
&-\frac{1}{\sqrt{2t^2+17t+29}}-\frac{1}{\sqrt{2t^2+15t+24}}+\frac{1}{\sqrt{3t^2+20t+29}}+\frac{1}{\sqrt{6t+35}}+\frac{1}{\sqrt{6t+35}}\\
&+\frac{1}{\sqrt{23}}+\frac{1}{\sqrt{16t^2+17t+9s+3st+1}}-\frac{1}{\sqrt{6t^2+23t+29}}-\frac{1}{\sqrt{4t^2+20t+24}}\\
&+\frac{1}{\sqrt{19}}+\frac{1}{\sqrt{29}}+\frac{1}{\sqrt{5t+29}}+\frac{1}{\sqrt{10t+29}}+\frac{1}{\sqrt{6t^2+2s^2+13t+5s+6st+3}}\\
&+\frac{1}{\sqrt{3s^2+2t^2+3t+9s+1}}-\frac{1}{\sqrt{4t^2+2s^2+6st+8t+7s+2}}+\frac{1}{\sqrt{6t^2+2st+19s+15t+9}}\\
&+\frac{1}{\sqrt{4t^2+4st+10s+12t+4}}-\frac{1}{2\sqrt{9t^2+24t+15}}+\frac{1}{\sqrt{9t^2+18t+4s+3st+7}}\\
&+\frac{1}{\sqrt{6t^2+6st+8s+11t+13}}\Bigg]+s^2t\Bigg[\frac{1}{\sqrt{2t^2+17t29}}+\frac{1}{\sqrt{2t^2+15t+29}}\\
&+\frac{1}{\sqrt{4t^2+2s^2+7s+8t+2}}\Bigg]+st^2\Bigg[\frac{1}{\sqrt{6t^2+23t+9}}+\frac{1}{\sqrt{4t^2+2st+5s+16t+14}}\\
&+\frac{1}{2\sqrt{9t^2+24t+15}}\Bigg]+s\Bigg[\frac{1}{\sqrt{3s+3t^2+10st+2t+6s-1}}+\frac{1}{\sqrt{4s^2+4t^2+8st+4s+4t}}\\
&-\frac{1}{\sqrt{6s^2+2t^2+8st+3s+t-1}}-\frac{1}{\sqrt{4s^2+4t^2+8st+4t+4s}}-\frac{1}{2\sqrt{9s^2+t^2+6st-1}}\\
&-\frac{1}{\sqrt{6s^2+2t^2+8st+3s+t-1}}+\frac{1}{\sqrt{6t^2+8st+7t+5s+1}}\Bigg]+s^3\\
&\cdot\left[\frac{1}{2\sqrt{4s^2+4t^2+8st+4s+4t}}\right]+s^2\Bigg[\frac{1}{\sqrt{6s^2+2t^2+8st+t+3s-1}}-\\
&\cdot\frac{3}{2\sqrt{4s^2+4t^2+8st+4s+4t}}+\frac{1}{\sqrt{4s^2+4t^2+8st+4s+4t}}
\end{aligned}
$$

$$+\frac{1}{\sqrt{2s^2+6t^2+8st+7s+5t}}+\frac{1}{2\sqrt{9s^2+6st+t^2-1}}+\frac{1}{\sqrt{6s^2+2t^2+8st+3s+t-1}}\Bigg]$$
$$-\left[\frac{1}{\sqrt{4s^2+4t^2+8st+4s+4t}}\right],$$
$$F(H)=st\left[84t^2+18s^2+40st+263t+8s+476\right]+s^2t\left[18t^2+5s^2+12st+58t+4s+77\right]$$
$$+s^2t\left[26t^2+2s^2+8st+52t+8s+26\right]+st^2\left[4t^2+16+16t\right]+s^3\left[4t^2+4s^2+8st\right]$$
$$+s\left[-17s^2+7t^2-6st+10t+14s-1\right]+s^2\left[20t^2+36s^2+40st-16s+4\right]$$
$$-\left[8t^2+8s^2+16st\right]. \tag{49}$$

Proof. For line distribution shown in Table 2, use of formulas (5)–(9), (11)–(18), and (22) will give desired above expressions. □

Theorem 6. *Let H be the line graph of total of Kragujevac tree; $K\in Kg_{q=s(2t+1)+1,s}$, then M-polynomial and certain topological indices that can be deducted from M-polynomial are*

$$M(H;u,v)=stu^{t+5}v^{t+4}+stu^{2t+4}v^{t+5}+stu^{t+5}v^{2t+s+2}+stu^{t+5}v^{3t+3}+st(s-1)u^{t+5}v^{2t+4}$$
$$+st(s-1)u^{t+4}v^{2t+4}+stu^{t+4}v^{3t+3}+stu^{t+4}v^{2t+s+2}+stu^{t+4}v^{5}+stu^{t+4}v^{3}+stu^{t+5}$$
$$\cdot v^{5}+stu^{5}v^{3}+stu^{2t+s+2}v^{3t+3}+st(s-1)u^{3t+3}v^{2t+4}+st(s-1)u^{2t+s+2}v^{2t+4}$$
$$+\frac{st(t-1)(t-2)}{2}u^{2t+4}v^{2t+4}+stu^{4}v^{3}+stu^{4}v^{5}+stu^{4}v^{t+5}+stu^{2t+4}v^{4}+stu^{2t+4}v^{5}$$
$$+stu^{2t+s+2}v^{s+3t+1}+\frac{st(t-1)}{2}u^{2t+s+2}v^{2t+s+2}+stu^{3s+t-1}v^{2t+s+2}$$
$$+su^{3s+t-1}v^{s+3t-1}+st(s-1)u^{2t+s+2}v^{2(s+t)}+\frac{s(s-1)(s-2)}{2}u^{2(s+t)}v^{2(s+t)} \tag{50}$$
$$+s(s-1)u^{3s+t-1}v^{2(s+t)}+s(s-1)u^{s+3t+1}v^{2(s+t)}+\frac{s(s-1)}{2}u^{3s+t-1}v^{3s+t-1}$$
$$+s(s-1)u^{2t+2s}v^{3s+t-1}+(s-1)u^{2t+2s}v^{2t+2s}+stu^{2t+4}v^{s+3t+1}+stu^{2t+4}v^{2t+2s}$$
$$+\frac{st(t-1)}{2}u^{3t+3}v^{3t+3}+stu^{3t+3}v^{s+3t+1}+stu^{3t+3}v^{2t+2s}$$
$$+su^{s+3t+1}v^{2t+2s}+s(s-1)u^{2t+2s}v^{2(t+s)},$$

$$M_1(H)=st(t+5)+st(2t+4)+st(t+5)+st(t+5)+st(s-1)(t+5)+st(s-1)(t+4)$$
$$+st(t+4)+st(t+4)+st(t+5)+st(5)+st(2t+s+2)+st(s-1)(3t+3)+st(s-1)$$
$$\cdot(2t+s+2)+\frac{st(t-1)(t-2)}{2}(2t+4)+st(4)+st(4)+st(4)+st(2t+4)$$
$$+st(2t+4)+st(2t+s+2)+\frac{st(t-1)}{2}(2t+s+2)+st(3s+t-1)+s(3s+t-1)$$
$$+st(s-1)(2t+s+2)+\frac{s(s-1)(s-2)}{2}(2s+2t)+s(s-1)(3s+t-1)+s(s-1)$$
$$\cdot(s+3t+1)+s(s-1)(2t+2s)+\frac{s(s-1)}{2}(3s+t-1)+s(s-1)(2t+2s)+(s-1)$$

$$\begin{aligned}
&\cdot (2t+2s) + st(2t+4) + st(2t+4) + \frac{st(t-1)}{2}(3t+3) + st(3t+3) + st(3t+3) \\
&+ s(s+3t+1) + st(t+4) + st(t+5) + st(2t+s+2) + st(3t+3) + st(s-1)(2t+4) \\
&+ st(s-1)(2t+4) + st(3t+3) + st(2t+s+2) + st(t+4) + st(t+4) + st(5) + st(3) \\
&+ st(5) + st(3) + st(3t+3) + st(s-1)(2t+4) + st(s-1)(2t+4) + \frac{st(t-1)(t-2)}{2} \\
&\cdot (2t+4) + st(3) + st(5) + st(t+5) + st(4) + st(5) + st(s+3t+1) \\
&+ \frac{st(t-1)}{2}(2t+s+2) + st(2t+s+2) + s(s+3t-1) + st(s-1)(2t+2s) \\
&+ \frac{s(s-1)(s-2)}{2}(2t+2s) + s(s-1)(2t+2s) + s(s-1)(2t+2s) + s(s-1)(2t+2s) \\
&+ \frac{s(s-1)}{2}(3s+t-1) + s(s-1)(3s+t-1) + (s-1)(2t+2s) + st(s+3t+1) + st(2t+2s) \\
&+ \frac{st(t-1)}{2}(3t+3) + st(s+3t+1) + st(2t+2s) + s(2t+2s), \\
M_2(H) = {} & st[(t+4)(t+5) + (t+5)(2t+4) + (2t+s+2)(t+5) + (3t+3)(t+5) + (s-1)(2t+4)(t+5) \\
&+ ((s-1)(t+4)(2t+4) + (3t+3)(t+4) + (2t+s+2)(t+4) + 5(t+4) \\
&+ 3(t+4) + 5(t+5) + 15 + (3t+3)(2t+s+2) \\
&\cdot (s-1)(2t+4)(3t+3) + (s-1)(2t+4)(2t+s+2) + \frac{(t-1)(t-2)}{2}(2t+4)(2t+4) \\
&+ 12 + 20 + (t+5)(4) + (4)(2t+4) + (5)(2t+4) + (s+3t+1)(2t+s+2) + \frac{(t-1)}{2} \\
&\cdot (2t+s+2)^2 + (3s+t-1)(2t+s+2) + (s-1)(2t+2s)(2t+s+2) + (s+3t+1) \\
&\left.\cdot (2t+4) + (2t+2s)(2t+4) + \frac{(t-1)}{2}(3t+3)^2 + (s+3t+1)(3t+3) + (2t+2s)(3t+3)\right] \\
&+ s[(s+3t-1)(3s+t-1) + (s-1)(2t+2s)(3s+t-1) + (s-1)(2t+2s)(s+3t+1)
\end{aligned} \tag{51}$$

$$\begin{aligned}
&+ (s-1)(2t+2s)(2s+2t) + \frac{(s-1)}{2}(3s+t-1)(3s+t-1) + (s-1)(2t+2s)(3s+t-1) \\
&\left.+ \frac{(s-1)(s-2)}{2}(2s+2t)(2t+2s) + (s+3t+1)(2s+2t)\right] + (s-1)(2t+2s)(2s+2t), \\
{}^{m}M_2(H) = {} & st\left[\frac{1}{(t+4)(t+5)} + \frac{1}{(t+5)(2t+4)} + \frac{1}{(2t+s+2)(t+5)} + \frac{1}{(3t+3)(t+5)} + (s-1)\right. \\
&\cdot \frac{1}{(2t+4)(t+5)} + (s-1)\frac{1}{(2t+4)(t+4)} + \frac{1}{(3t+3)(t+4)} + \frac{1}{(2t+s+2)(t+4)} + \frac{1}{(5)(t+4)} \\
&+ \frac{1}{(3)(t+4)} + \frac{1}{(5)(t+5)} + \frac{1}{(3)(5)} + \frac{1}{(3t+3)(2t+s+2)} + (s-1)\frac{1}{(2t+4)(3t+3)} + (s-1) \\
&\cdot \frac{1}{(2t+4)(2t+s+2)} + \frac{(t-1)(t-2)}{2}\frac{1}{(2t+4)(2t+4)} + \frac{1}{12} + \frac{1}{20} + \frac{1}{4(t+5)} + \frac{1}{4(2t+4)} \\
&+ \frac{1}{5(2t+4)} + \frac{1}{(s+3t+1)(2t+s+2)} + \frac{(t-1)}{2}\frac{1}{(2t+s+2)(2t+s+2)} + \frac{1}{(2s+2t)(2t+4)} \\
&+ \frac{1}{(2t+s+2)(3s+t-1)} + (s-1)\frac{1}{(2s+2t)(2t+s+2)} + \frac{1}{(s+3t+1)(2t+4)} + \frac{(t-1)}{2}
\end{aligned} \tag{52}$$

$$
\begin{aligned}
&\cdot \frac{1}{(3t+3)(3t+3)} + \frac{1}{(s+3t+1)(3t+3)} + \frac{1}{(2s+2t)(3t+3)}\Bigg] + s\Bigg[(s-1)\frac{1}{(2s+2t)(3s+t-1)} \\
&+ (s-1)\frac{1}{(2s+2t)(s+3t+1)} + (s-1)\frac{1}{(2s+2t)(2s+2t)} + \frac{(s-1)}{2}\frac{1}{(3s+t-1)(3s+t-1)} \\
&+ (s-1)\frac{1}{(3s+t-1)(2t+2s)} + \frac{1}{(s+3t-1)(s+3t-1)} + \frac{(s-1)(s-2)}{2} \\
&\cdot \frac{1}{(2s+2t)(2s+2t)} + \frac{1}{(2s+2t)(s+3t+1)}\Bigg] + (s-1)\frac{1}{(2s+2t)(2t+2s)}, \\
R_\alpha(H) = st\Bigg[&\frac{1}{(t+4)^\alpha(t+5)^\alpha} + \frac{1}{(t+5)^\alpha(2t+4)^\alpha} + \frac{1}{(2t+s+2)^\alpha(t+5)^\alpha} \\
&+ \frac{1}{(3t+3)^\alpha(t+5)^\alpha} + (s-1)\frac{1}{(2t+4)^\alpha(t+5)^\alpha} + (s-1)\frac{1}{(2t+4)^\alpha(t+4)^\alpha} \\
&+ \frac{1}{(3t+3)^\alpha(t+4)^\alpha} + \frac{1}{(2t+s+2)^\alpha(t+4)^\alpha} + \frac{1}{(5)^\alpha(t+4)^\alpha} \\
&+ \frac{1}{(3)^\alpha(t+4)^\alpha} + \frac{1}{(5)^\alpha(t+5)^\alpha} + \frac{1}{(3)^\alpha(5)^\alpha} \\
&+ \frac{1}{(3t+3)^\alpha(2t+s+2)^\alpha} + (s-1)\frac{1}{(2t+4)^\alpha(3t+3)^\alpha} + (s-1)\frac{1}{(2t+4)^\alpha(2t+s+2)^\alpha} \\
&+ \frac{(t-1)(t-2)}{2}\frac{1}{(2t+4)^\alpha(2t+4)^\alpha} + \frac{1}{(3)^\alpha(4)^\alpha} + \frac{1}{(5)^\alpha(4)^\alpha} + \frac{1}{(t+5)^\alpha(4)^\alpha} \\
&+ \frac{1}{(4)^\alpha(2t+4)^\alpha} + \frac{1}{(5)^\alpha(2t+4)^\alpha} + \frac{1}{(s+3t+1)^\alpha(2t+s+2)^\alpha} \\
&+ \frac{(t-1)}{2}\frac{1}{(2t+s+2)^\alpha(2t+s+2)^\alpha} + \frac{1}{(2t+s+2)^\alpha(3s+t-1)^\alpha} \\
&+ (s-1)\frac{1}{(2s+2t)^\alpha(2t+s+2)^\alpha} + \frac{1}{(s+3t+1)^\alpha(2t+4)^\alpha} + \frac{1}{(2s+2t)^\alpha(2t+4)^\alpha} \\
&+ \frac{(t-1)}{2}\frac{1}{(3t+3)^\alpha(3t+3)^\alpha} + \frac{1}{(s+3t+1)^\alpha(3t+3)^\alpha} + \frac{1}{(2s+2t)^\alpha(3t+3)^\alpha}\Bigg] \\
&+ s\Bigg[\frac{1}{(2s+2t)^\alpha(s+3t+1)^\alpha} + (s-1)\frac{1}{(2s+2t)^\alpha(3s+t-1)^\alpha} + (s-1) \\
&\cdot \frac{1}{(2s+2t)^\alpha(s+3t+1)^\alpha} + (s-1)\frac{1}{(2s+2t)^\alpha(2s+2t)^\alpha} + \frac{(s-1)}{2} \\
&\cdot \frac{1}{(3s+t-1)^\alpha(3s+t-1)^\alpha} + (s-1)\frac{1}{(3s+t-1)^\alpha(2s+2t)^\alpha} \\
&+ \frac{(s-1)(s-2)}{2}\frac{1}{(2s+2t)^\alpha(2s+2t)^\alpha} + \frac{1}{(s+3t-1)^\alpha(s+3t-1)^\alpha}\Bigg]
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
&+ (s-1)\frac{1}{(2s+2t)^\alpha(2s+2t)^\alpha}, \\
\mathrm{RR}_\alpha(H) = st\big[&(t+4)^\alpha(t+5)^\alpha + (t+5)(2t+4)^\alpha + (2t+s+2)(t+5)^\alpha + (3t+3)^\alpha(t+5)^\alpha \\
&+ (s-1)(2t+4)^\alpha(t+5)^\alpha + (s-1)(2t+4)^\alpha(t+4)^\alpha + (3t+3)^\alpha(t+4)^\alpha + (2t+s+2)^\alpha \\
&\cdot (2t+s+2)^\alpha + (5)^\alpha(t+4)^\alpha + (3)^\alpha(t+4)^\alpha + (5)^\alpha(t+5)^\alpha + (3)^\alpha(5)^\alpha + (3t+3)^\alpha \\
&\cdot (2t+s+2)^\alpha + (s-1)(2t+4)^\alpha(3t+3)^\alpha + (s-1)(2t+4)^\alpha(2t+s+2)^\alpha \\
&+ \frac{(t-1)(t-2)}{2}(2t+4)^\alpha(2t+4)^\alpha + (3)^\alpha(4)^\alpha + (5)^\alpha(4)^\alpha + (t+5)^\alpha(4)^\alpha + (4)^\alpha(2t+4)^\alpha \\
&+ (5)^\alpha(2t+4)^\alpha + (s+3t+1)^\alpha(2t+s+2)^\alpha + \frac{(t-1)}{2}(2p+s+2)^\alpha(2t+s+2)^\alpha \\
&+ (2t+s+2)^\alpha(3s+t-1)^\alpha + (s-1)(2t+2s)^\alpha(2t+s+2)^\alpha + (s+3t+1)^\alpha(2t+4)^\alpha
\end{aligned}
$$

$$+(2t+2s)^{\alpha}(2t+4)^{\alpha}+\frac{(t-1)}{2}(3t+3)^{\alpha}(3t+3)^{\alpha}+(s+3t+1)^{\alpha}(3t+3)^{\alpha}$$
$$+(2t+2s)^{\alpha}(3t+3)^{\alpha}\Big]+s\Big[(s+3t-1)^{\alpha}(3s+t-1)^{\alpha}+\frac{(s-1)(s-2)}{2}$$
$$\cdot(2t+2s)^{\alpha}(2s+2t)^{\alpha}+(s-1)(2t+2s)^{\alpha}(3s+t-1)^{\alpha}+(s-1)(2t+2s)^{\alpha}$$
$$\cdot(s+3t+1)^{\alpha}+(s-1)(2t+2s)^{\alpha}(2s+2t)^{\alpha}+\frac{(s-1)}{2}(3s+t-1)^{\alpha}(3s+t-1)^{\alpha}$$
$$+(s-1)(3s+t-1)^{\alpha}(2s+2t)^{\alpha}+(2t+2s)^{\alpha}(s+3t+1)^{\alpha}\Big]+(s-1)(2t+2s)^{\alpha}(2s+2t)^{\alpha},$$

$$\mathrm{SDD}(H)=st\Big[\frac{(t+5)}{t+4}+\frac{(2t+4)}{t+5}+\frac{(t+5)}{2t+s+2}+\frac{(t+5)}{3t+3}+(s-1)\frac{(t+5)}{2t+4}+(s-1)\frac{(t+4)}{2t+4}+\frac{(t+4)}{3t+3}$$
$$+\frac{(t+4)}{2t+s+2}+\frac{(t+4)}{5}+\frac{(t+4)}{3}+\frac{(t+5)}{5}+\frac{(5)}{3}+\frac{(2t+s+2)}{3t+3}+(s-1)\frac{(3t+3)}{2t+4}+(s-1)$$
$$\cdot\frac{(2t+s+2)}{2t+4}+\frac{(t-1)(t-2)}{2}+\frac{(4)}{3}+\frac{4}{5}+\frac{4}{t+5}+\frac{(2t+4)}{4}+\frac{(2t+4)}{5}+\frac{(2t+s+2)}{s+3t+1}$$
$$+\frac{(t-1)}{2}\frac{(2t+s+2)}{2t+s+2}+\frac{(3s+t-1)}{2t+s+2}+(s-1)\frac{(2t+s+2)}{2s+2t}+\frac{(2t+4)}{s+3t+1}+\frac{(2t+4)}{2s+2t}$$
$$+\frac{(t-1)}{2}+\frac{(3t+3)}{s+3t+1}+\frac{(3t+3)}{2s+2t}+(t+4)\frac{1}{t+5}+(t+5)\frac{1}{2t+4}+(2t+s+2)\frac{1}{t+5}$$
$$+(3t+3)\frac{1}{t+5}+(s-1)(2t+4)\frac{1}{t+5}+(s+3t+1)\frac{1}{2t+4}$$
$$+(s-1)(2t+4)\frac{1}{t+4}+(3t+3)\frac{1}{t+4}+(2t+s+2)\frac{1}{t+4}+(5)\frac{1}{t+4}+(3)\frac{1}{t+4}+(5)\frac{1}{t+5}$$
$$+(3)\frac{1}{5}+(3t+3)\frac{1}{2t+s+2}+(s-1)(2t+4)\frac{1}{3t+3}+(s-1)(2t+4)\frac{1}{2t+s+2}+\frac{(t-1)(t-2)}{2}$$
$$\cdot(2t+4)\frac{1}{2t+4}+(3)\frac{1}{4}+(5)\frac{1}{4}+(t+5)\frac{1}{4}+(4)\frac{1}{2t+4}+(5)\frac{1}{2t+4}+(s+3t+1)\frac{1}{2t+s+2}$$
$$+\frac{(t-1)}{2}(2t+s+2)\frac{1}{2t+s+2}+(2t+s+2)\frac{1}{3s+t-1}+(s-1)(2t+2s)\frac{1}{2t+s+2}$$
$$\cdot\frac{1}{2t+4}+(2t+2s)\frac{1}{2t+4}+\frac{(t-1)}{2}(3t+3)\frac{1}{3t+3}+(s+3t+1)\frac{1}{3t+3}+(2t+2s)\frac{1}{3t+3}\Big]$$
$$+s\Big[\frac{(3s+t-1)}{s+3t-1}+\frac{(s-1)(s-2)}{2}\frac{(2s+2t)}{2s+2t}+(s-1)\frac{(3s+t-1)}{2s+2t}+(s-1)\frac{(s+3t+1)}{2s+2t}$$
$$+\frac{(s-1)}{2}+(s-1)\frac{(2s+2t)}{3s+t-1}+\frac{(s+3t+1)}{2s+2t}+1+\frac{(s-1)(s-2)}{2}+(s-1)(2t+2s)$$
$$\cdot\frac{1}{3s+t-1}+(s-1)(2t+2s)\frac{1}{s+3t+1}+(s-1)(2t+2s)\frac{1}{2s+2t}+\frac{(s-1)}{2}+(s-1)$$
$$\cdot(3s+t-1)\frac{1}{2s+2t}+(2t+2s)\frac{1}{s+3t+1}\Big]+(s-1)[2], \tag{54}$$

$$H(H)=2st\Big[\frac{1}{2t+9}+\frac{1}{3t+9}+\frac{1}{3t+s+7}+\frac{1}{4t+8}+(s-1)\frac{1}{3t+9}+(s-1)\frac{1}{3t+8}+\frac{1}{4t+7}$$
$$+\frac{1}{3t+s+6}+\frac{1}{t+9}+\frac{1}{t+7}+\frac{1}{t+10}+\frac{1}{8}+\frac{1}{5t+s+5}+(s-1)\frac{1}{5t+7}+(s-1)\frac{1}{4t+s+6}$$
$$+\frac{(t-1)(t-2)}{2}\frac{1}{4t+8}+\frac{1}{7}+\frac{1}{9}+\frac{1}{t+9}+\frac{1}{2t+8}+\frac{1}{2t+9}+\frac{1}{5t+2s+3}+\frac{(t-1)}{2}\frac{1}{4t+2s+4}$$

$$
\begin{aligned}
&+\frac{1}{4s+3t+1}+(s-1)\frac{1}{4t+3s+2}+\frac{1}{s+5t+5}+\frac{1}{2s+4t+4}+\frac{(t-1)}{2}\frac{1}{6t+6}+\frac{1}{s+6t+4}\\
&+\frac{1}{5t+2s+3}\Bigg]+2s\Bigg[\frac{1}{4s+4t-2}+\frac{(s-1)(s-2)}{2}\frac{1}{4s+4t}+(s-1)\frac{1}{5s+3t-1}+(s-1)\frac{1}{3s+3t+1}\\
&+(s-1)\frac{1}{4s+4t}+\frac{(s-1)}{2}\frac{1}{6s+2t-2}+(s-1)\frac{1}{3t+5s-1}+s\frac{1}{3s+5t+1}\Bigg]+2(s-1)\frac{1}{4s+4t},\\
\mathrm{ISI}(H)={}&st\Bigg[\frac{(t+4)(t+5)}{2t+9}+\frac{(t+5)(2t+4)}{3t+9}+\frac{(2t+s+2)(2t+s+2)}{3t+s+7}+\frac{(3t+3)(t+5)}{4t+8}\\
&+\frac{(s-1)(2t+4)(t+5)}{3t+9}+\frac{(s-1)(2t+4)(t+4)}{3t+8}+\frac{(3t+3)(t+4)}{4t+7}+\frac{(2t+s+2)(t+4)}{3t+s+6}\\
&+\frac{5(t+4)}{t+9}+\frac{3(t+4)}{t+7}+\frac{(5)(t+5)}{t+10}+\frac{15}{8}+\frac{12}{7}+\frac{(3t+3)(2t+s+2)}{5t+s+5}\\
&+\frac{(s-1)(2t+4)(3t+3)}{5t+7}+\frac{(s-1)(2t+4)(2t+s+2)}{4t+s+6}+\frac{(t-1)(t-2)}{2}\frac{(2t+4)(2t+4)}{4t+8}\\
&+\frac{20}{9}+\frac{(t+5)(4)}{t+9}+\frac{(4)(2t+4)}{2t+8}+\frac{(5)(2t+4)}{2t+9}+\frac{(s+3t+1)(2t+s+2)}{5t+2s+3}\\
&+\frac{(t-1)}{2}\frac{(2t+s+2)(2t+s+2)}{4t+2s+4}+\frac{(2t+s+2)(3s+t-1)}{4s+3t+1}+\frac{(s-1)(2t+2s)(2t+s+2)}{3t+3s+2}\\
&+\frac{(s+3t+1)(2t+4)}{s+5t+5}+\frac{(2t+2s)(2t+4)}{4t+2s+4}+\frac{(t-1)}{2}\frac{(3t+3)(3t+3)}{6t+6}
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
&+\frac{(s+3t+1)(3t+3)}{s+6t+4}+\frac{(2t+2s)(3t+3)}{5t+2s+3}\Bigg]+s\Bigg[\frac{(2t+2s)(s+3t+1)}{3s+5t+1}\\
&+\frac{(s-1)(s-2)(2s+2t)(2t+2s)}{2(4s+4t)}+\frac{(s-1)(2t+2s)(3s+t-1)}{5s+3t-1}+\frac{(s-1)(2t+2s)(s+3t+1)}{3s+5t+1}\\
&+\frac{(s-1)(2t+2s)(2s+2t)}{4s+4t}+\frac{(s-1)(3s+t-1)(3s+t-1)}{2(6s+2t-2)}+\frac{(s-1)(3s+t-1)(2s+2t)}{3t+5s+1}\\
&+\frac{(s+3t-1)(3s+t-1)}{6s+4t-2}\Bigg]+\frac{(s-1)(2t+2s)(2s+2t)}{4s+4t},\\
A(H)={}&st\Bigg[\frac{(t+4)^3(t+5)^3}{(2t+7)^3}+\frac{(t+5)^3(2t+4)^3}{(3t+7)^3}+\frac{(2t+s+2)^3(t+5)^3}{(3t+s+5)^3}+\frac{(3t+3)^3(t+5)^3}{(4t+6)^3}+\frac{(3)^3(4)^3}{(5)^3}\\
&+\frac{(s-1)(2t+4)^3(t+5)^3}{(3t+7)^3}+\frac{(s-1)(2t+4)^3(t+4)^3}{(3t+6)^3}+\frac{(3t+3)^3(t+4)^3}{(4t+5)^3}+\frac{(2t+s+2)^3(2t+s+2)^3}{(3t+s+4)^3}\\
&+\frac{(5)^3(t+4)^3}{(t+7)^3}+\frac{(3)^3(t+4)^3}{(t+5)^3}+\frac{(5)^3(t+5)^3}{(t+8)^3}+\frac{(3)^3(5)^3}{(6)^3}+\frac{(3t+3)^3(2t+s+2)^3}{(5t+s+3)^3}+\frac{(5)^3(2t+4)^3}{(2t+7)^3}\\
&+\frac{(s-1)(2t+4)^3(3t+3)^3}{(5t+5)^3}+\frac{(s-1)(2t+4)^3(2t+s+2)^3}{(4t+s+4)^3}+\frac{(2t+4)^3(2t+4)^3(t-1)(t-2)}{8(4t+6)^3}\\
&+\frac{(5)^3(4)^3}{(7)^3}+\frac{(t+5)^3(4)^3}{(t+7)^3}+\frac{(4)^3(2t+4)^3}{(2t+6)^3}+\frac{(2t+s+2)^3(2t+s+2)^3(t-1)}{8(4t+2s+2)^3}+\frac{(2t+2s)^3(2t+4)^3}{(2s+4t+2)^3}\\
&+\frac{(s+3t+1)^3(2t+s+2)^3}{(5t+2s+1)^3}+\frac{(2t+s+2)^3(3s+t-1)^3}{(4s+3t-1)^3}+\frac{(s-1)(2t+2s)^3(2t+s+2)^3}{(4t+3s)^3}\\
&+\frac{(s+3t+1)^3(2t+4)^3}{(s+5t+3)^3}+\frac{(3t+3)^3(3t+3)^3(t-1)}{8(6t+4)^3}+\frac{(s+3t+1)^3(3t+3)^3}{(s+6t+2)^3}+\frac{(2s+2s)^3(3t+3)^3}{(2s+5t+1)^3}\Bigg]
\end{aligned}
$$

$$+ s\left[\frac{(2t+2s)^3(s+3t+1)^3}{(3s+5t-1)^3} + \frac{(2t+2s)^3(2s+2t)^3(s-1)(s-2)}{8(4s+4t-2)^3} + \frac{(s-1)(2t+2s)^3(3s+t-1)^3}{(5s+3t-5)^3} + \frac{(s-1)(2t+2s)^3(s+3t+1)^3}{(3s+5t-1)^3} + \frac{(3s+t-1)^3(3s+t-1)^3(s-1)}{8(6s+2t-4)^3} + \frac{(s-1)(3s+t-1)^3(2s+2t)^3}{(3t+5s-3)^3} + \frac{(s-1)(2t+2s)^3(2s+2t)^3}{(4t+4s-2)^3} + \frac{(s+3t-1)^3(3s+t-1)^3}{(4s+4t-2)^3}\right] + \frac{(s-1)(2t+2s)^3(2s+2t)^3}{(4t+4s-2)^3}. \tag{56}$$

Proof. For line distribution shown in Table 2, use of formulas given in (19)–(28) will give the above desired expressions. □

4. Conclusion

In this study, we computed different topological indices and polynomials for total graph and the line graph of total graph of the considered tree, i.e., Kragujevac tree. It would be fascinating in the future to discover the same outcomes for those chemical invariants that have not yet been evaluated.

Data Availability

The data used to support the findings of this study are cited at relevant places within the text as references.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

This work was equally contributed by all writers.

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