Reverse Engineering on Planar Contours: An Algorithmic Review

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ABSTRACT

Reverse Engineering on image contours plays a significant role in data reduction, transformation and deformation of shapes without the loss of reasonable quality. The mathematical basis of captured contours are used in various applied areas such as facial modeling, animations, real time image processing applications, automatic sketching, shape matching, capturing hand drawn images and designing civil engineering, bio-medical and mechanical engineering objects etc. The authors do hope that this article can effectively provides algorithmic review for approximating outlines of planar images. Furthermore, it offers a sound analysis of pros and cons of specified techniques. It also delivers enhancements and optimizations via exploring the weaknesses of existing algorithms and attacking them through hybrid solutions.

CCS CONCEPTS

• **Data Visualization Contour Extraction**;

KEYWORDS

GA,Boundary Extraction, Contour Regeneration, Image Processing, NURBS, Curve Fitting, Spline Function, LSM

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1 INTRODUCTION

Planar images are used to store visual information of object contours. Human visual capabilities prefer data to be in image form but for any machine effectively extracted information and meaningful translation of this data is useful. An object is represented by its boundary or it's interior. Preservation of complete shape of an object is done by capturing outline of any object. Different algorithms have been proposed for capturingobject outlines with the aim that these algorithms work well for different computer vision applications with less heavy computational time.

Various capturing techniques have been proposed by different researchers. Each one using variousspline models like Bezier splines, Rational Cubic interpolation, B-splines, NURBS and also few other techniques (including use of control parameters, genetic algorithms and wavelets). The usual basis that are used in regenerated contour fitting includes identification of significant points, least square method, interpolation of edge points and feature boundary extraction techniques [1]. Each technique used for capturing outline has its own advantages and drawbacks. At the base of each method preprocessing is important in removing noise, thinning the outline after binary conversion of image and boundary detection [3], corner detection etc. Either the algorithms use already existing methods for this purpose, or they propose their own techniques. The curves which are robust gives more local control and do not cost heavy computation to the designer, are attractive to be used in such methods [2]. Bezier cubic curves fitted as splines can effectively define splines with inflections and spaces.The process includes obtaining a digitized image, detecting corner points of the planar object(s) involved after preprocessing of the input image and then fitting desirable spline in the best possible way to achieve optimal accuracy in less cost of computation.

2 ALGORITHMIC REVIEW OF DIFFERENT APPROACHES

2.1 Genetic Approach

In evolutionary algorithms we search for our optimal solution in the state space given with the help of mentioned parameters: (i) Each generation is produced better on average by introducing evolutionary pressures of selection and reproduction. (ii) Competition between solutions is developed to optimize the final solution [6]. Genetic Approach(based on B-Splines) can be summarized as follows:

Step1: Boundary Detection: Approach[14] is used. A .bmp file is input to this approach and its outputs includes (i) Segment Pieces (ii) Contour points and weights of those contour points.

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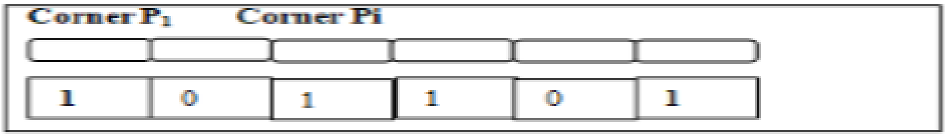

Figure 1: Boundary translated into Chromosome bit strings

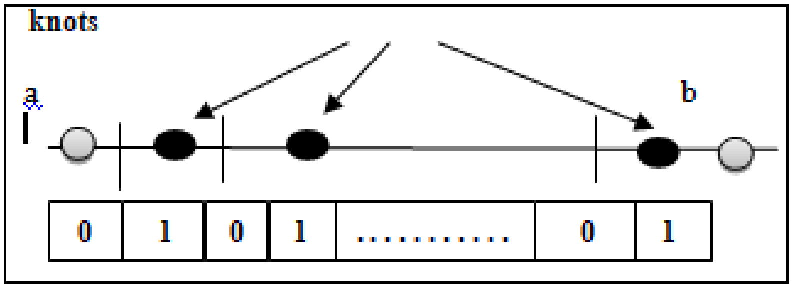

Figure 2: individual Chromosome

Step2: Corner Detection: Corner points (here called significant points) are detected using [15], where significant points represent high curvature points of an object.
Step3: Set the three control parameters to these values: **(i)** Probability (of Cross Over, C) = 0.7 for double point cross-over **(ii)** Probability (of Mutation, M) = 0.001, the range be 0 <= R (Knot Ratio) < 0.5, as uniform B-Splines.
Step4: Create initial population using random numbers. Initial population segments of curve loop(s)) consists of K individuals (segments). Individuals (segments) are of gene length L. Genes (data points) in chromosomes are randomly set to either 0 or 1. One value is assigned to significant corner point's genes.
Step5: For each contour piece, assign bit value equals 1 to each important point.
Step6: Given m+1 significant points, equation B-Spline Y(T) given by:

$$Y(T) = \sum_{i=0}^{m} M_{j,r}(T)\, c_i \qquad Mi, \mathrm{p}\,(\mathrm{T}) \text{ can be calculated as follows :}$$

$$M_{j,0}(v) = \begin{cases} 1 & if\ v_j \le v < v_{j+1} \\ 0 & othewise \end{cases} M_{j.r}$$
$$(v) = \frac{v - v_j}{v_{j+r} - v_j} M_{j,q-1}(v) + \frac{v_{j+r+1} - v}{v_{j+r+1} - v_{j+1}} M_{j+1,r-1}(v)$$

V = $\{v_0, v_1, v_2 .... v_m\}$, V is the knot vector.If (gene = 1), include the respective data point (on the curve) in the knot vector.

Step7: For each contour in the population, (i) Evaluate Best Fit value (ii) Fitness Measure is calculated via Akaike's Information Criterion (AIC) model. (iii) AIC is helpful to pick best model from existing potential models. AIC is given

$$AIC = M\log_e Q_1 + 2(2n + m)$$

The smaller the calculated value the better the curve fits.where, M = Number of Raw Points, n = Number of inner points, m = Curve order. Q_1 is calculated by the formula:

$Q_1 = \sum_{i=1}^{n} w_j\{\{Sx_j(t) - x_j(t)\}^2 + \{Sy_j(t) - y(t)\}^2\}$Y(T) = Estimated Curve F.

Step8: If Total number of generations exhausted then Terminate Computational Process & Report Best chromosome (segment) of last generation else follow Step 9.**Step9:** Pick estimates with highest fitness.

Step 10: Perform crossover and produce next population [17].**10.1.** Two separate segments are selected form generated population using above specified selection process.**10.2**. Bit sequences with two points are randomly selected.**10.3**. Two bit sequences are swapped e.g. If first sequence is 111000 and second sequence is 000001 and exchange point is 3 and 5 then sequence 1'110000 is and sequence 2' is 001001.**10.4.** The new strings are selected to be a part of next population.
Step11: Mutation process. **11.1.** There is a low probability that some of the units of our bit sequences are flipped. This will help in the maintenance of diversity.**11.2.** Goto step5

2.2 Pros and Cons of Genetic Approach

Pros include (i) Local control (for local optima) is obtained in case of B-Splines as polynomial coefficients of points (Mj,r(T)). Only a small sample of the segment is influenced by moving control points. (ii) Non-dependency on the degree of spline.Con is that knots of each segment is calculated and stored side by side so its comparatively space inefficient [7].

2.3 Recommended Transformative Directions

These directions can be seen as future possibilities: (i) NURBS can be used incorporating the optimization of weights using real number (fractional) genes. (ii) In this approach the load of computation is concentrated in step 5th, 6th & 7th. We can apply parallel computing here to save computational time. (ii)Non-Uniform Rational B-Splines can incooperate mesh enhancement [9].

2.4 Evolutionary Intelligence Approach Using NURBS

To get an optimized curve, knots should be selected in a way that avoid possible local optima. Furthermore an iterative solution can be effectively found. The algorithm can be summarized as follows: Step1:Finding Image Contour(s):Take a grey scale image and extract contour points of its object(s) via Quddus[18].
Step2: Corner Points Detection[25]:Corner points are detected using triangular method proposed by Chetrikov, explained in random knot insertion algorithm in this draft. This approach assesses a parameter of corner weight in two iteration.
Step3:Non-Uniform Rational B-Splinesof order q is defined as:

$$C(v) = \frac{1}{\sum_{i=0}^{n} N_{i.q}(v)\, w_i} \sum_{i=0}^{n} N_{i.q}(v)\, w_i Q_i$$

$$N_{i,0}(v) = \begin{cases} 0 & if\ v_i \le v < v_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,q}(v) = \frac{v - v_i}{v_{i+q} - v_i} N_{i,q-1}(v) + \frac{v_{i+q+1} - v}{v_{i+q+1} - v_{i+1}} N_{i+1,q-1}(v)$$

For each segment**3.1.**Use Least Square Method to identify signify cant points. **3.2.** Identify Vector. **3.3.**Calculate weights. **3.4.**Estimate Contour.**Step4**: StartGenerationQ, Bias factor = -0.5.**Step5**: For iterator value equals 1 initially perform at least base iterations.
5.1. Assessment includes for each segment j in Q(generation)calculate goodness factor (yi) from,

$$y_i = \frac{2\left(2l_j + k\right)}{AIC_j}$$

$$AIC = M\left|\ln Q\right| + 2\left(2l + k\right)$$

l = The number of points in vector, k = degree of spline, M = Frequence of data points in each curve part.
5.2. Selection For each segment i in P, If Random [0,1] <= 1-gi+B then Ps = Ps U {i} otherwise Pr = Pr U {i}.
5.3. Allocation For each segment i of least gi in PsInsert a knot at midpoint of the segment iEnd For, P = Pr U Ps
5.4. If Ps <= round (£Pr) then stop the process.

2.5 Pros and Cons of NURBS Approach

Pros include (i)increased effectiveness for complicated shapes (i.e. shapes with large measurement data) as optimal knots are selected for representing actual curves. (ii) Low computation cost. (iii) Provision of more local control (support) on the shape of the curve thus better approximation for the underlying data.Cons are (i) Strong Convex Hull property, (ii) Variation diminishing property. (iii) Invariance under affined geometric transformations. (iv) Weights are used for extra degree of freedom.

2.6 Stochastic Evolution Approach

An iterative approach to solve combinatorial problems is Stochastic Evolution. This approach (basedon Cubic-Splines) described in the can be summarized as follows:
Step1:Outline Extraction: Outline of an object is captured via Chain Code technique. Result is called data point set [10].
Step2: Corner Detection: Corner points are detected using triangular method proposed byChetrikov, explained in random knot insertion algorithm in this draft. Two iterations are applied to evaluate corner strength of each contour point.
Step3: Iterate each segment S, 3.1. Find intermediate points (Vi and Wi)/tangent vectors (Di and Di+1).This method is explained in detail in Least Square Method given in this draft.
$P_i(t) = (1-t)^3 F_i + 3t(1-t)^2 V_i + 3t^2(1-t) W_i + t^3 F_{i+1},\ 0 \le t \le 1$
Vi = Fi + viDi, Wi = Fi+1 - wiDi+1
3.2. Find initial values of shape parameters for the intermediate tangent vectors. wi is found by putting all other values in the aboveequation. vi is found in the same way. **3.3.** Fit an initial curve with the above points (Fi, Vi, Wi, Fi+1).
Step4: Iterate for each segment Si,**4.1.** Set the controlp with No. of iterations equals R Stochastic Evolution.**4.2.** Repeat the process.**4.2.1.** Find new values of shape parameters (vi& wi) from current one randomly.**4.2.2.** Evaluate the error E (via least square method) and compute the gain Y.**4.2.3.**If Y>Random (-p,0)then Pick updated solution otherwisepick old solution.**4.2.4.** Ifno improvement in Solution is Observed As to the the value of the control p otherwisereinitialize the control p.**4.2.5.** IfExisting Solution Best Fits**4.2.5.1.** Maintain Existing Solution.**4.2.5.2.** Add to the value of R.**4.3.**Iferror increases than the defined valueRecursively divide subsegments.
Step5: Extract the fitted last curve.The shape parameters vi and wi can loose or tight the curve.

2.7 Pros and Cons of Stochastic Evolution

Pros include (i) Efficient than hermite spline form. (ii) Added control on contour boundary (iii)Flexible curve fit solution due to usage of modified hermite Cubic Spline.(iii) Powerful yet simple technique to implement. (iv) Computationally efficient than least square approach. (Here only initial fit is done via LSM). Cons include (i) Extra cost of having more intermediate points is incurred with consequently the cost of more iterations. (ii) Ellipses approximations take much more time than other objects. The algorithm needs to incorporate such shaped objects as well [6].

2.8 Recommended Transformative Direction

The algorithm can incorporate circles and ellipses if implemented with rational cubic spline.

2.9 Least Square Method

Algorithm (based on Bezier Cubics) can be summarized as follows:
Step1: Contour Extraction: Take a grey scale image and extract contour of its object(s) via Avrahami algorithm. [20]
Step2: For each point Pi =(xi,yi) on the contour, calculate the approximate curvature Ck (i) via Davis algorithm [21].

$$C^k(i) = \frac{a_{ik}.b_{ik}}{|a_{ik}|\,.\,|b_{ik}|} a_{ik} = \left(x_i - x_{i+k}, y_i - y_{i+k}\right) b_{ik} = \left(x_i - x_{i-k}, y_i - y_{iik}\right).$$

2.1. Set a threshold Tfor Ck (i).**2.2.** If Ck (i) of point Pi(xi, yi) = Local Maximum and Ck (i) >T thenpick the point.
Step 3: Divide the contour points into groups at supposed cornerpoints.
Step4: Piecewise curve fitting (of singlesegment) using Least Square Method.Power basis expression of Bezier curve is used B(t) = (Bx(t), By(t)). Suppose an ordered set of contour points Pi = (xi,yi). i =1,....,n

$$B_x(t) = a_x t^3 + b_x t^2 + c_x t + d_x$$

$$B_y(t) = a_y t^3 + b_y t^2 + c_y t + d_y, 0 \le t \le 1$$

4.1.Determine initial parameter ti for each contour point of the segment using chord-length parameterization

$$t1 = \begin{cases} 0, & i = 1 \\ \frac{\text{lengthofpolygonalline} P_1 P_2 \ldots\ldots P_i}{\text{lengthofpolygonalline} P_1 P_2 \ldots\ldots P_n} & 1 < i \le n \end{cases}$$

(i) At i=1, t=0 i.e. the 1st contour point of segment, (ii) At i=n, t=1 means last contour point of segment, (iii) For rest of the points, its ratio having value between 0 and 1.

4.2. Produce the values of 8 unknowns (ax, bx, cx, dx, ay, by, cy and dy) that make S(squared sum distance) minimum, from the 8 equations that can be generated by taking the following partial derivatives∂S/∂ax,S/∂bx,S/∂cx,S/∂dx,S/∂ay,S/∂by,S/∂cy,andS/∂dyand equating them to zero. Derivatives will be taken.

$$\begin{aligned} S &= \sum_{i=1}^{n} [\text{distancebetween} B(t_i) \text{ and} p_i]^2 \\ &= \sum_{i=1}^{n} \left(a_x t_i^3 + b_x t_i^2 + c_x t_i + d_x - x_i\right)^2 \\ &+ \sum_{i=1}^{n} \left(a_y t_i^3 + b_y t_i^2 + c_y t_i + d_y - y_i\right)^2 \end{aligned}$$

This computation generates two systems of linear equations. One for x and y each. The system of equations for x are:

$$\begin{aligned} &a_x \sum_{i=1}^{n} t_i^6 + b_x \sum_{i=1}^{n} t_i^5 + c_x \sum_{i=1}^{n} t_i^4 + d_x \sum_{i=1}^{n} t_i^3 = \sum_{i=1}^{n} x_i t_i^3 \ , \\ &a_x \sum_{i=1}^{n} t_i^5 + b_x \sum_{i=1}^{n} t_i^4 + c_x \sum_{i=1}^{n} t_i^3 + d_x \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} x_i t_i^2 a_x \sum_{i=1}^{n} t_i^4 \\ &+ b_x \sum_{i=1}^{n} t_i^3 + c_x \sum_{i=1}^{n} t_i^2 + d_x \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} x_i t_i a_x \sum_{i=1}^{n} t_i^3 + b_x \sum_{i=1}^{n} t_i^2 \\ &+ c_x \sum_{i=1}^{n} t_i + d_x n \qquad = \sum_{i=1}^{n} x_i \end{aligned}$$

Step5: If the fit is good goto step 7otherwise goto step 6.

Step 6: **6.1.** Apply re-parameterization, for each contour point Pi, do reparameterization for increased curve accuracy. Solve the following quintic equation in t by Newton-Raphson's method.Solution t will be used as new ti.

$$[B(t) - P_i] . B^{'(t)} = 0$$

6.2. Normalize the new parameter values.**6.3**. If (fit is good) goto step 7, **6.4**. Repeat Step 6.1 for specified number of repetitions, if still good fit not obtained, Fit 2 Bezier curves to the same segment (segment subdivision). Two Bezier curves B1(t) and B2(t) are expressed as:

$$\begin{cases} B_x^{(1)}(t) = a_x^{(1)} t^3 + b_x^{(1)} t^2 + c_x^{(1)} t + d_x^{(1)} \\ B_y^{(1)}(t) = a_y^{(1)} t^3 + b_y^{(1)} t^2 + c_y^{(1)} t + d_y^{(1)}, \quad 0 \le t \le 1 \end{cases},$$
$$\begin{cases} B_x^{(2)}(t) = a_x^{(2)} t^3 + b_x^{(2)} t^2 + c_x^{(2)} t + d_x^{(2)} \\ B_y^{(2)}(t) = a_y^{(2)} t^3 + b_y^{(2)} t^2 + c_y^{(2)} t + d_y^{(2)}, \quad 0 \le t \le 1 \end{cases}$$

Applying G0 and G1,

$$\begin{cases} B_x^{(1)} = B_x^{(2)}(0) \\ B_y^{(1)} = B_y^{(2)}(0) \end{cases} \begin{cases} \alpha B_x^{(1)'}(1) = B_x^{(2)'}(0) \\ \alpha B_y^{(1)'}(1) = B_y^{(2)'}(0) \end{cases}$$
$$where\ \alpha = \frac{\text{lengthofpolygonalline} P_m \ldots P_n}{\text{lengthofpolygonalline} P_1 \ldots P_m}$$

After Gaussian elimination and reparametrization Bezier curves connected with G1 continuity is achieved.

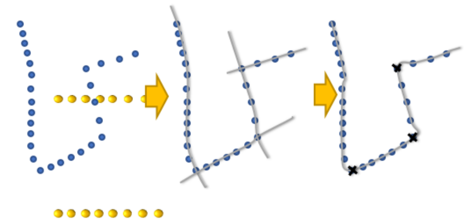

Figure 3: Graphical Review of LSM

Step 6: Ifthe fit with 2 Bezier curves does not give good results. Increase No. of Bezier Curves and goto above step.

Step 7: If it is not the last segmentGoto step 4.**Step 8:** Determine end points after obtaining Bezier curves. Extend each curve in both directions so that each pair of adjoining segments has an intersection point.The approach is shown in Figure 3Threshold is included because without it, the algorithm is too sensitive to small variations of Ck (i). Before this algorithm was proposed, the curve fitting algorithms determine the end points first and then fit a curve by fixing the end points. It creates the scenario as given below which results in ill-fitted curves.

2.10 Pros and Cons of Least Square

Pros include (i) The degradation around the corner of 2-D objects is greatly reduced.(ii) Degree of freedom is not reduced. (iii) Less No. of unnecessary curves. (iv) Contour points are extracted in sub-pixel precision. Contour is extracted via Avrahami and Pratt algorithm which avoids the effect of error introduced by conversion of grey scale image to a bi-level image.The modified Avrahmi algorithm is used to avoid its two disadvantages: (i) human intervention for specification of initial trace points, (ii) degradation around corner points. Conn is heavy computations[8].

2.11 Random Knot Insertion Approach

Algorithm (based on Bezier Cubics) can be summarized as follows:

Step1: Convert a digitized image (Scanned/MS-Paint/Adobe Photoshop) to binary image.

Step 2: Boundary Tracing: Iterate each boundary curve,Use Freeman's chain code algorithm to detect all the boundary points of loop.

Step 3: Corner Points: **3.1.** Iterate each curve (boundary) point p, **3.1.a** Many admissible triangles (p-, p, p+) are defined. These triangle varies between a minimum and maximum square distance on the curve from p- to p , from p to p+ and the angle alpha <= alpha max .Triangles are selected starting from p outwards and stopped at the conditions mentioned above. These constrained rules are as follows:

$$\begin{aligned} &d_{min}^2 \le \left|p - p^+\right|^2 \le d_{max}^2 \ , \\ &d_{min}^2 \le \left|p - p^-\right|^2 \le d_{max}^2 \\ &where \quad \alpha \le \alpha_{max} \ \alpha = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \end{aligned}$$

3.1.b. The triangle having the smallest value of alpha is selected out of all admissible triangles.**3.1.c.**A value of sharpness according to smallest alpha is assigned to the point p.

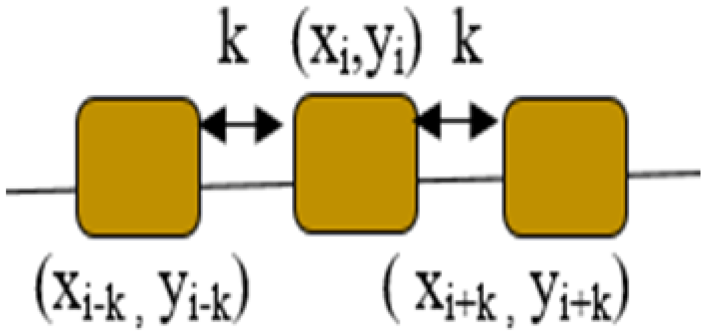

Figure 4: Application of k

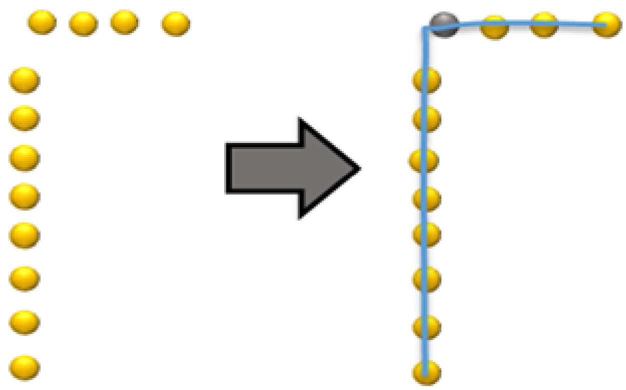

Figure 5: Review Curve Extraction

Figure 6: Input and Output of Step 2

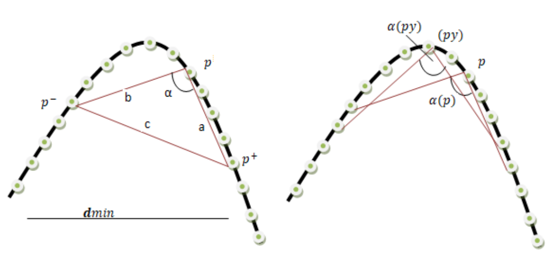

Figure 7: Graphical Representation of Random Knot

3.2. For each curve (boundary) point p,**3.2.a.**The points which have sharpness greater than their neighbors are selected as corner points as these are the points of heavy curvatures. A potential point p is ignores if it has a neighbor pv such that angle alpha(p) >angle alpha(pv), where p's neighborhood can be defined as

$$|p - p_v|^2 \le d_{min}^2$$

A point in p's neighborhood will be considered ifthe above condition is satisfied or if it is adjacent to p. Corner points detected by this approach are shown in Figure 8**Step 4**: For each segment the intermediate control points P1 and P2are found by imposing C1 constraints onparametric cubic splines.

$$P_i\,(t) = (1-t)^3 F_i + 3t(1-t)^2 V_i + 3t^2\,(1-t)\,W_i + t^3 F_{i+1},\ 0 \le t \le 1$$

Step 5: For each curve segment,**5.1.** Select a random spline point. Rand() function is used to get this randompoint x in between end points. This randompoint can be traced for its location from the following formula:

$$LOC = \left(\frac{x}{100}\right) \times n$$

Figure 8: Results of Random Knot Insert

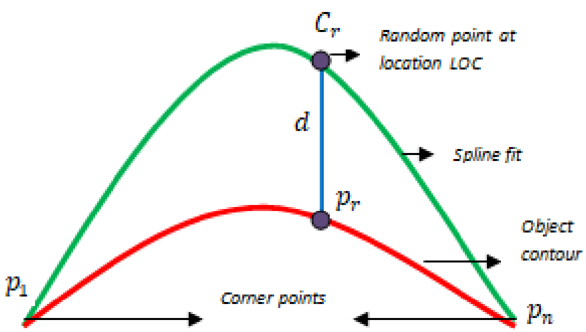

Figure 9: Object Contour vs. Spline Fit

5.2. Calculate the Euclidian distance d on actual boundary. **5.3.** If d > € Add into the sequence of corner points w.r.t its poistion otherwise check the next curve segment.**Step 6:** If flag is set goto step 5.

2.12 Pros and Cons of Knot Insertion

Pros include (i) Computational efficiency(ii) Chain Code algorithm is used for corner detection. Though it works in intensive stack formulation but at the cost of giving the direction of boundary points for each boundary loop. This information is very helpful to approximate curve at high curvature points. (iii) Global spline formation is C1 continuous. (iv) Number of inserted knots has no co-relation to the total number boundary points. (v) Number of knots inserted has no mapping to the complexity of the shape of the object. Cons are increased segments due to increased numberof knots.

2.13 Recommended Transformative Directions

The algorithm uses 3-point derivatives. Approximation derivatives of such kind take a point before data point Fi and a point after it to calculate tangent values at end points of the curve segment. For better approximation, 5-point derivatives can be used which in turn increases curve approximation accuracy thus decreasing the knot insertion overhead.At Step 4:Calculate mid-term control points Pi and Pj for each segment, by imposing C1 constraints using 5-point derivatives.

2.14 Curve Approximation Slope and Relation Usage Approach

Algorithm (based on Bezier Cubics) can be summarized as follows:

Figure 10: Control points estimation of cubic Bezier curve

Step 1: Normalize the grey scale intensity of the input image in the range [0,1]**Step 2:** Threshold the normalized intensities to convert the image to binary. If threshold=0.4,pixel intensity less than0.4is white pixel and others are black.
Step 3: Boundary Tracing:Iterate each boundary using Freeman's chain code algorithm to detect all the boundary points.
Step 4: Corner Points:**4.1.** For each curve (boundary) point ***p***,**4.1.a.** Many admissible triangles *(p-, p, p+)* are defined. These triangle varies between a minimum and maximum square distance on the curve from *p- to p* , from *p to p+* and the angle *alpha* <= $alpha_{max}$.Triangles are selected starting from *p* outwards and stopped at the conditions mentioned above.**4.1.b.** The triangle having the smallest value of *alpha* is selected out of all admissible triangles.**4.1.c.**A value of sharpness according to smallest alpha is assigned to the point *p*.**4.2.**Iterate each curve (boundary) point ***p***,**4.2.a.**The points which have sharpness greater than their neighbors are selected as corner points as these are the high curvature.
Step 5: Control Points calculation- Control point P_1 should be searched along the slope *m*.**5.1.** P_1=P_0. **5.2.** Repeat. **5.2.a.** P1 = P1 + m, initially P1=P0 and P1 has to be searched along the slope.**5.2.b**. P2 = Ꝁ P1 **5.2.c.** Determine cubic Bezier curve using calculated control points.**5.3.**Whileapproximation error from original cubic Bezier curve reduces. (i) Dots show the sequence of search of control points P_1 and P_2. Circled points are the detected control points. (ii) The accuracy of the approximated Bezier curve is shown by drawing it over the original cubic Bezier curve.
Step 6: Curve Approximation. Iterate each curve segment,Move both the control points (P_1 and P_2) along the slopes (at the start and end of the curves) towards or away from the control points P_0 and P_3 , respectively till the approximation error is minimized.
Step 7: Segment Subdivision. IftheMaximum Approximation Error > Given Threshold then Subdivide the segment into two segments at the point of maximum approximation error. Recursively process the divided sub-segment.

2.15 Pros and Cons of Curve Approximation Slope and Relation Usage Approach

Pros include (i) Efficient approximation curve at high curvature points. (ii) Continuity order of the original segments is followed here.Cons include two passes of control point approximations for curve best fit.

2.16 Recommended Transformative Directions

If recursion levels are greater thanthreshold limit then instead of individually processing the subdivided segments (going through the whole algorithm and calculations recursively), use rational cubic spline with default weights for other segments while with adjusted weights for this segment.

3 CONCLUSION AND FUTURE WORK

Conclusively, there are various methodologies proposed by researchers having their own pros and cons. The same problem is attacked with genetic algorithms, simulated annealing, stochastic evolution, parameterization and many other methods using various types of splines. (i) **Spiral Usage for curve fitting** of objects in an image is a potential area of research needed to be explored. Until now curves are automated using other spline models. If end points data, curvature extrema, inflection points and angle theta between two tangents is recoded, spirals can be easily plotted between given data points. Idea is to break the segments at the points where angle theta mentioned above is greater than equals to Π. (ii)**Distributed and parallel processing** can be used to resolve the problem understudy. For each piece breaking the segments and curve fitting can be done separately [24] [26]. (iii) **Open Curves**, Inorder to handle any type of outline, the above methods can be changed with slight variations to handle open curves. (iv) **Higher Order Curves**, It will also be interesting to explore at the cost of comparatively heavy computational time, higher order polynomials can reduce the number of segments required to capture the given outline. (v) **3-D Curves,** The proposed improvements may be extended to work for 3-D surface models where a third dimension (z(t)) comes into the scene. (vi) **Colored Images,** A potential area of research is objects extraction from colored images. In detection of the edges (contours) of the objects significant work will be needed to fit in the above algorithms[4].

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