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ALFVÉN SOLITONS IN THE SOLAR WIND

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Abstract The interaction of circularly-polarized Alfvén waves with the surrounding plasma in high speed solar wind streams is investigated. Alfvén wave modulational instability is discussed and nonlinear envelope soliton solutions of the magnetohydrodynamic equations are introduced. The characteristics of these Alfvén solitons are compared with observational results obtained from Helios I and II. A model of the expected turbulent spectrum due to a collection of such solitons is constructed and its radial dependence is investigated, again along with comparison to Helios data.

1. Introduction

Magnetic field fluctuations in high speed solar wind streams have been attributed mainly to the presence of parallel propagating Alfvén waves [see Belcher et al., 1969; Belcher and Davis, 1971]. The predominance of nearly incompressive Alfvén waves in solar wind streams has also been confirmed by Daily [1973], Parker [1980a, b], and Bavassano et al. [1981]. A detailed discussion of Alfvén wave dominated solar wind phenomena is given by Denskat and Burlaga [1977].

Investigations by Burlaga and Turner [1976] and Neugebauer et al. [1978], however, revealed that variations in density, magnetic field energy, and hydrodynamic velocity were also present in regions previously associated with Alfvén waves. This result contrasted with the theoretical constancy of these quantities in the presence of pure Alfvénic fluctuations. Consequently, it was argued that what was observed was a mixture of Alfvén and fast magnetosonic waves propagating at small angles with respect to the background magnetic field. However, the above mentioned authors were unable to observe any direct correlation between density and magnetic field which would be expected from such a mixture of waves.

In order to explain the discrepancy between theoretical results and observations, it was suggested [Burlaga and Turner, 1976; Neugebauer et al. 1978] that this was due to the presence of, e.g., nonlinear Alfvén waves propagating nearly parallel to the background magnetic field, or some similar nonlinear mechanism involving primarily an Alfvén wave.

In the present paper we pursue this suggestion by demonstrating that nonlinear Alfvén waves can interact with the background plasma to form soliton fields. These fields appear to be an interesting description of nonlinear Alfvénic fluctuations which may result from the inherent modulational instability of Alfvén waves [Derby, 1978; Goldstein, 1978]. To a limited extent these solutions can be used to describe Alfvénic turbulence by considering a collection of solitons.

For simplicity, we shall make the following assumptions:

1. We use the one-fluid isotropic isothermal magnetohydrodynamic (MHD) equations with the generalized Ohm's law. This prevents us from treating anisotropic temperatures and restricts us to frequencies below the proton gyrofrequency. We thus also ignore the alpha particle component of the solar wind, although this may significantly affect the wave propagation characteristics [e.g., Isenberg and Hollweg, 1982].
2. We consider parallel propagation only, which is motivated by mathematical ease and earlier interpretations of the apparent wave-vectors (see, however, Barnes [1981]). In the soliton case this restriction to one dimension may be severe, as two-dimensional Langmuir solitons are known to collapse while the one-dimensional ones propagate undistorted [cf. Galeev et al., 1977].
3. Only small fluctuations in density and longitudinal velocity are considered. This is in keeping with the dominant Alfvénic nature of solar wind observations.
4. A uniform background state is assumed, although we estimate the evolution of our resulting power spectrum in a slowly varying solar wind.

Section 2 presents the mathematical formulation. Section 3 is devoted to the basic equations necessary for a general instability analysis of Alfvén waves. In section 4 we look for soliton solutions and compare some of their properties with observations. In section 5 we construct a power spectrum, assuming that the turbulence can be represented as a set of solitons. We compare these results with the analysis made by Bavassano et al. [1982] and Denskat and Neubauer [1982]. Section 6 is devoted to a discussion of the radial dependence of the power spectrum under some simple hypotheses. The final section presents our conclusions.

2. Mathematical Formulation

The relevant MHD equations are [Boyd and Sanderson, 1969]

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho' \underset{\sim}{v}) = 0 \tag{1}$$

$$\rho' \frac{d \underset{\sim}{v}}{dt} = - \nabla P' + \frac{1}{c} (\underset{\sim}{j} \times \underset{\sim}{B}') \tag{2}$$

$$\underset{\sim}{E}' = - \frac{\underset{\sim}{v} \times \underset{\sim}{B}'}{c} + \frac{m^+}{\rho' e c} (\underset{\sim}{j} \times \underset{\sim}{B}') + \frac{m^+}{2\rho' e} \nabla P' \tag{3}$$

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$$\nabla \times \underset{\rightarrow}{B}' = \frac{4\pi}{c} \underset{\rightarrow}{j} \quad (4)$$

$$\nabla \times \underset{\rightarrow}{E}' = -\frac{1}{c}\frac{\partial \underset{\rightarrow}{B}'}{\partial t} \quad (5)$$

and

$$\frac{P'}{\rho'} = \text{const} \quad (6)$$

where $d/dt \equiv \partial/\partial t + (\underset{\rightarrow}{v} \cdot \nabla)$. Equations (1) - (3) are the equations of continuity, motion, and generalized Ohm's law, respectively. Equations (4) and (5) are Maxwell's equations, and equation (6) is the isothermal equation of state. It might be noted here that, although it would be preferable to use the adiabatic equation of state, we use the isothermal one in order to avoid unnecessarily complicated analytical expressions.

In equations (1)-(6) ρ', $\underset{\rightarrow}{v}$, and P' are the mass density, hydrodynamic velocity, and pressure respectively; m^+ and e are the ion mass and electronic charge; $\underset{\rightarrow}{E}'$ and $\underset{\rightarrow}{B}'$ are the electric and magnetic field vectors and $\underset{\rightarrow}{j}$ is the current density.

Substituting equation (4) into (2), we get

$$\frac{d\underset{\rightarrow}{v}}{dt} = -\frac{1}{\rho}\nabla P + \frac{v_A{}^2}{\rho}(\nabla \times \underset{\rightarrow}{B}) \times \underset{\rightarrow}{B} \quad (7)$$

where $v_A = \sqrt{(B_o{}^2/4\pi\rho_o)}$ is the Alfvén speed. Next, we eliminate $\underset{\rightarrow}{E}'$ from equation (3) by using (5) and (6) to obtain

$$\frac{\partial \underset{\rightarrow}{B}}{\partial t} = \nabla \times (\underset{\rightarrow}{v} \times \underset{\rightarrow}{B})$$

$$-\frac{v_A^2}{\Omega_i}\nabla \times \frac{1}{\rho}\left[(\nabla \times \underset{\rightarrow}{B}) \times \underset{\rightarrow}{B}\right] \quad (8)$$

Equations (7) and (8) use the nondimensional variables $\rho = \rho'/\rho_o$, $\underset{\rightarrow}{B} = \underset{\rightarrow}{B}'/B_o$, and $P = P'/P_o$, where the subscript o refers to background quantities. $\Omega_i = eB_o/m^+c$ is the ion gyro-frequency.

If the background magnetic field is taken to be in the z direction, viz $\underset{\rightarrow}{B}_o = B_o\underset{\rightarrow}{z}$, and the assumption of parallel propagation ($\partial/\partial x = 0 = \partial/\partial y$) is used, equations (7) and (8) combine to yield

$$\frac{\partial^2 B_\pm}{\partial t^2} - \frac{\partial}{\partial z}\left[\frac{v_A^2}{\rho}\frac{\partial B_\pm}{\partial z}\right]$$

$$+ \frac{\partial}{\partial z}\left[v_z\frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(v_z B_\pm)\right]$$

$$\pm \frac{i}{\Omega_i}v_A^2\frac{\partial}{\partial z}\left[\frac{d}{dt}\left[\frac{1}{\rho}\frac{\partial B_\pm}{\partial t}\right]\right] = 0 \quad (9)$$

where $B_\pm = B_x \pm iB_y$.

From equations (1) and (6) and the z component of equation (7) we derive

$$\frac{\partial^2\rho}{\partial t^2} - c_s^2\frac{\partial^2\rho}{\partial z^2} = \frac{v_A{}^2}{2}\frac{\partial^2}{\partial z^2}|B_\pm|^2 + \frac{\partial}{\partial z}(\rho v_z^2) \quad (10)$$

where $c_s = \sqrt{(P_o/\rho_o)}$ is the speed of sound. The quantities v_z and ρ are related by the continuity equation

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial z}(\rho v_z) = 0 \quad (11)$$

Equations (9)-(11) are fully nonlinear, relating together $B_\pm$, ρ, and v_z. In the case when ρ and v_z are constant, equation (9) becomes

$$\left[\frac{\partial^2}{\partial t^2} - v_A^2\frac{\partial^2}{\partial z^2} \pm \frac{i\,v_A^2}{\Omega_i}\frac{\partial^3}{\partial z^2\partial t}\right]B_\pm = 0 \quad (12)$$

This yields a dispersion relation for wave solutions with arbitrary amplitudes which is given by

$$\omega^2 \mp \frac{\omega\omega_A^2}{\Omega_i} - \omega_A^2 = 0 \quad (13)$$

where $\omega_A = k_A v_A$ with ω and k_A being the wave frequency and number. For low-frequency waves, i.e., $\omega_A/\Omega_i \ll 1$, equation (13) yields

$$\omega_\pm = \omega_A\left[1 \mp \frac{\omega_A}{2\Omega_i}\right] \quad (14)$$

This is a dispersion relation for finite amplitude ion-cyclotron modes [see Abraham-Shrauner and Feldman, 1977], where the upper sign corresponds to the rotation of the field vector in the same sense as the ions. It may be noted that the wave traveling in the opposite direction,

$$\omega_\pm = -\omega_A\left[1 \mp \frac{\omega_A}{2\Omega_i}\right]$$

is not considered here and that the identification of ω_+ or ω_- with B_+ or B_- depends on the convention used in describing the traveling wave although, of course, the physical interpretation does not.

3. Basic Equations

Equations (9)-(11) are now linearized with respect to ρ and v_z in order to determine changes brought about in a predominantly Alfvénic fluctuation due to variations in these quantities. The linearizing procedure yields the following equations

$$\frac{\partial^2 B_\pm}{\partial t^2} - v_A^2\frac{\partial}{\partial z}\left[(1-\delta\rho)\frac{\partial B_\pm}{\partial z}\right]$$

$$+ \frac{\partial}{\partial z}\left[\delta v\frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(\delta v\, B_\pm)\right]$$

$$\pm \frac{i\,v_A^2}{\Omega_i}\frac{\partial^2}{\partial z\,\partial t}\left[(1-\delta\rho)\frac{\partial B_\pm}{\partial z}\right] = 0 \quad (15)$$

$$\left[\frac{\partial^2}{\partial t^2} - c_s^2\frac{\partial^2}{\partial z^2}\right]\delta\rho = \frac{v_A{}^2}{2}\frac{\partial^2}{\partial z^2}(|B_\pm|^2) \quad (16)$$

and

$$\frac{\partial\,\delta\rho}{\partial t} + \frac{\partial\,\delta v}{\partial z} = 0 \quad (17)$$

where we have taken $\rho = 1 + \delta\rho$ and $v_z = 0 + \delta v$ (recall that ρ and $B_\pm$ are normalized to their background values).

In addition to the soliton solution of section 4, equations (15)-(17) can be used to investigate a wide range of Alfvén wave-related phenomena, such as the decay instability [Sagdeev and Galeev, 1969] and the modulational instability [Derby, 1978; Goldstein, 1978], where variations in ρ and v_z affect the wave amplitude, which in turn couples to the plasma through equations (16) and (17). All of these analyses begin by inserting an Alfvén wave,

$$B_\pm = b(z,t) \exp [i (k_A z - \omega_\pm t)]$$

into equations (15)-(17). Once again using the inequality $\omega_A/\Omega_i << 1$ and assuming a scaling according to

$$\frac{\partial b}{\partial t} \propto \frac{b}{\tau} \qquad \frac{\partial b}{\partial z} \propto \frac{b}{v_A \tau} \qquad \delta v \propto v_A b^2$$

$$\delta\rho \propto b^2$$

where $1/\tau << \omega_\pm \simeq \omega_A$, we obtain the following expressions to order b^3 and $1/\tau$:

$$i \frac{\partial b}{\partial t} + i v_g \frac{\partial b}{\partial z} + \frac{\omega_A \delta\rho\, b}{2} - \delta v\, k_A b \mp \frac{v_A^2}{2\Omega_i} \frac{\partial^2 b}{\partial z^2} = 0 \qquad (18)$$

$$\left[\frac{\partial^2}{\partial t^2} - c_s^2 \frac{\partial^2}{\partial z^2}\right] \delta\rho = \frac{v_A^2}{2} \frac{\partial^2}{\partial z^2} (|b|^2) \qquad (19)$$

and

$$\frac{\partial\, \delta\rho}{\partial t} + \frac{\partial\, \delta v}{\partial z} = 0 \qquad (20)$$

where $v_g = v_A(1 \mp \omega_A/\Omega_i)$ is the group velocity of the original wave.

Equations (18)-(20) (but including the higher order terms in $1/\tau$, which we shall neglect in our study of a slowly varying envelope) form the backbone of various calculations. For example, the modulational instability analysis proceeds by perturbing b as $b = b_o + \delta b$ with δb, δv, $\delta\rho \propto \exp[i(k'z - \Omega' t)]$ and searching for the daughter wave dispersion relation $\Omega'(k')$. In general, such daughter waves grow, at the expense of the parent Alfvén wave [cf. Goldstein, 1978; Derby, 1978], over time scales sufficiently short to be important in the solar wind. As a special case, the decay instability assumes the daughter waves to also be propagating normal modes (i.e., Alfvén, fast, or slow waves), and this too leads to potentially important growth times. However, it appears that the total spectrum of fluctuations in the solar wind is stable against the decay instability [Cohen and Dewar, 1974] except, perhaps, the very lowest frequency components in regions very close to the sun ($\simeq$0.3 AU) where the spectrum flattens [Bavassano et al., 1982]. Thus we proceed by supposing that the amplitude of the Alfvén waves in the solar wind varies as a result of the modulational instability. What is now required is the ultimate shape of this slowly varying envelope which must self-consistently satisfy (18)-(20). It is possible that the soliton solution given in the next section, which fulfills these requirements, does indeed represent the fate of a modulationally unstable Alfvén wave.

4. Soliton Solutions

Mio et al. [1976a, b] and MjΦlhus [1976] have analyzed modulational instability and soliton envelopes for nonlinear Alfvén waves propagating along a static magnetic field. These authors, however, used a double perturbation technique which we feel obscures the physical aspects of the problem. We, instead, look for soliton solutions for equations (18)-(20), noting that these equations are similar to Zakharov's [1972] equations which govern Langmuir turbulence, which in turn are related to the nonlinear Schrodinger equation with known soliton solutions.

Accordingly, we assume a wave amplitude of the form

$$b(z,t) = b_o \operatorname{sech} [\kappa(z - Vt)] e^{i\delta\omega t} \qquad (21)$$

where b_o, κ, V, and $\delta\omega$, all of which we shall take as constants, are the maximum field amplitude, inverse width, velocity, and nonlinear frequency shift, respectively, of the soliton envelope. These constants are determined in a self-consistent manner from equations (18)-(20) and (21); we find

$$V = v_A (1 - \omega_A/\Omega_i) \qquad (22)$$

$$\kappa = \frac{k_A b_o}{2} \left[\frac{\Omega_i/\omega_A}{1 - \beta} \right]^{1/2} \qquad (23)$$

and

$$\delta\omega = - \frac{\omega_A |b_o|^2}{8 (1 - \beta)} \qquad (24)$$

where $\beta = c_s^2/v_A^2$. Here we have only considered the left hand polarized wave (corresponding to ω_+), since the right hand wave (ω_-) is always modulationally stable [Mio et al., 1976a]. In evaluating (22)-(24), we have approximated $\delta\rho$ and δv in (18) by their values at the center of the soliton envelope. This yields reasonable results near this center, although, in general, V, κ, and $\delta\omega$ are functions of z and t. The mass density and velocity variations are given by

$$\delta\rho = \frac{|b(z,t)|^2}{2 (1 - \beta)} \qquad (25)$$

and

$$\delta v = V \delta\rho \cong v_g \delta\rho \qquad (26)$$

Physically, the increased density (25) leads to a lower local Alfvén speed causing the associated buildup in wave energy in this region. The energy transport is achieved by forcing this material to move at the wave group velocity (equation (22)), which results in the unchanged soliton envelope. In this way the wave is modified continuously and a balance maintained between energy entering and leaving the region. We may also note from equation (23) that the width of the soliton, κ^{-1}, is inversely proportional to the amplitude b_o. Since these features are similar to those of the solutions of the nonlinear Schrodinger equation, most of the analysis performed upon the latter may be relevant to our solution.

Numerical solutions of equations (9)-(11) by Mio et al. [1976b] show that a perturbation in the field amplitude tends to form a localized field enhancement moving with a velocity in excess of the Alfvén speed and could be interpreted as the formation of a soliton, although other interpretations (e.g., shock wave formation) could also be made. On the other hand, the solar wind observations referred to in the introduction provide encouragement for the existence of Alfvén solitons. Burlaga and Turner [1976] pointed out that for amplitudes in the range $0.1 \leq b_o < 0.5$, 83% of the events observed had $|\underline{B}|$ and ρ not constant. These observed amplitudes were in a frequency range 10^{-4}-10^{-1} Hz (with $\Omega_i \simeq 10^{-1}$ Hz at 1 AU), and these data lie within the limits of our theory, particularly after removing the Doppler shift to higher frequencies due to the solar wind flow. A similar range of observations by Neugebauer et al. [1978] revealed hydrodynamic velocity variations $\delta v \simeq v_A \delta\rho$ in agreement with equation (26). In view of this, it is not inconsistent to suggest that the above set of observations contained Alfvén solitons or similar structures which relate variations in the wave field to variations in the plasma properties.

5. Turbulence Spectrum

In order to construct a turbulence spectrum which consists of a collection of soliton fields, we follow the work of Kingsep et al. [1973] and Yu and Spatschek [1976], who consider an ensemble of solitons with a common energy density W (normalized to $B_o^2/8\pi$) and system of total length L. This energy is equally distributed between N solitons.

Now, the soliton solution can be written as

$$b(\xi) = b_o \operatorname{sech} \kappa\xi$$

where $\xi = z - Vt$. The total energy per unit cross sectional area is

$$E = \frac{1}{2\kappa} \int_{-\infty}^{\infty} \frac{|b|^2}{8\pi} \, d(\kappa\xi) = \frac{b_o^2}{8\pi\kappa} \qquad (27)$$

From equation (23) we can define a quantity κ' which is independent of the soliton amplitude via

$$\kappa = \kappa' b_o$$

where

$$\kappa' = \frac{k_A}{2} \left[\frac{\Omega_i / \omega_A}{1 - \beta} \right]^{1/2} \qquad (28)$$

By using (28), equation (27) becomes

$$E = \frac{b_o}{8\pi\kappa'} \qquad (29)$$

Thus, dividing up the total energy per area WL gives

$$WL = NE = \frac{N b_o}{8\pi\kappa'} \qquad (30)$$

From this equation we see that the amplitude of each soliton depends on the number of solitons, N, according to

$$b_o = \frac{b_o'}{N} \qquad (31)$$

where we have defined another soliton-independent parameter $b_o' = 8\pi\kappa' WL$.

The maximum number of solitons allowed in the system is $N_{max} = \kappa L$, which results from the assumption that all the solitons are identical, nonoverlapping, and each soliton occupies a width $\simeq\kappa^{-1}$. The minimum number is $N_{min} = 1$. Thus, assuming any value of N in the range $N_{min} < N < N_{max}$ is equally likely, the probability that the system is in an N soliton state is

$$P(N) = \frac{1}{N_{max}} \qquad (32)$$

The power spectrum of each soliton is given by

$$P_k = \frac{2\pi}{L} |b_k|^2 \qquad (33)$$

where $b_k = (b_o/2\kappa) \operatorname{sech}(\pi k/2\kappa)$ is the Fourier transform of $b(\xi)$. Therefore the power spectrum of randomly spread solitons is

$$NP_k \equiv P_k(N) = \frac{\pi}{2} \frac{N}{L\kappa'^2} \operatorname{sech}^2 \left[\frac{\pi k N}{2\kappa' b_o'}\right] \qquad (34)$$

The mean power spectrum is obtained by ensemble averaging over the states of the system and is given by

$$\langle P_k(N) \rangle = \frac{\pi}{2L\kappa'^2} \int_0^{N_{max}} N \operatorname{sech}^2 \left[\frac{\pi k N}{2\kappa' b_o'} \right] P(N) dN$$

$$= \frac{b_o'}{\kappa'\eta L k^2} \left[\eta k \tanh(\eta k) - \log \cosh(\eta k)\right] \qquad (35)$$

where

$$\eta = \frac{\pi}{2} \left[\frac{L}{\kappa' b_o'} \right]^{1/2}$$

and we have assumed $N_{max}/N_{min} \gg 1$ in order to replace the lower integration limit by 0.

In order to analyze analytically the ensemble averaged power spectrum, we divide the spectrum into two regions depending on the value of ηk. In the case when $\eta k \gg 1$ we obtain from (35)

$$\langle P_k(N) \rangle \propto k^{-2} \qquad (36)$$

in this limit of large k. In the other extreme, at low k when $\eta k \ll 1$, we find

$$\langle P_k(N) \rangle \simeq \text{constant} \qquad (37)$$

Thus, as can be seen from the plot of (35) in Figure 1, or directly from (36)-(37), this spectrum is flat at low frequencies and falls off steeply at higher ones. This is qualitatively similar to solar wind observations [e.g., Bavassano et al., 1982; Denskat and Neubauer, 1982], particularly for measurements well inside 1 AU. It is not yet clear whether the quantitative agreement, with observed spectral indices $\simeq$ 1.6,

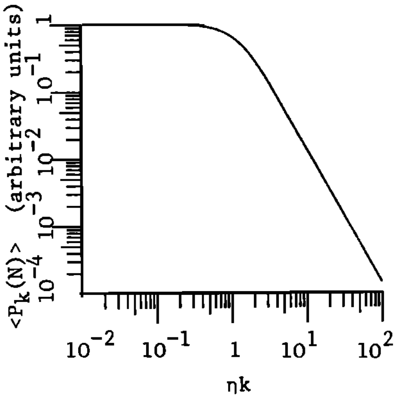

Fig. 1. Power spectrum of ensemble-averaged soliton turbulence.

can be improved within the soliton description (e,g., an alternative ensemble average, collections of unequal solitons, etc.).

6. Radial Dependence

The radial dependence of the observed solar wind magnetic field power spectrum has only recently received detailed investigation. Results from Helios I and II have been presented by Bavassano et al., [1982] and Denskat and Neubauer [1982], who have shown that as heliocentric distances increase toward 1 AU, the low-frequency flattening disappears as the whole spectrum becomes encompassed by the steeper power law. This implies that the apparent breakpoint, $k = k_{br}$, where the two different power laws of the spectrum in Figure 1 meet, moves to lower k with increasing heliocentric distances. Bavassano et al. have interpreted this in terms of a k-dependent damping length.

We can now use the results in the previous section to search for an alternative theoretical description of this behavior. From equation (35) and the definition of η which follows it, we see that the breakpoint k_{br} in the soliton spectrum occurs when

$$\eta\, k_{br} \propto 1$$

Thus

$$k_{br} = \frac{2}{\pi}\left[\frac{\kappa' b_o'}{L}\right]^{1/2} \quad (38)$$

By using the definition of b_o' from equation (31), we find

$$k_{br} = 4\left[\frac{2}{\pi}\right]^{1/2} \kappa' \sqrt{W} \quad (39)$$

where we recall $W = \delta b^2/B_o^2$ is the normalized energy density in the fluctuations. An estimate of the numerical value of k_{br} depends sensitively on the parent wave and β via κ' (28) and W, making a meaningful value difficult to obtain in our simple model.

In order to determine the radial dependence of k_{br}, we thus need to know the radial dependences of W and κ', which in turn demands a knowledge of the radial dependences of k_A, Ω_i, ω_A, and β (see equation (28)).

Since the Doppler shifted frequency of a wave, $\omega_o = \omega_A + k_A v_{sw}$, is constant [cf. Schwartz et al., 1981] and the solar wind speed $v_{sw} \propto$ const with $v_A/v_{sw} \ll 1$, we find that

$$k_A \propto \text{const} \quad (40)$$

Taking for simplicity a purely radial field $B_o \propto 1/r^2$, while $\rho \propto 1/v_{sw}r^2 \propto 1/r^2$, then $v_A \propto 1/r$ and thus

$$\omega_A = k_A v_A \propto \frac{1}{r} \quad (41)$$

and

$$\Omega_i \propto \frac{1}{r^2} \quad (42)$$

Finally, from the definition of β following (24), we have

$$\beta = \frac{c_s^2}{v_A^2} \propto \frac{T^2}{1/r^2} \propto \frac{(1/r)^2}{1/r^2} = \text{const} \quad (43)$$

where, for simplicity, we have assumed that the temperature falls off as r^{-1}, i.e., somewhat less rapidly than adiabatically.

By using equations (40)-(43), we reach

$$\kappa' \propto \left[\frac{1}{r}\right]^{1/2} \quad (44)$$

Finally, from (39) and (44), we can write

$$k_{br} \propto \left[\frac{W}{r}\right]^{1/2} \quad (45)$$

It remains to determine the radial dependence of W. One approach is via WKB theory which gives [cf. Schwartz et al., 1981]

$$W = \frac{\delta b^2}{B_o^2} \propto r \quad (46)$$

Combining (45) and (46), we obtain

$$k_{br} \propto \text{const} \quad (47)$$

which is not in particularly good qualitative agreement with the observational results.

As an alternative example, we illustrate the suggestion [Hollweg, 1973] that the ratio $\delta b^2/B_o^2$ eventually saturates, thus

$$W = \frac{\delta b^2}{B_o^2} \propto \text{const} \quad (48)$$

Now, from (45) and (48), we have that

$$k_{br} \propto r^{-1/2} \quad (49)$$

Equation (49) predicts that with increasing heliocentric distance the entire spectrum will eventually fall into the steeper part of the curve shown in Figure 1, in closer qualitative agreement with the observations. Of course, whether the more detailed behavior of individual frequency components can be matched within this hypothesis remains an open question. Moreover, ultimately the nonlinear saturation must be incorporated into a self-consistent theory of soliton propagation in a fully inhomogeneous medium.

7. Conclusions

We have suggested that circularly polarized Alfvén waves, which should be modulationally unstable in the solar wind, evolve as a result of this instability into the localized field solutions known as solitons. The observations of Burlaga and Turner [1976] and Neugebauer et al. [1978] indicate that high speed solar wind streams are Alfvén wave dominated, and a nonlinear description is necessary to account for the fluctuations in energy density, mass density, and hydrodynamic velocity. Our results of section 4 show that Alfvén solitons might provide a useful theoretical description of the above- mentioned fluctuations and do yield nonzero fluctuations in density and longitudinal velocity.

In section 5 we constructed a power spectrum by assuming that it can be represented as a collection of solitons. The spectra obtained from this model show some qualitative similarities with the observational results from Helios I and II [e.g., Bavassano et al., 1982; Denskat and Neubauer, 1982] although the precise power laws are not quantitatively explained by this simple ensemble averaging technique. In section 6 we have investigated the radial evolution of the break in the power spectrum. Our analysis shows better qualitative agreement with observational results if the Alfvén fluctuation amplitude saturates [e.g., Hollweg, 1973] than if WKB theory is used, although we have not solved properly the more difficult problem of soliton propagation in an inhomogeneous medium.

It is not yet clear how useful the soliton fields are in describing the overall turbulent state of the solar wind. Different forms of soliton ensembles, etc., may improve the quantitative analysis. In any event, the nonlinear solution itself does provide a potentially interesting tool for further investigation. Although it is possible that this solution is not stable, the numerical results of Mio et al. [1976b] suggest that, if their structure is indeed a soliton, it might be stable. It would be interesting to examine the character of nonparallelly propagating solitons to search for soliton collapse, as in the case of Langmuir solitons, although this will obviously involve considerable effort. Equally intriguing are the behavior of particles in the soliton fields and the consequences of such soliton-particle interactions for the nonthermal features of the solar wind ion distributions.

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