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Obliquely propagating whistler instability in an inhomogeneous plasma penetrated by a helical electron beam

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The excitation of the whistler instability due to normal gyroresonant interaction is studied in a warm inhomogeneous plasma in the presence of a helical electron beam. The results show that it is possible to excite oblique whistler waves propagating at large angles with respect to the ambient magnetic field. It is found that the inhomogeneity increases the frequency and growth rate of the instability.

I. INTRODUCTION

The excitation of the whistler mode instability ($\omega < \omega_{ce}$) in a plasma penetrated by a helical electron beam has been investigated in a number of studies.[1–4] It is well known that a whistler wave propagating in the direction opposite to the beam can be unstable as a result of normal gyroresonant interaction ($m = 1$) which is characterized by the resonance condition $\omega - k_{\parallel} V_b \approx m\omega_{ce}$. So far the studies of this interaction have been restricted to the configuration where both plasma and the beam are assumed to be homogeneous. Here we consider the case of a warm inhomogenous plasma penetrated by a helical electron beam and study the effect of the inhomogeneity on the excitation of the whistler instability in the electrostatic limit. The effect of inhomogeneity appears through the gradient in the number density of the plasma electrons.

II. DISPERSION RELATION

The dispersion relation for the system under consideration can be written as $D_p + D_b = 0$, where D_p and D_b are the dispersion relations for the plasma and the beam, respectively. In order to determine D_p, we first evaluate the dielectric tensor elements for the plasma. Since we are specifically interested in the excitation of whistler instability due to normal gyroresonant interaction, only $m = 1$ terms are of relevance in our computations. The externally applied magnetic field $\mathbf{B}_0$ is taken in the z direction. The inhomogeneity is considered to be weak, and the density gradient is assumed to be in the y direction. We assume the distribution function for the plasma electrons to be of the form:

$$f_0 = n_p(y)\left(\frac{m}{2\pi T_\perp}\right)\left(\frac{m}{2\pi T_\parallel}\right)^{1/2} \exp\left(\frac{mv_\perp^2}{2T_\perp} - \frac{mv_\parallel^2}{2T_\parallel}\right),$$

where n_p is the plasma density and $T_\parallel$ and $T_\perp$ are the parallel and perpendicular temperatures, respectively. The general expression for the dielectric tensor $\epsilon_{\alpha\beta}$ for such a plasma can be written as[5]

$$\epsilon_{\alpha\beta} = \delta_{\alpha\beta} + \frac{4\pi e^2}{m\omega}\left[2\pi \int dv_\perp^2\, dv_\parallel \left(\Phi Q_{\alpha\beta} - \frac{1}{\omega_{ce}}\frac{\partial \Phi}{\partial y} P_{\alpha\beta}\right) + \frac{\delta_{\alpha x}\delta_{\beta x}}{\omega\omega_{ce}^2}\int \frac{v_\perp^2}{2}\frac{\partial^2 f_0}{\partial y^2}\, d\mathbf{v}\right], \quad (1)$$

where

$$\Phi = -m\left(\frac{T_\parallel + T_\perp}{T_\parallel T_\perp}\right) f_0 - \frac{k_x}{\omega\omega_{ce}}\frac{1}{L} f_0, \quad (2)$$

$$Q_{\alpha\beta} = q_\alpha^* q_\beta/(\omega - k_\parallel v_\parallel - \omega_{ce}),$$
$$P_{\alpha\beta} = p_\alpha q_\beta/(\omega - k_\parallel v_\parallel - \omega_{ce}). \quad (3)$$

The components of the vectors q and p are given by

$$q_x = (\omega_{ce}/k_x) J_1,$$
$$q_y = iv_\perp [J_0 - (\omega_{ce}/k_x v_\perp) J_1],$$
$$q_z = v_\parallel J_1, \quad (4)$$

$$p_x = (v_\perp^2/4)(J_1 + J_3),$$
$$P_y = -iv_\perp(\omega_{ce}/k_x)[J_0 - (2\omega_{ce}/k_x v_\perp) J_1],$$
$$P_z = (\omega_{ce}/k_x) v_\parallel J_1, \quad (5)$$

where $J_m = J_m(k_x v_\perp/\omega_{ce})$ is the Bessel function, ω_{ce} is the cyclotron frequency, $L = \{[(1/n)(\partial n/\partial y)]^{-1}\}$ is the scale length of the inhomogeneity, m and e are the mass and charge of electrons, respectively, and $d\mathbf{v} = dv_\perp^2\, dv_z\, d\varphi$.

In order to carry out the integration over $dv_\parallel$ in (1) we make use of the following relation

$$\left(\frac{m}{2\pi T_\parallel}\right)^{1/2} \int_{-\infty}^{\infty} \frac{dv_\parallel \exp(-mv_\parallel^2/T_\parallel)}{(\omega - k_\parallel v_\parallel - \omega_{ce})} = -\frac{1}{k_\parallel v_{t\parallel}} Z_p\left(\frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}}\right), \quad (6)$$

where Z_p is the Fried–Conte dispersion function[6] and $v_{t\parallel} = (2T_\parallel/m)^{1/2}$. The integration over transverse velocities in Eq. (1) is carried out by using the following two relations[7]:

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$$\int_0^\infty \exp(-\sigma^2 x^2) J_m(\alpha x) J_m(\beta x) dx = \frac{1}{2\sigma^2} \exp\left(-\frac{\alpha^2+\beta^2}{4\sigma^2}\right) I_m\left(\frac{\alpha\beta}{2\sigma^2}\right), \quad (7)$$

where I_m is the modified Bessel function, and

$$\int_0^\infty x^{\lambda-1} \exp(-\alpha x^2) J_s(\beta x) J_t(\beta x) dx = 2^{-s-t-1} \alpha^{-(1/2)(s+t+\lambda)} \beta^{(s+t)} \frac{\Gamma[(s+t+\lambda)]}{\Gamma(s+1)\Gamma(t+1)}$$
$$\times {}_3F_3\left(\frac{s+t+1}{2}; \frac{s+t+1}{2}; \frac{s+t+\lambda}{2}; s+1, t+1, s+t+1; -\frac{\beta^2}{\alpha}\right), \quad (8)$$

where ${}_3F_3$ is the hypergeometric series. In our specific case where only $m=1$ interaction is considered, expression (8) simplifies and ${}_3F_3$ is replaced by ${}_2F_2$ hypergeometric function and ${}_1F_1$ confluent hypergeometric function. It may be noted that the generalized hypergeometric series[8] is given by the expression

$$pF_q(\alpha_1, \alpha_2 \cdots \alpha_p; \beta_1, \beta_2, \ldots, \beta_q; Z) = \sum_{r=0}^{\infty} \left[\prod_{h=1}^{p} (\alpha_h)_r Z^r \left(\prod_{h=1}^{q} (\beta_h)_r r! \right)^{-1} \right],$$

where

$$(a)_r = \Gamma(a+r)/\Gamma(a).$$

Using relations (6)–(8) we obtain the following expressions for the elements of the dielectric tensor of the plasma:

$$\epsilon_{xx} = 1 + \frac{2\omega_p^2}{\omega k_\parallel v_{t\parallel}} \left(\frac{A'}{\xi^2} \Lambda_1 - \frac{B'\xi^2}{8Lk_x} g_1 \right),$$

$$\epsilon_{xy} = \frac{2i\omega_p^2 Z_p}{\omega k_\parallel v_{t\parallel}} \left[A'\left(g_2 - \frac{\Lambda_1}{\xi^2}\right) - \frac{B'\xi^2}{4Lk_x} g_3 \right],$$

$$\epsilon_{xz} = \frac{2\omega_p^2}{\omega k_\parallel v_{t\perp}} \left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}} Z_p\right) \left(\frac{A'}{\xi} \Lambda_1 - \frac{B'}{8Lk_x} g_4 \right),$$

$$\epsilon_{yx} = \frac{-2i\omega_p^2}{\omega k_\parallel v_{t\parallel}} Z_p \left(A' g_5 + \frac{B'}{k_x L} g_6 \right),$$

$$\epsilon_{yy} = 1 + \frac{2\omega_p^2 Z_p}{\omega k_\parallel v_{t\parallel}} \left(A' g_7 - \frac{B'}{k_x L} g_8 \right),$$

$$\epsilon_{yz} = \frac{-2i\omega_p^2}{\omega k_\parallel v_{t\perp}} \left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}} Z_p\right) \left(A' g_9 - \frac{B'}{k_x L} g_{10} \right),$$

$$\epsilon_{zx} = \frac{2\omega_p^2 \Lambda_1}{\omega k_\parallel v_{t\perp} \xi} \left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}} Z_p\right) \left(A' - \frac{B'}{L k_x} \right),$$

$$\epsilon_{zy} = \frac{2i\omega_p^2}{\omega k_\parallel v_{t\perp}} \left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}} Z_p\right) \left[g_9 \left(A' - \frac{B'}{k_x L} \right) \right],$$

$$\epsilon_{zz} = 1 + \frac{2\omega_p^2 (\omega - \omega_{ce})}{\omega k_\parallel^2 v_{t\perp}^2} \left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}} Z_p\right) \times (A' - B' k_x L) \Lambda_1,$$

where

$$A' = 2\left(1 + \frac{v_{t\perp}^2}{v_{t\parallel}^2}\right) - \frac{k_x v_{t\perp}^2}{\omega \omega_{ce} L},$$

$$B' = 2\left(1 + \frac{v_{t\perp}^2}{v_{t\parallel}^2}\right),$$

$$g_1 = {}_1F_1(\tfrac{3}{2}; 2; -\xi^2) + \tfrac{1}{8} \xi^2 {}_2F_2(\tfrac{5}{2}; 3; 2; 5; -\xi^2),$$

$$g_2 = \tfrac{1}{2} {}_1F_1(\tfrac{3}{2}; 2; -\xi^2),$$

$$g_3 = {}_2F_2(\tfrac{3}{2}; 3; 2; 2; -\xi^2) + \tfrac{1}{8} \xi^2 {}_2F_2(2; \tfrac{5}{2}; 1; 4; -\xi^2) - \tfrac{1}{2} {}_1F_1(\tfrac{3}{2}; 2; -\xi^2) - \tfrac{1}{16} \xi^2 {}_2F_2(\tfrac{5}{2}; 3; 2; 5; -\xi^2),$$

$$g_4 = \xi^3 {}_1F_1(\tfrac{3}{2}; 2; -\xi^2) - \tfrac{1}{8} \xi^5 {}_2F_2(\tfrac{5}{2}; 3; 2; 5; -\xi^2),$$

$$g_5 = -(1/\xi^2)\Lambda_1 + \tfrac{1}{2} {}_1F_1(\tfrac{3}{2}; 2; -\xi^2),$$

$$g_6 = -(2/\xi^2)\Lambda_1 + \tfrac{1}{2} {}_1F_1(\tfrac{3}{2}; 2; -\xi^2),$$

$$g_7 = {}_2F_2(\tfrac{1}{2}; 2; 1; 1; -\xi^2) + \Lambda_1/\xi^2 - {}_1F_1(\tfrac{3}{2}; 2; -\xi^2),$$

$$g_8 = {}_2F_2(\tfrac{1}{2}; 2; 1; 1; -\xi^2) + 2\Lambda_1/\xi^2 - \tfrac{3}{2} {}_1F_1(\tfrac{3}{2}; 2; -\xi^2),$$

$$g_9 = \tfrac{1}{2}\xi \, {}_1F_1(\tfrac{3}{2}; 2; -\xi^2) - (1/\xi)\Lambda_1,$$

$$g_{10} = \tfrac{1}{2}\xi \, {}_1F_1(\tfrac{3}{2}; 2; -\xi^2) - (2/\xi)\Lambda_1,$$

$$\xi = \frac{k_x v_{t\perp}}{\omega_{ce}},$$

$$\Lambda_1 = \exp\left(-\frac{k_x^2 v_{t\perp}^2}{2\omega_{ce}^2}\right) I_1\left(-\frac{k_x^2 v_{t\perp}^2}{2\omega_{ce}^2}\right).$$

Having obtained the elements of the dielectric tensor,the dispersion relation for the plasma including electromagnetic effects can be written as

$$D_p = A\eta^4 + B\eta^2 + C = 0, \quad (9)$$

where $\eta = ck/\omega$ is the refractive index and the coefficients A, B, C are given by the following expressions[9]:

$$A = \epsilon_{xx} \sin^2\theta + \epsilon_{zz} \cos^2\theta + (\epsilon_{xz} + \epsilon_{zx}) \sin\theta \cos\theta - \frac{i}{(k_\parallel^2 + k_x^2)^{1/2}} \left(\sin\theta \frac{\partial \epsilon_{yx}}{\partial y} + \cos\theta \frac{\partial \epsilon_{yz}}{\partial y} \right);$$

$$\theta = \tan^{-1}\left(\frac{k_x}{k_\parallel}\right),$$

$$
\begin{aligned}
B = & -\epsilon_{xx}\epsilon_{zz} - (\epsilon_{yy}\epsilon_{zz} - \epsilon_{yz}\epsilon_{zy})\cos^2\theta + \epsilon_{xz}\epsilon_{zx} \\
& - (\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}\epsilon_{yx})\sin^2\theta \\
& + [\epsilon_{xy}\epsilon_{yz} + \epsilon_{yx}\epsilon_{zy} - \epsilon_{yy}(\epsilon_{xz} + \epsilon_{zx})]\sin\theta\cos\theta \\
& + \frac{i}{k_x}\biggl((\epsilon_{zz} + \epsilon_{yy}\sin^2\theta)\frac{\partial}{\partial y}\epsilon_{yx} \\
& + (\epsilon_{yy}\sin\theta\cos\theta - \epsilon_{zx})\frac{\partial}{\partial y}\epsilon_{yz} \\
& - (\epsilon_{yx}\sin^2\theta + \epsilon_{yz}\sin\theta\cos\theta)\frac{\partial}{\partial y}\epsilon_{yy}\biggr),
\end{aligned}
$$

$$
\begin{aligned}
C = & \epsilon_{zz}(\epsilon_{xx}\epsilon_{yy} - \epsilon_{xy}\epsilon_{yx}) - \epsilon_{xx}\epsilon_{yz}\epsilon_{zy} - \epsilon_{yy}\epsilon_{xz}\epsilon_{zx} \\
& + \epsilon_{xy}\epsilon_{yz}\epsilon_{zx} + \epsilon_{yx}\epsilon_{zy}\epsilon_{xz} \\
& + \frac{i}{k_x}\biggl((\epsilon_{yz}\epsilon_{zy} - \epsilon_{yy}\epsilon_{zz})\frac{\partial\epsilon_{yx}}{\partial y} \\
& + (\epsilon_{yy}\epsilon_{zx} - \epsilon_{yx}\epsilon_{zy})\frac{\partial\epsilon_{yz}}{\partial y} + (\epsilon_{zz}\epsilon_{yx} - \epsilon_{yz}\epsilon_{zx})\frac{\partial\epsilon_{yy}}{\partial y}\biggr).
\end{aligned}
$$

In the electrostatic limit the dispersion relation reduces to $A = 0$. For the helical beam with parallel velocity V_Z and longitudinal spread v_{tb} we assume distribution of the form

$$
f_b = n_b f_\perp(v_\perp)\left(\frac{1}{\pi v_{tb}}\right)^{1/2}\exp\left[-\left(\frac{v_\parallel - V_Z}{v_{tb}}\right)^2\right],
$$

where the transverse velocity distribution $f_\perp(v_\perp)$ is assumed to be peaked at some velocity $V_\perp$ and is of the form

$$
f_\perp(v_\perp) = \frac{1}{\pi V_\perp^2}\left(\frac{v_\perp}{V_\perp}\right)^2\exp\left(-\frac{v_\perp^2}{V_\perp^2}\right), \qquad (10)
$$

where n_b is the density of the beam. In the electrostatic approximation, the susceptibility of the beam can be written as[10]

$$
\begin{aligned}
\epsilon_b = & \frac{2\omega_b^2}{k^2 v_{tb}^2}\sum_{m=-\infty}^{\infty} S_m + \left(S_m\frac{\omega - k_\parallel V_z - m\omega_{ce}}{|k_\parallel| v_{tb}}\right. \\
& \left. + T_m\frac{m\omega_{ce}}{|k_\parallel| v_{tb}}\frac{k_x^2 v_{tb}^2}{2\omega_{ce}^2}\right) Z\left(\frac{\omega - k_\parallel V_z - m\omega_{ce}}{|k_\parallel| v_{tb}}\right), \qquad (11)
\end{aligned}
$$

where ω_b is the beam plasma frequency, S_m and T_m are given by expressions

$$
\begin{aligned}
S_m &= 2\pi\int_0^\infty v_\perp f_\perp J_m^2\left(\frac{k_x v_\perp}{\omega_{ce}}\right) dv_\perp, \\
T_m &= -2\pi\int_0^\infty \frac{\omega_{ce}^2}{k_\perp^2}\frac{\partial f_\perp}{\partial v_\perp} J_m^2\left(\frac{k_x v_\perp}{\omega_{ce}}\right) dv_\perp. \qquad (12)
\end{aligned}
$$

Since $m = 1$ interaction is of specific interest here, we retain only $m = 1$ in terms in (11). Substituting the value of $f_\perp$ from (10) into (12), we obtain the following expressions for S_1 and T_1:

$$
\begin{aligned}
S_1 &= \frac{V_\perp^2 k_x^2}{2\omega_{ce}^2}\,{}_1F_1\left(\frac{3}{2};2;-\frac{k_x^2 V_\perp^2}{\omega_{ce}^2}\right), \\
T_1 &= -\frac{1}{2}\left[{}_1F_1\left(\frac{3}{2};3;-\frac{k_x^2 V_\perp^2}{\omega_{ce}^2}\right)\right. \\
& \left. - 2\,{}_1F_1\left(\frac{3}{2};2;-\frac{k_x^2 V_\perp^2}{\omega_{ce}^2}\right)\right),
\end{aligned} \qquad (13)
$$

where ${}_1F_1$ is the confluent hypergeometric function.

The dispersion relation for the system in the electrostatic aproximation is considerably simplified if we assume $k_x^2 V_\perp^2/\omega_{ce}^2 \gg 1$ and $\xi \ll 1$. In the limit $k_x^2 V_\perp^2/\omega_{ce}^2 \gg 1$, the hypergeometric functions appearing in (13) reduce to

$$
{}_1F_1\left(\frac{3}{2};2;-\frac{k_x^2 V_\perp^2}{\omega_{ce}^2}\right) = \frac{1}{\Gamma(\frac{1}{2})}\left(\frac{k_x V_\perp}{\omega_{ce}}\right)^{3},
$$

and

$$
{}_1F_1\left(\frac{3}{2};3;-\frac{k_x^2 V_\perp^2}{\omega_{ce}^2}\right) = \frac{2}{\Gamma(\frac{1}{2})}\left(\frac{k_x v_\perp}{\omega_{ce}}\right)^{-3}. \qquad (14)
$$

Thus

$$
S_1 = \frac{1}{2\sqrt{\pi}}\frac{\omega_{ce}}{k_x V_\perp}, \qquad (15)
$$

and

$$
T_1 = 0.
$$

Retaining the terms up to ξ^2 only the dispersion relation in the above limits can then be written

$$
\begin{aligned}
D = & \frac{k_x^2}{k_\parallel^2}\left(\xi^2 + \frac{2\omega_p^2 Z_p}{\omega k_\parallel v_{t\parallel}}A'A_1\right) + \xi^2 + \frac{2\omega_p^2(\omega - \omega_{ce})}{k_\parallel^2 v_{t\perp}^2\omega} \\
& \times\xi^2 A_1\left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}}Z_p\right)\left(A' - \frac{B'}{Lk_x}\right) \\
& + \frac{2\omega_p^2 k_x A_1\xi}{k_\parallel^2 v_{t\perp}\omega}\left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}}Z_p\right)\left(2A' - \frac{B'}{Lk_x}\right) \\
& + \frac{2\omega_p^2 A'}{k_\parallel^2 v_{t\parallel}\omega L}\left[\frac{k_\perp}{k_\parallel}\left(-A_1 + \frac{\xi^2}{2}\,{}_1F_1\right)Z_p\right. \\
& \left. + \frac{v_{t\parallel}}{v_{t\perp}}\left(1 + \frac{\omega - \omega_{ce}}{k_\parallel v_{t\parallel}}Z_p\right)(-A_1\xi)\right] \\
& + \frac{2\omega_b^2 S_1}{(k_\parallel^2 + k_x^2)v_{tb}^2}\left[1 + \frac{\omega - k_\parallel V_z - \omega_{ce}}{k_\parallel v_{tb}}\right. \\
& \left. \times Z\left(\frac{\omega - k_\parallel V_Z - \omega_{ce}}{k_\parallel v_{tb}}\right)\right] = 0. \qquad (16)
\end{aligned}
$$

III. NUMERICAL ANALYSIS AND DISCUSSION

The solution of Eq. (16) is obtained for complex ω $(= \omega_r + i\omega_i)$ and real $k_\parallel$. The parallel velocity of the beam (V_Z) is taken opposite to the parallel wavenumber $(k_\parallel)$ of the wave. Figures 1(a) and 1(b) show the real and imaginary part of the frequency as a function of $ck_\parallel/\omega_{ce}$ with inhomogeneity as a parameter for $\omega_p/\omega_c = 5.0$, $\omega_b/\omega_c = 1.0$, $V_z/c = -0.05$, and $ck_x/\omega_{ce} = 100$. It may be noted that the frequencies, velocities, and wavenumbers have been normalized to electron cyclotron frequency, velocity of light, and ω_{ce}/c, respectively. In Figs. 1(a) and 1(b), the solid lines represent the complete solutions for $c/L\omega_{ce} = 20$. Growth occurs only on the dispersion branch labeled "A." The value of $ck_\parallel/\omega_{ce}$ corresponding to the maximum growth rate is found to be 11.2, which means that instabilities propagating at large angles $\theta \simeq \tan^{-1}(k_x/k_\parallel) \approx 81.8°$ with respect to the

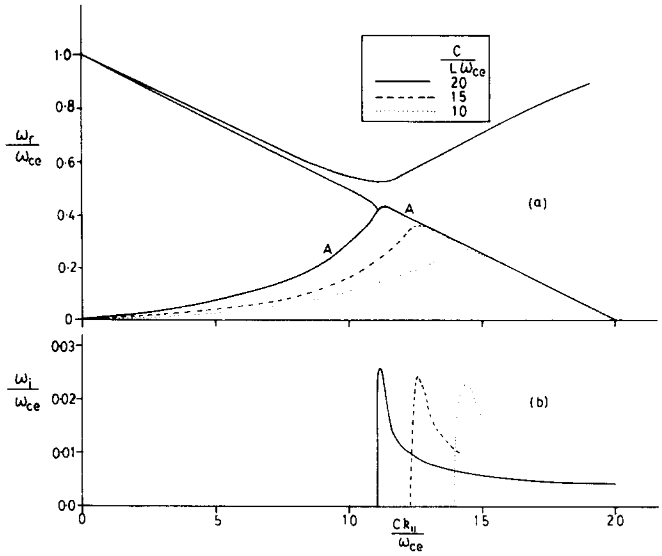

FIG. 1. Whistler instability excitation due to normal Doppler effect interaction with a helical beam. (a) The frequency ω_r/ω_c and (b) the growth rate ω_i/ω_{ce} as a function of parallel wavenumber $ck_{\parallel}/\omega_{ce}$ with inhomogeneity as a parameter $(C/L\omega_{ce})$ for the parameters: $\omega_p/\omega_c = 5.0$, $\omega_b/\omega_c = 1.0$, $V_z/c = -0.05$, $V_{\perp}/c = 0.025$, $ck_x/\omega_{ce} = 100$, $V_{tb}/c = 0.0001$, $v_{t\parallel} \approx v_{t\perp} \approx 0.0001$.

magnetic field can be excited by a helical beam. The solutions for $c/L\omega_c = 10$, 15 are shown by dotted and dashed lines. It is seen that the frequency and growth rate of the instability increases with $c/L\omega_{ce}$.

It is also noted [not shown in Figs. 1(a) and 1(b)] that an increase in beam thermal spread drastically reduces the growth rate of the instability as found earlier in the case of a homogeneous plasma penetrated by a helical beam.[3]

To summarize we have considered a helical electron beam interacting with an inhomogeneous plasma. The inhomogeneity appears through the number density of the plasma electrons. In the electrostatic limit it is shown that it is possible to excite oblique whistler instabilities propagating at large angles with respect to the magnetic field. It is found that the effect of inhomogeneity is to increase the frequency and the growth rate of the instability. The results of the above analysis are of special interest in relation to the excitation of whistler mode waves in the earth's magnetosphere where inhomogeneous plasma penetrated by charged particles with helical beam type distribution may well exist.

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