PARTICLE INTERACTIONS WITH ALFVÉN SOLITONS

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Abstract. We compute velocity and the corresponding energy changes due to non-resonant interactions of protons with Alfvén solitons. It is seen that the protons heat in the perpendicular direction but associated with this is a cooling in the parallel direction.

1. Introduction

Observations of proton distributions and temperatures in high-speed solar wind streams have shown that the proton distributions exhibit various degrees of anisotropy (Bame *et al.*, 1975; Goodrich and Lazarus, 1976; Marsch *et al.*, 1982a, b). In these papers it was also shown that core protons in fast speed solar wind streams have an anisotropy of the form $T_\perp / T_\parallel \sim 2$ (the subscripts $\parallel$ and $\perp$ refer to the parallel and perpendicular directions with respect to the background magnetic field).

Since high-speed streams appear to be dominated to a large extent by parallel propagating Alfvén waves (Belcher and Davis, 1971), it is possible that these waves may be responsible for local proton heating in the perpendicular direction. Schwartz *et al.* (1981) found that due to cyclotron damping of Alfvén waves the energy converted to perpendicular proton heating was insufficient to explain the observed proton anisotropy. In this paper we consider non-resonant proton interactions with Alfén solitons postulating that this could be possible way of explaining the observed results for the anisotropy of proton temperatures.

In a previous paper (Ovendon *et al.*, 1983) (hereafter referred to as Paper I) we showed that left hand circularly polarized Alfvén waves are modulationally unstable and this instability could lead to soliton formation. Soliton solutions were obtained via Zakharov's equations, and using these as elementary building blocks of turbulence it was shown that a good qualitative agreement could be obtained between this theory and observational results, for mass density, velocity, magnetic field fluctuations and the shape and radial evolution of the power spectrum. Below we summarize some of the results of Paper I which would be used in the present work. The self consistent solution of Zakharov's equations for $\omega/\Omega_p \ll 1$ (where ω is the frequency of propagation of the wave and Ω_p is the gyrofrequency of the protons) showed that the magnetic field

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fluctuations normalized to the background magnetic field are given by

$$b(z, t) = b_0 \operatorname{sech}[K(z - c_g t)]\, e^{i\delta\omega t}, \tag{1}$$

where K is the inverse width of the soliton, $\delta\omega$ is the nonlinear frequency shift, and c_g is the group velocy of the propagating solitary wave. Expressions for K, $\delta\omega$, and c_g are

$$K = \frac{k_A b_0}{2} \left[\frac{\Omega_p/\omega_A}{1 - \beta}\right]^{1/2},$$

$$\delta\omega = -\frac{\omega_A |b_0|^2}{8(1 - \beta)},$$

$$c_g = v_A(1 - \omega_A/\Omega_p), \tag{2}$$

respectively. Here $\beta = c_s^2/v_A^2$ and v_A is the Alfvén speed, c_s is the velocity of sound and $\omega_A = k_A v_A$, where k_A is the wave number associated with the Alfvén wave. The mass density (normalized to its background value) and parallel velocity fluctuations are given by

$$\delta\rho = \frac{|b|^2}{2(1 - \beta)},$$

$$\delta v = v_A \delta\rho. \tag{3}$$

In the present paper we consider particle (proton) interactions with a turbulent state consisting of solitons. Only non-resonant interactions are taken into account. In the following sections parallel and perpendicular velocity are computed and using phase averaged values of these, corresponding energy changes are found. Such interactions could lead to the observed temperature anisotropy in high-speed solar wind streams.

2. Mathematical Formulation

The equation of motion of a particle of mass m and charge q in electric and magnetic fields is given by

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\left[\mathbf{E}' + \frac{1}{c}\,\mathbf{v} \times \mathbf{B}'\right], \tag{4}$$

where $\mathbf{E}'$ and $\mathbf{B}'$ are related to one another via Maxwell's equation

$$\nabla \times \mathbf{E}' = -\frac{1}{c}\frac{\partial \mathbf{B}'}{\partial t}. \tag{5}$$

We assume that $\mathbf{E}'$ and $\mathbf{B}'$ are set up by a localized Alfvén wave (soliton) propagating along the background magnetic field $\mathbf{B}_0$ which is taken along the z-axis. From

Equation (5) we have (see Paper I)

$$E'_x = \frac{v_A}{c} B'_y ,$$

$$E'_y = -\frac{v_A}{c} B'_x .$$

This allows us to write Equation (4) as two equations, namely

$$\frac{dv_+}{dt} = i\Omega_p[(v_z - v_A)B_+ - v_+] , \tag{6}$$

$$\frac{dv_z}{dt} = i\frac{\Omega_p}{2} (v_+ B_+^* - v_+^* B_+) , \tag{7}$$

where $\Omega_p = qB_0/mc$ is the gyrofrequency of the proton, $v_+ = v_x + iv_y$ and $B_+ = B_x + iB_y$ $(B_{x,y} = B'_{x,y}/B_0)$, where v_+ and B_+ correspond to the left-hand polarized wave and * is the complex conjugate. We now let

$$v_+ = v_\perp(z, t)\, e^{-i(\Omega_p t + \phi_0)} ,$$

where ϕ_0 is a constant phase factor containing information about the initial conditions. We further divide $v_\perp$ into its real and imaginary parts

$$v_\perp = u + iw .$$

Thus Equation (6) becomes

$$\frac{du}{dt} = i\frac{\Omega_p}{2} (v_z - v_A)B_+ e^{i(\Omega_p t + \phi_0)} + \text{c.c.} , \tag{8}$$

$$\frac{dw}{dt} = \frac{\Omega_p}{2} (v_z - v_A)B_+ e^{i(\Omega_p t + \phi_0)} + \text{c.c.} \tag{9}$$

Here c.c. is the complex conjugate part. Equations (8) and (9) govern the evolution of the amplitude of the perpendicular particle velocity. For B_+ we use the solution found in Paper I, i.e.,

$$B_+ = b\, e^{i(kz - \omega_A t)} , \tag{10}$$

where b is given by Equation (1). In all further calculations we make use of the fact that ω/Ω_p is a small quantity and thus neglect the nonlinear frequency shift and approximate the group velocity c_g by v_A. This is done in view of the fact that small alterations brought about by including terms of the order ω/Ω_p would only hinder the calculations without greatly affecting the results. Thus Equation (1) in this level of approximation becomes

$$b = b_0 \operatorname{sech}[K(z - v_A t)] .$$

We substitute Equation (10) into Equations (8), (9), and (7) to obtain

$$\frac{\mathrm{d}u}{\mathrm{d}t} = i\,\frac{\lambda K}{\pi}\, v\,\mathrm{sech}\,Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.}\,, \tag{11}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{\lambda K}{\pi}\, v\,\mathrm{sech}\,Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.}\,, \tag{12}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -i\,\frac{\lambda K}{\pi}\,(u - iw)\,\mathrm{sech}\,Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.}\,, \tag{13}$$

where we have shifted to a frame of reference in which the field is at rest, i.e., $v = v_z - v_A$, and $z(t) = v(t - t_0)$, and have taken

$$\frac{\lambda K}{\pi} = \frac{\Omega_p b_0}{2}\,.$$

3. Particle Velocity Changes

Equations (11)–(13) are coupled (via the trajectory $z(t)$) integro-differential equations and in general such equations cannot be solved exactly. Thus if we wish to remain within the analytical frame work some approximate method has to be used in order to solve these equations. We make use here, of the 'Born Approximation', which is a method of handling integral equations by use of successive approximations and is essentially equivalent with Neumann's series expansion. The essence of this approach is to consider the particle and fields separate and to integrate the equations of motion of a particle in the given field assuming that the resulting changes in the velocity are small compared to the initial velocity (for more details see for, e.g., Roman, 1965). In the context of plasma physics this approximation has been used to solve similar problems (see below).

The Born approximation was used by Berger *et al.* (1958) to discuss transit time field dumping whilst dealing with the problem of heating of confined plasmas by oscillating electromagnetic fields. In a later paper Morales and Lee (1974) used the same approximation to evaluate the effect of localized fields on the evolution of electron distribution function, in an unmagnetized plasma. The localized fields oscillated with a frequency comparable to electron plasma frequency. Morales and Lee (1974) compared the analytic results of the Born approximation with results of numerical integration of the equations of motion and found close agreement between the two. In this paper (Morales and Lee, 1974) pointed out that what was most important was the 'phase averaged velocity kick $\langle \Delta V \rangle$ imparted to the particles by the localized fields'. Those authors also noted that the effect of particles spending several periods inside the localized field was not taken into account by the Born approximation, and in fact in this level of approximation was not necessary.

Thus in view of the above mentioned it should be borne in mind that such an approach neglects the coupling of the field to the particles in the system of Maxwell–Vlasov equations. Consequently, wherever such an approximation is used it should be seen as an indication rather than as an accurate solution of the problem.

In order to solve Equations (11)–(13) we begin by expanding the velocities about the unperturbed velocity in the general form,

$$a = a_0 + \Delta a^{(1)} + \Delta a^{(2)},$$

and further in accordance with Born's approximation we assume that $a_0 \gg \Delta a^{(1)} \gg \Delta a^{(2)}$. This allows us to use as first approximation the unperturbed trajectory,

$$z(t) = v_0(t - t_0),$$

where v_0 and t_0 are constants. Substituting the above in the set of Equations (11)–(13) we get for example for

$$\Delta u^{(1)} = i \int_{\infty}^{\infty} \frac{\lambda}{v_0} (v_0 + \Delta v^{(1)}) \operatorname{sech} \pi\tau \exp i(\Omega_p + kv_0) \frac{\pi\tau}{Kv_0} e^{i\phi} \,\mathrm{d}\tau + \text{c.c.},$$

where $\pi\tau = Kz(t) = Kv_0(t - t_0)$ and now we take $1 + \Delta(v^{(1)}/v_0) \approx 1$ (which is the first Born approximation), and $\phi = \phi_0 + \Omega_p t_0$ is the phase factor.

Similar expressions are obtained for $\Delta w^{(1)}$ and $\Delta v^{(1)}$ and integration over the whole range $-\infty, \infty$ yields

$$\Delta u^{(1)} = i\lambda \operatorname{sech}\left\{\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right\} e^{i\phi} + \text{c.c.}, \tag{14}$$

$$\Delta w^{(1)} = \lambda \operatorname{sech}\left\{\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right\} e^{i\phi} + \text{c.c.}, \tag{15}$$

$$\Delta v^{(1)} = -\frac{(u_0 - iw_0)}{v_0} \Delta u^{(1)}. \tag{16}$$

Thus we see that in the first approximation velocity changes take place but if we phase average equations (14)–(16) the net result is zero. This is to be expected from an unperturbed orbit approximation since, for any particle with initial conditions such that it is accelerated by a field we find another particle whose initial conditions would produce the opposite results – the net effect being zero. This is not so when higher order perturbed orbit velocity changes are included. Firstly we note that the phase averages of the squares of the quantities in Equations (14)–(16) are given by (which are second-order quantities)

$$\langle(\Delta u^{(1)})^2\rangle_\phi = 2\lambda^2 \operatorname{sech}^2\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right],$$

$$\langle(\Delta w^{(1)})^2\rangle_\phi = \langle(\Delta u^{(1)})^2\rangle_\phi, \tag{17}$$

$$\langle(\Delta v^{(1)})^2\rangle_\phi = \frac{(u_0^2 + w_0^2)}{v_0^2}\langle(\Delta u^{(1)})^2\rangle_\phi .$$

We now proceed to compute the second-order velocity changes taking into account perturbed orbits for the trajectory. From Equations (11)–(13) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Delta u^{(1)} + \Delta u^{(2)}) = \frac{i\lambda K}{\pi} v_0 \operatorname{sech} Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.} + + \frac{i\lambda K}{\pi}\Delta v^{(1)} \operatorname{sech} kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.}, \tag{18}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Delta w^{(1)} + \Delta w^{(2)}) = \frac{\lambda K v_0}{\pi} \operatorname{sech} Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.} + + \frac{\lambda K}{\pi}\Delta v^{(1)} \operatorname{sech} Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.}, \tag{19}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\Delta v^{(1)} + \Delta v^{(2)}) = -i\frac{\lambda K}{\pi}(u_0 - iw_0) \operatorname{sech} Kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.} - - i\frac{\lambda K}{\pi}(\Delta u^{(1)} - i\Delta w^{(1)}) \operatorname{sech} kz(t)\, e^{i(\Omega_p t + kz(t) + \phi_0)} + \text{c.c.} \tag{20}$$

We first consider the second pair of terms on the right-hand sides of Equations (18)–(20). For these terms we use the unperturbed trajectory $z(t) = v_0(t - t_0)$, which is valid for the present level of approximation. It can be easily shown that for the second pair of terms on the right-hand side of Equation (18) we have, using Equation (16), the following relationship:

$$i\frac{\lambda K}{\pi}\Delta v^{(1)} \operatorname{sech} Kv_0(t - t_0)\, e^{i(\Omega_p t + Kv_0(t - t_0) + \phi_0)} + \text{c.c.} = = -\frac{1}{(u_0 - iw_0)}\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{(\Delta v^{(1)})^2}{2}\right\}. \tag{21}$$

Similarly it can be shown that the second pair of terms on the right-hand side of

Equation (20), by using Equations (14)–(16), reduce to

$$-i\,\frac{\lambda K}{\pi}\,(\Delta u^{(1)} - i\Delta w^{(1)})\,\mathrm{sech}\,Kv_0(t - t_0)\,e^{i(\Omega_p t + kv_0(t-t_0)+\phi_0)} + \text{c.c.} =$$

$$= -\frac{1}{2v_0}\frac{\mathrm{d}}{\mathrm{d}t}\,((\Delta w^{(1)})^2 + (\Delta u^{(1)})^2)\,. \tag{22}$$

We do not consider Equation (19) as it will be shown later that it is not used in latter calculations.

We now consider the first pair of terms in Equations (18) and (20), noting that here it is necessary to take into account the perturbed trajectory:

$$z(t) = v_0(t - t_0) + \Delta z^{(1)}(t)\,, \tag{23}$$

where

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\Delta z^{(1)} = \Delta v^{(1)}\,.$$

The first pair of terms on the right-hand side of Equation (18) is

$$i\,\frac{\lambda K}{\pi}\,v_0\,\mathrm{sech}\,K[v_0(t - t_0) + \Delta z^{(1)}(t)]\,e^{i[\Omega_p t + k(v_0(t-t_0)+\Delta z^{(1)})]} + \text{c.c.}$$

By a change of variables to

$$K[v_0(t - t_0) + \Delta z^{(1)}] = \pi\tau\,,$$

we can integrate the above expression and after phase averaging obtain

$$2\lambda^2\,\frac{u_0}{v_0^2}\,\mathrm{sech}^2\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right]. \tag{24}$$

Here $\Delta V^{(1)}$ has been neglected in the sech terms.

Similarly we get from the first pair of terms of Equation (20) (after phase averaging)

$$-i\,\frac{\lambda K}{\pi}\,(u_0 - iw_0)\,\mathrm{sech}\,[K(v_0(t - t_0) + \Delta z^{(1)}(t)]\,e^{i(\Omega_p t + kv_0(t-t_0)+\Delta z^{(1)}+\phi_0)} + \text{c.c.} =$$

$$= -2\lambda^2\,\frac{(u_0^2 + w_0^2)}{v_0^3}\,\mathrm{sech}^2\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right]. \tag{25}$$

Now combining Equations (21) and (22) after phase averaging and integration with Equations (24) and (25), respectively, we obtain expressions for second-order velocity changes $\langle \Delta u^{(2)} \rangle$ and $\langle \Delta v^{(2)} \rangle$ given by

$$\langle \Delta u^{(2)} \rangle_\phi = 2\lambda^2 \frac{u_0}{v_0^2} \operatorname{sech}\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right] - \frac{1}{2(u_0 - iw_0)} \langle (\Delta v^{(1)})^2 \rangle_\phi , \quad (26)$$

$$\langle \Delta v^{(2)} \rangle_\phi = -2\lambda^2 \frac{(u_0^2 + w_0^2)}{v_0^3} \operatorname{sech}^2\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right] - $$

$$- \frac{1}{2v_0} \{ \langle (\Delta w^{(1)})^2 \rangle_\phi + \langle (\Delta u^{(1)})^2 \rangle_\phi \} . \quad (27)$$

4. Energy Changes

From the results obtained in the preceding section we can now compute the gain or loss of energy of the protons in both the perpendicular and parallel directions, due to a single-proton soliton interaction.

We first calculate the energy change in the perpendicular direction. Without loss of generality we set $w_0 = 0$, thus the phase-averaged energy change is given by

$$\langle \Delta \varepsilon_\perp \rangle_\phi = \tfrac{1}{2} m [2u_0 \langle \Delta u^{(2)} \rangle_\phi + \langle (\Delta u^{(1)})^2 \rangle_\phi + \langle (\Delta w^{(1)})^2 \rangle_\phi] ;$$

using Equations (17) and (26) in the above expression we obtain

$$\langle \Delta \varepsilon_\perp \rangle_\phi = m\lambda^2 \left(2 + \frac{u_0^2}{v_0^2}\right) \operatorname{sech}^2\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right] . \quad (28)$$

Similarly for the parallel direction we get

$$\langle \Delta \varepsilon_\parallel \rangle_\phi = -m\lambda^2 \left(2 + \frac{u_0^2}{v_0^2}\right) \operatorname{sech}^2\left[\frac{\pi}{2}\left(\frac{\Omega_p + kv_0}{Kv_0}\right)\right] . \quad (29)$$

From the above two expressions we see that the proton gains energy in the perpendicular direction from the soliton field thereby increasing its perpendicular temperature. On the other hand the proton lowers its parallel temperature by losing energy to the field. The soliton field provides a mechanism for the transfer of energy from the parallel to the perpendicular direction. The loss and gain in the parallel and perpendicular directions are equal. The result obtained here is qualitatively similar to the one obtained by Arunasalam (1976) for the case of ion cyclotron resonance heating, where it was shown, using quasi-linear theory that with the heating in the perpendiculat direction is associated cooling in the parallel direction.

If we know the number of solitons per unit length (N/cm) then the interaction rate between protons and Alfvén solitons can be written as $N |v_0|$. If the background distribution for the protons is known (or can be approximated by a shifted bi-Maxwellian in the case of the solar wind) then the rate of gain of energy in the perpendicular direction will be given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(nk_B T_\perp) = 2\pi \int_{-\infty}^{\infty}\int_0^{\infty} \mathrm{d}v_z\, \mathrm{d}u_0^2 \langle \Delta\varepsilon_\perp \rangle_\phi f_0 N |v_0| ,$$

where n is the number density and $T_\perp$ is the perpendicular temperature of the protons. A similar expression can be written for the rate of loss of energy in the parallel direction.

Such a transfer of energy could perhaps explain the observed proton temperature anistropy in fast speed solar wind streams. The results of the paper by Schwartz *et al.* (1981), where cyclotron resonance was used, showed enough energy could not be transferred in the perpendicular direction, since at higher frequencies (near the proton gyrofrequency) only a small amount of energy could be transferred to the protons. Non-resonant interactions taking place essentially at low frequency ($\omega/\Omega_p \ll 1$), where more energy is contained, could perhaps be better candidates for accounting for the above mentioned anisotropy. A knowledge of the interaction rate $N\,|v_0|$ would, however, be needed to test the validity of this model – observations till now present no information on this.

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