Coupled nonlinear helicon-acoustic waves in a semiconductor piezoelectric plasma – via a new type of KdV equation

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For a semiconductor piezoelectric plasma coupled helicon-acoustic waves are investigated by using the reductive perturbation method. The equation governing the nonlinear coupled wave is a mixed modified KdV and BBM equation along with an additional term. A one-soliton solution of this equation is obtained.

The propagation of nonlinear waves in piezoelectric semiconductor plasmas has been studied extensively in recent years [1–5]. In the above mentioned papers Brillouin and Raman scattering have been studied as well as modulational instability via the nonlinear Schrödinger equation, and the Benjamin–Bona–Mahoney equation has been obtained in the investigation of coupled electron-acoustic waves in such plasmas.

We consider it an n-type piezoelectric semiconductor plasma in order to investigate the nonlinear behaviour of coupled helicon-acoustic waves. The basic equations necessary for carrying out this analysis for the one-dimensional case are the following [5,6],

$$\partial_t n + \partial_z n v_z = 0 , \tag{1}$$

$$\partial_t v_\pm + v_z \partial_z v_\pm = -\frac{e}{m} E_\pm \mp \mathrm{i} \frac{e}{m} (v_z B_\pm - \omega_c v_\pm) , \tag{2}$$

$$\partial_t v_z + v_z \partial_z v_z = -\frac{e}{m} (v_x B_y - v_y B_x) - \frac{v_T^2}{n} \partial_z n , \tag{3}$$

$$(\rho \partial_{tt} - c_e \partial_{zz}) u_\mp = \beta \partial_z E_\pm , \tag{4}$$

$$\partial_z E_\pm = \pm \mathrm{i} \partial_t B_\pm , \tag{5}$$

$$\partial_{zz} E_\pm = \mu_0 \partial_{tt} D_\pm + \mu_0 \partial_t j_\pm , \tag{6}$$

$$D_\pm = \epsilon' E_\pm - \beta \partial_z u_\mp , \tag{7}$$

$$j_\pm = -n e v_\pm . \tag{8}$$

Eq. (1) is the electron continuity equation and eqs. (2), (3) are the electronic equations of motion in the perpendicular and parallel directions respectively. Eq. (4) is the lattice equation of motion and eqs. (5)–(8) are Maxwell's equations. We note here that since helicons are circularly polarized waves propagating parallel to the ambient magnetic field the fluctuating quantities in the perpendicular directions have all been expressed in the form $a_\pm = a_x \pm \mathrm{i} a_y$, where $\pm$ corresponds to the left and right handed circularly polarized waves respectively. The quantities n, v, u, E and B represent the electronic number density, the electron velocity, the lattice displacement and the electric and magnetic field intensities respectively. We further note that ρ, β, ϵ' and μ_0 are the lattice ion mass density, the piezoelectric coupling constant, the dielectric tensor and the magnetic susceptibility respectively. Finally ω_c and v_T are the electron gyrofrequency and the electron thermal velocity respectively. In order to apply the reductive perturbation method we expand in the following manner,

$$n = n_0 + \epsilon^2 n_1 + \ldots , \tag{9a}$$

$$v_z = \epsilon^2 v_{z1} + \dots . \tag{9b}$$

Here n_0 is the background number density which is taken to be homogeneous and constant. All quantities with subscripts $\pm$ are expanded in the following way,

$$a_\pm = \epsilon a_{\pm 1} + \epsilon^3 a_{\pm 2} + \dots . \tag{10}$$

We note here that in eq. (9) the leading order terms for fluctuations in the number density and the parallel velocity are of order ϵ^2, this is because for helicon waves fluctuations in the above mentioned quantities do not contribute to the linear dispersion relation. We further note that for quantities which have subscripts $\pm$ the lowest order (i.e. order ϵ) determines the linear dispersion relation for helicons. Further we introduce stretched coordinates:

$$\zeta = z - \lambda t, \quad \tau = \epsilon^2 t . \tag{11}$$

Here λ is a velocity parameter which is determined later. We note that this ordering is the same as that used in ref. [7] for obtaining the modified KdV equation for Alfvén waves. We now substitute the ordering scheme given by expressions (9)–(11) in the set of equations (1)–(8) and collect terms in different orders of ϵ. In lowest order, i.e. in order ϵ, we obtain the linear dispersion for circularly polarized coupled helicon-acoustic waves. We assume that the first order fluctuating quantities a_{+1} are proportional to $\exp(ik\zeta)$ and obtain the expression

$$(\rho\lambda^2 - c_e)[(\lambda^2 - c^2)(\lambda \pm \omega_c/k) - \omega_p^2 \lambda/k] = \beta^2 \lambda^2 \omega_c/\epsilon' k . \tag{12}$$

In expression (12) $\lambda = \omega/k$ is the phase velocity of the coupled helicon-acoustic wave.

From terms of order ϵ^2 we obtain an expression relating v_{z1} and $v_{\pm 1}$. We obtain

$$\partial_\zeta v_{z1} = \lambda/[2(\lambda^2 - v_T^2)]\partial_\zeta |v|^2 , \tag{13}$$

where $|v|^2 = v_x^2 + v_y^2$.

Finally in order ϵ^3 we obtain the equation for the evolution of the amplitude of the coupled helicon-acoustic wave. In obtaining this equation the compatibility and secularity conditions have been used [8–10]. This equation has the form

$$\partial_\tau v_\pm + \alpha_1 \partial_{\tau\zeta\zeta} v_\pm + \alpha_2 \partial_\zeta |v|^2 v_\pm + i\alpha_3 \partial_{\zeta\zeta} |v|^2 v_\pm = 0 . \tag{14}$$

The coefficients α_1, α_2 and α_3 are given by the following expressions respectively,

$$\alpha_1 = -\frac{[(\lambda^2 - c^2)(\rho\lambda^2 - c_e) - \lambda^2\beta^2/\epsilon']^2}{2\lambda^2\omega_p^2(\rho\lambda^2 - c_e + \beta^2 c_e/\epsilon')} ,$$

$$\alpha_2 = (\rho\lambda^2 - c_e)\frac{(\lambda^2 - c^2)(\rho\lambda^2 - c_e) - \lambda^2\beta^2/\epsilon'}{2\lambda(\rho\lambda^2 - c_e + \beta^2 c_e/\epsilon')(\lambda^2 - v_T^2)} ,$$

$$\alpha_3 = \omega_c \alpha_1/\lambda , \tag{15}$$

where ω_p is the electron plasma frequency and $c = (\epsilon'\mu_0)^{-1/2}$ is the velocity of light in the piezoelectric semiconductor.

It can be seen that eq. (13) resembles the modified KdV and Benjamin–Bona–Mahoney (BBM) equation [10,11]. The BBM equation is considered to be more appropriate in describing the nonlinear evolution of long wavelength waves, it is also referred to as the regularized long wave equation. Had the last term on the left hand side been missing the equation could probably have been called a "modified BBM equation" and it has been checked that a solution of the form $A_0 \operatorname{sech} \kappa\eta$ fits the equation. However, the presence of the last term alters this situation and below we have attempted to write a stationary one-soliton solution of eq. (14). Following ref. [7] we move to a frame of reference moving with the wave. This is done by changing variables: $\eta = \zeta - \mu\tau$ where μ is the arbitrary velocity with which the solitary wave propagates. Substituting this in eq. (14) we can integrate once. Further we separate into real and imaginary parts by taking

$$v_\pm = A(\eta)\exp(\pm i\phi) .$$

We finally obtain from the real and imaginary parts

$$d_{\eta\eta} A + \beta_1 A + \beta_2 A^3 \beta_3 A^5 = 0 , \tag{16}$$

$$d_\eta \phi = \gamma A^2 , \tag{17}$$

where

$$\beta_1 = 1/\alpha_1 ,$$

$$\beta_2 = (\rho\lambda^2 - c_e)\omega_p^2\lambda\,(\{2\mu[(\lambda^2 - c^2)(\rho\lambda^2 - c_e) - \lambda^2\beta^2/\epsilon']\}(\lambda^2 - v_T^2))^{-1} ,$$

$$\beta_3 = \tfrac{3}{4}[\omega_c/(\lambda^2 - v_T^2)\mu]^2 ,$$

$$\gamma = 2\omega_c/\mu(\lambda^2 - v_T^2) .$$

In order to find a solution to eqs. (16) and (17) we have tried the solution of the generalized KdV equation [12] which is of the form

$$A=A_0(\lambda_0+\cosh\kappa\eta)^{-n}\,, \qquad (18)$$

where A_0, λ_0, κ and n are determined below after putting eq. (18) into eq. (16). We take $n=\frac{1}{2}$ and collect terms in different orders of $\cosh\kappa\eta$ and after some algebra we obtain the following expressions,

$$\kappa=\lambda\omega_p\frac{[2(\rho\lambda^2-c_e+\beta^2c_e/\epsilon')]^{1/2}}{(\lambda^2-c^2)(\rho\lambda^2-c_e)-\lambda^2\beta^2/\epsilon'}\,,$$

$$A_0=\frac{2}{\omega_p}\left(\frac{2\mu(\lambda^2-v_T^2)}{\rho\lambda^2-c_e}\right)^{1/2}\times\left[1+\frac{2\omega_c^2}{\omega_p^2}\left(1+\frac{\beta^2c_e}{\epsilon'(\rho\lambda^2-c_e)}\right)\right]^{-1/4}\,,$$

$$\lambda_0=-\left(1+\frac{\omega_c^2(\rho\lambda^2-c_e+\beta^2c_e/\epsilon')}{(\rho\lambda^2-c_e)^2\omega_p^2}\right)^{-1/2}\,, \qquad (19)$$

Now from eqs. (17)–(19) after integration we get the following expression for ϕ,

$$\phi=\frac{3\omega_cA_0^2}{8\mu(\lambda^2-v_T^2)}\int\frac{d\eta}{\lambda_0+\cosh\kappa\eta}\,. \qquad (20)$$

Here we note that $\lambda_0+\cosh\kappa\eta\neq0$. Thus the solution for the nonlinear equation (eq. (14)) can be written as

$$v_\pm=A_0(\lambda_0+\cosh\kappa\eta)^{-1/2}\exp(\pm i\phi)\,, \qquad (21)$$

where κ, A_0 and λ_0 are given by expressions (19) and ϕ by expression (20).

In conclusion we note that this model of considering nonlinear coupled helicon-acoustic waves was a simplified one since collisions between the carriers and the lattice were neglected. If, however, these collisions are included then perhaps an externally applied electric field has also to be included in the calculations – this would counter balance the wave form damping out immediately (due to collisions). The inclusion of collisions and the external electric field would of course lead to the right hand side of eq. (14) to be nonzero. This in turn would imply that an exact (one-soliton) solution would not exist, and a perturbative technique would have to be used to give an approximate solution. We hope to report on this investigation soon elsewhere.

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