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1993 Phys. Scr. 47 209

(http://iopscience.iop.org/1402-4896/47/2/014)

Physica Scripta. Vol. 47, 209–213, 1993.

Stationary Shock-Front of a Relativistically Strong Electromagnetic Radiation in an Underdense Plasma

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Received April 27, 1992; accepted August 24, 1992

Abstract

The propagation of a relativistically strong electromagnetic pulse in an underdense homogeneous plasma is considered with the focus on the possible existence of stationary structure of a pulse front. The analytical stationary shock-like solutions are obtained and analyzed. These solutions correspond to the conversion of the pulse electromagnetic energy to an electron plasma wave in the narrow region of the considered stationary front. Our theoretical analysis is supported by the PIC simulations, which demonstrate the formation and existence of the shock-like structure of the pulse leading front and extremely fast pulse depletion.

1. Introduction

The recent progress in producing extremely powerful laser pulses [1, 2] has prompted researches into investigations of relativistically strong laser pulse interaction with plasmas. For a pulse of such high intensity the kinetic energy of oscillating electrons can exceed their rest energy mc^2. Thus, the interaction has to be treated as strongly nonlinear. In fact, in the ultrarelativistic limit only the pulse edges can be considered as regions of strongly nonlinear interaction. It is so, since inside the main body of the pulse the interaction is suppressed as a result of relativistic electron mass growth and this part of the laser pulse propagates just as in a vacuum.

Therefore, the matter of primary interest is to study the interaction on the pulse edges. The polar cases of this study are the case of sharp pulse edges and the case of the pulse with adiabatically slow growth and decrease of amplitude. The first case was studied in Refs [3, 4] where it was shown that the leading edge of the pulse excites an intense electron plasma wave. The shape of the rear front of the pulse with the sharp leading edge appears to be of minor importance for the plasma wave excitation. The smooth pulses were considered in Refs [5, 6]. It was shown there that the leading part of the pulse undergoes steepening as a result of various nonlinear processes. For example, for a sufficiently long pulse the leading front steepening is driven by the intense stimulated backward Raman scattering. Once the steepening has appeared the excitation of the plasma wake is triggered off and we again return to the case of a sharp leading edge.

We assume here that the leading edge of the pulse is a stationary one. Neglecting the interaction inside the main body of the pulse we shall study the structure of the pulse leading edge. Previously there were several studies of envelope shock-fronts (for example in Ref. [7]), but all of them addressed primarily the weakly relativistic case while the subject of our interest is the pulses of ultrarelativistic intensity. In the present paper the analytical theory of strong envelope shock-like fronts is developed. Computer simulations are carried out to demonstrate the formation of a shock-front and its further propagation.

2. Mathematical formulation

Due to the ultra fast nature of the process under consideration, the motion of the ions can be neglected, and these only form an immobile neutralizing background. We shall look for the solution of the fully relativistic nonlinear electron fluid equations along with Maxwell's equations to describe the pulse front motion. We consider the one-dimensional case and circularly polarized radiation. The potential of charge separation ϕ (normalized to mc^2) is taken as a variable characterizing the low frequency plasma motion. For the description of the high frequency field we take the transverse electron momentum $\boldsymbol{q}_{\rm tr}$ (normalized to mc). This is natural as in one-dimensional case this momentum is exactly equal to the normalized vector potential of the pulse field $eA_\perp/mc^2$. Since we are looking for the stationary solution for the pulse front propagating at the velocity V, we assume the following form of $\boldsymbol{q}_{\rm tr}$:

$$\boldsymbol{q}_{\rm tr} = \tfrac{1}{2}[\boldsymbol{e}_0 a \exp(-\mathrm{i}\omega t + \mathrm{i}kx) + \boldsymbol{e}_0^* a^* \exp(\mathrm{i}\omega t - \mathrm{i}kx)]. \quad (1)$$

Here a is the complex amplitude of the pulse field which depends only on the combination of coordinates $\xi = x - Vt$, vector $\boldsymbol{e}_0 = (\boldsymbol{y}_0 + \mathrm{i}\boldsymbol{z}_0)/\sqrt{2}$ is a unit vector of circular polarization, ω and k are the high frequency and wave number, respectively, which are related only to another by

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the expression $k = V\omega/c^2$. The coupled equations for variables a and ϕ can be obtained in the form [8]

$$\frac{\partial^2 a}{\partial \xi^2} + \left\{\frac{\omega^2}{c^2} - \frac{\omega_p^2 \gamma_0^2}{c^2(1+\phi)}\beta \times [1 - (1 + |a|^2/2)/(1+\phi)^2\gamma_0^2]^{-1/2}\right\} a = 0, \quad (2)$$

$$\frac{\partial^2 \phi}{\partial \xi^2} - \frac{\omega_p^2 \gamma_0^2}{c^2}\{1 - \beta[1 - (1 + |a|^2/2)/(1+\phi)^2\gamma_0^2]^{-1/2}\} = 0, \quad (3)$$

where $\beta = V/c$, $\gamma_0^2 = 1/(1-\beta^2)$, $\omega_p = (4\pi e^2 n_0/m)^{1/2}$ is the electron plasma frequency, and n_0 is the unperturbed electron density.

Since we consider the case of an underdense plasma ($\omega_p^2 \ll \omega^2$), the normalized velocity β can be assumed to be close to 1. Further on, considering in addition the case $(1 + |a|^2/2)/[(1+\phi)^2\gamma_0^2] \ll 1$ and expanding the square root terms in eqs (2) and (3) we finally obtain the equations

$$\frac{\partial^2 a}{\partial \xi^2} + \frac{\omega_p^2}{c^2}\left\{\frac{\omega^2}{\omega_p^2} - \frac{\gamma_0^2}{(1+\phi)}\right\} a = 0, \quad (4)$$

$$\frac{\partial^2 \phi}{\partial \xi^2} - \frac{\omega_p^2}{2c^2}[(1 + |a|^2/2)/(1+\phi)^2 - 1] = 0, \quad (5)$$

which are already simple enough to be studied analytically.

3. Analytical and numerical analysis

We note that for $\gamma_0^2 < \omega^2/\omega_p^2$ only periodical solutions of the coupled equations [(4) and (5)] exist. As such solutions are out of scope of our study, we restrict ourselves to the opposite case $\gamma_0^2 > \omega^2/\omega_p^2$. We assume that the field inside the pulse is relativistically strong. The results of numerical integration for this case which demonstrate a typical behavior of the solution of the coupled equations [(4) and (5)] in the leading part of the pulse are presented in Fig. 1.

In the region ahead of the pulse (for $\xi \to +\infty$) both a^2 and ϕ vanish as $\exp(-2\xi/\hat{\xi})$, where $\hat{\xi}$ is a corresponding characteristic scale length

$$\hat{\xi} = \frac{c}{\omega_p}\left\{\gamma_0^2 - \frac{\omega^2}{\omega_p^2}\right\}^{-1/2} \quad (6)$$

Behind the front (in the region where the electrostatic potential ϕ becomes larger than $\gamma_0^2\omega_p^2/\omega^2 - 1$) the amplitude a demonstrates the oscillating behavior:

$$a = a_m(\xi)\cos\left(\int_{\xi_1}^{\xi} \kappa(\xi')\, d\xi'\right). \quad (7)$$

Here a_m is a slowly varying amplitude. The expression for $\kappa(\xi)$ is defined by:

$$\kappa(\xi) = \frac{\omega_p}{c}\left\{\frac{\omega^2}{\omega_p^2} - \frac{\gamma_0^2}{[1+\phi(\xi)]}\right\}^{1/2}, \quad (8)$$

and ξ_1 is the coordinate of the point on the front where $\kappa(\xi)$ is equal to zero.

The oscillations of a can be interpreted as being produced by the beating of two light waves:

$$a\exp(-i\omega t + ikx) = a_+ \exp(i\varphi_+) + a_- \exp(i\varphi_-), \quad (9)$$

where $a_\pm = a_m(\xi)/2$ and

$$\varphi_\pm = \pm \int_{\xi_1}^{\xi} \kappa(\xi')\, d\xi' - \omega t + kx \quad (10)$$

are amplitudes and phases of the waves, respectively. The local frequencies and wave numbers of the waves are defined as $\omega_\pm = -\partial\varphi_\pm/\partial t$ and $k_\pm = \partial\varphi_\pm/\partial x$ and are given by

$$\omega_\pm(\xi) = \omega \pm V\kappa(\xi), \quad k_\pm(\xi) = k \pm \kappa(\xi). \quad (11)$$

One of the waves (with the subscript "+") can be associated with the "fresh" radiation supplied to the front region from the bulk of the pulse. The other wave ("−") is associated with the portion of the radiation which has already lost its energy on the leading front and is shifting backwards in the frame of the leading edge.

The electrostatic potential ϕ behind the leading front increases and, as the result, for $\phi \gg 1$ eq. (4) becomes decoupled from eq. (5). Thus, the plus-wave propagates in this region as in a vacuum so that ω_+ and k_+ there become equal to $\omega_0 \equiv \omega(1+\beta) \simeq 2\omega$ and $k_0 \equiv \omega(1+\beta)/c \simeq 2k$, respectively. As the main body of the pulse (where $\phi \gg 1$) is fairly long in comparison with the front structure, it carries the main pulse energy. Therefore, ω_0 and k_0 can be considered as the carrying frequency and the wave number of the pulse.

As the potential ϕ behind the front varies slowly in comparison with the oscillations of a, in this region the value $a_m^2\kappa$ is conserved ($a_m^2\kappa = \text{const}$) and can be treated as an adiabatic invariant. To find the constant that it is equal to, we study the solution of eqs (4) and (5) near the shock front where $\phi \ll 1$. Assuming that $\hat{\xi} \ll c/\omega_p$, eqs (4) and (5) can be reduced in this region to the equation without parameters

$$\frac{d^2 u}{d\eta^2} + (f - 1)u = 0, \quad (12)$$

where

$$u = a/\hat{a}, \quad \eta = \xi/\hat{\xi}, \quad f = \phi/\hat{\phi} = \int_\infty^\eta d\eta' \int_\infty^{\eta'} d\eta''[u(\eta'')]^2.$$

Vector and scalar potentials are normalized to the values $\hat{a} = 2\gamma_0(1 - \omega^2/\omega_p^2\gamma_0^2)$ and $\hat{\phi} = (1 - \omega^2/\omega_p^2\gamma_0^2)$, respectively. For parameters which correspond to $\hat{\phi} \ll 1$, eq. (12) can be valid if $f \gg 1$. In the region where $f \gg 1$ the fast oscillations of u with a slow varying amplitude u_m occur. The amplitude u_m is related to f and constant C_0 [obtained here by the

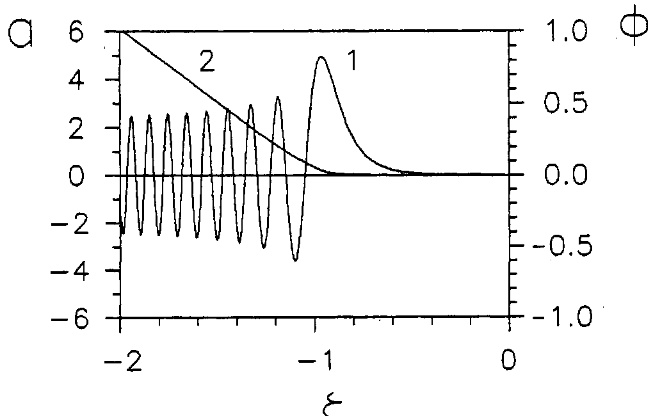

Fig. 1. Solution of the coupled equations [(4) and (5)] in the front part of the pulse for $\omega/\omega_p = 100$ and $\gamma_0^2 = 1.01\omega^2/\omega_p^2$. Curves 1 and 2 correspond to profiles of a and ϕ, respectively. The coordinate ξ is normalized to c/ω_p

numerical integration of eq. (12)] by expression

$$u_{\mathrm{m}}^2(f-1)^{1/2} = C_0 \simeq 10.43. \tag{13}$$

For ϕ greater than unity eq. (4) should be used to describe the variations of a. Since the adiabatic invariant is still conserved we find that $a_{\mathrm{m}}^2 \kappa = C_0 \hat{a}^2/\hat{\xi}$ and thus for a_+^2 we can obtain

$$a_+^2(\xi) = (C_0 \hat{a}^2/4\hat{\xi})/\kappa(\xi). \tag{14}$$

When $\phi \to \infty$, $a_+^2(\xi) \to a_0^2 \gg 1$. Here

$$a_0^2 = \frac{C_0 \omega_0^4}{16\gamma_0^2 \omega_{\mathrm{p}}^4}\left\{\frac{4\gamma_0^2\omega_{\mathrm{p}}^2}{\omega_0^2} - 1\right\}^{5/2} \tag{15}$$

is a constant value which can be associated with the amplitude of the main part of the pulse.

Thus, via eq. (15) we can find γ_0 [and, consequently, the velocity of the leading shock-like front $V \equiv c(1 - 1/\gamma_0^2)^{1/2}$] as function of the pulse frequency and amplitude:

$$\gamma_0^2 \simeq \frac{\omega_0^2}{4\omega_{\mathrm{p}}^2}\left\{1 + \left(\frac{4a_0^2\omega_{\mathrm{p}}^2}{C_0\omega_0^2}\right)^{2/5}\right\}. \tag{16}$$

Note that in the above analysis [where $\hat{\phi} = (1 - \omega^2/\omega_{\mathrm{p}}^2\gamma_0^2) \ll 1$] the range of considered amplitudes is restricted to the case when $1 \ll a_0^2 \ll \omega_0^2/\omega_{\mathrm{p}}^2$. For greater amplitude ($a_0^2 \geqslant \omega_0^2/\omega_{\mathrm{p}}^2$) a more sophisticated analysis without expansion in eqs (2) and (3) should be carried out to find out that the growth of γ_0^2 with the increase of amplitude is limited by a certain value. This value can be obtained by the numerical integration and is approximately equal to $0.4\omega_0^2/\omega_{\mathrm{p}}^2$. This means that even a pulse of an infinitely large amplitude with a stationary leading front propagates at the velocity not too close to the speed of light in free space ($1 - V/c \to \simeq 1.2\omega_{\mathrm{p}}^2/\omega_0^2$).

The results of numerical investigation of eqs (2) and (3) are presented in Fig. 2. It shows the velocity of the stationary front as a function of a_0 for two values of the parameter $\omega_0/\omega_{\mathrm{p}}$. It can be seen that the velocity of the leading front for a relativistically strong pulse ($a_0 \gg 1$) depends rather weakly on the pulse amplitude: with the growth of the amplitude the value of $V/c - 1$ increases from $-2\omega_{\mathrm{p}}^2/\omega_0^2$ (for $1 \leqslant a_0 \ll \omega_0/\omega_{\mathrm{p}}$) up to the value $\simeq -1.2\omega_{\mathrm{p}}^2/\omega_0^2$ (as $a_0 \to \infty$).

The group velocity of radiation inside the pulse $v_{\mathrm{g}} \simeq c[1 - 0.5\omega_{\mathrm{p}}^2/\omega_0^2(1 + \phi)]$ (where $\phi \gg 1$) tends to the speed of light in free space as $\phi \to \infty$, meanwhile the pulse leading front travels at the speed V which is below c. As the group velocity is the velocity of energy transport, there is a permanent energy input into the front region, where the supplied energy is converted to longitudinal plasma oscillations. The rate of this conversion can be estimated as

$$W \simeq (v_{\mathrm{g}} - V)\frac{k_0^2 a_0^2}{8\pi} \simeq c\,\frac{k_0^2 a_0^2}{16\pi\gamma_0^2}. \tag{17}$$

Therefore one can suggest that the pulse evolution [5, 6] and the above-described solutions can be joined in the following scenario of the fast depletion of the fairly long ($\tau \gg \omega_{\mathrm{p}}^{-1}$) and relativistically strong electromagnetic pulse. The jump of amplitude on the leading front (formed due to the stimulated backward Raman scattering or some other nonlinear phenomena [6]) leads to the excitation of the plasma wake wave and to the formation of the shock-like structure of the front described above. The velocity of the shock V is slightly less than the speed of light ($c - V \simeq c\omega_{\mathrm{p}}^2/\omega_0^2$) and weakly depends on the pulse amplitude behind the front region. Since the main part of the pulse propagates as in vacuum, the process of the pulse depletion looks as the progressive cut-off of its leading part. The cut-off point moves backward into the main body of the pulse with a relative velocity $V - c$ until it reaches the pulse rear front. So, the total time of the pulse depletion can be estimated as

$$t_{\mathrm{dep}} \simeq \tau\omega_0^2/\omega_{\mathrm{p}}^2. \tag{18}$$

Using a simplified model, we should be aware, that in addition to the nonlinear effects mentioned above (which provide the formation of the jump of amplitude on the leading front), there are other physical processes which can have some effect on the evolution of the pulse. For example, it can be breaking of nonlinear plasma wake or trapping of thermal plasma electrons. Being important even for the case of nonrelativistic initial electron plasma temperatures [9] these processes can result in heating of plasma electrons behind the shock front. The forward Raman scattering and modulational instability could also play some role in the pulse evolution (though both of them are to be suppressed for an electromagnetic wave of ultrarelativistic amplitude).

In order to verify our theoretical predictions we used particle-in-cell simulations. The initially smooth and circularly polarized electromagnetic pulse with a duration $\tau \simeq 240\pi\omega_0^{-1}$ and the maximum amplitude $a_0 = 3$ penetrated into the plasma (where $\omega_{\mathrm{p}} = \omega_0/3$) with a sharp boundary. We used 1(2/2)-D fully self-consistent relativistic electromagnetic code [6]. The cell size was equal to $0.25c/\omega_0$ and the number of particles in the cell was equal to 10. The ions were considered as an immobile neutralizing background.

The results of the simulations are given in Fig. 3, which displays the profiles of the absolute value of the transversal electric field $E_\perp = (E_y^2 + E_z^2)^{1/2}$ normalized on $mc\omega_0/e$ for five successive moments. Here we do not discuss the reasons that lead to the formation of the jump of a pulse amplitude on the leading front, as details of the shock formation process can be found in Ref. [6]. We only mark that the bulk of the pulse travels just at the speed of light in free space, while the pulse shock front has the velocity approximately $1.5c(\omega_{\mathrm{p}}^2/\omega_0^2)$ less. So, the numerical experiment demonstrates a fairly good agreement with our theory.

Looking upon the picture we can also see that the main part of the pulse energy is transferred to plasma electrons.

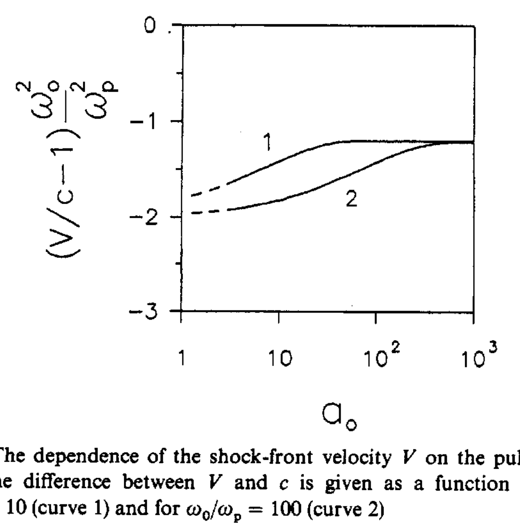

Fig. 2. The dependence of the shock-front velocity V on the pulse amplitude. The difference between V and c is given as a function of a_0 for $\omega_0/\omega_{\mathrm{p}} = 10$ (curve 1) and for $\omega_0/\omega_{\mathrm{p}} = 100$ (curve 2)

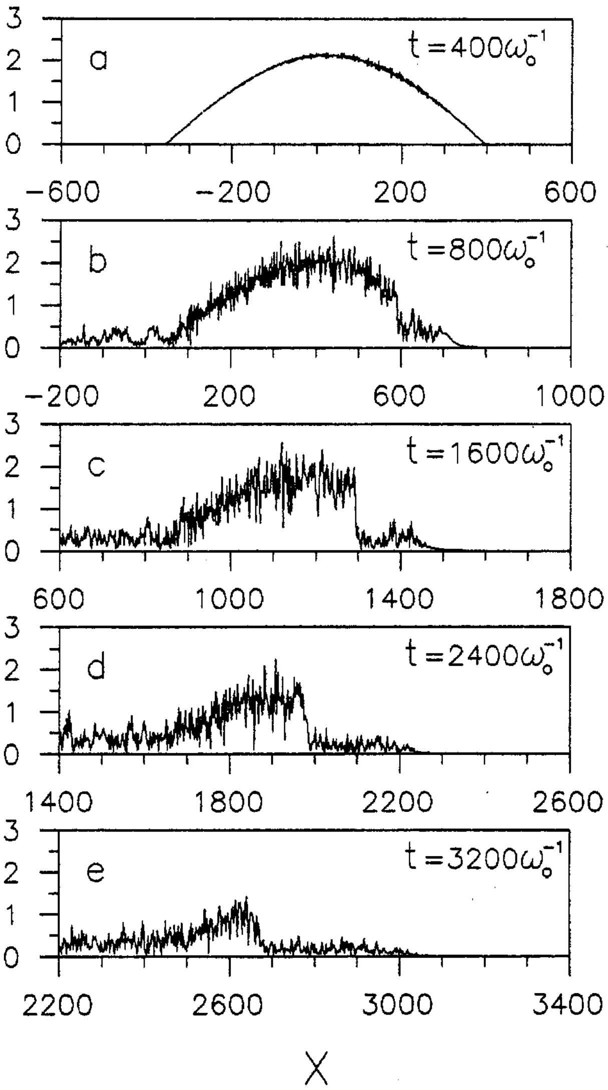

Fig. 3. A typical scenario of the depletion for a fairly long and relativistically strong pulse in an underdense plasma. Results of PIC simulations for a circularly polarized pulse with $t_p = 240\pi\omega_p^{-1}$ and $|a|_{max} = 3$ for $\omega_p = \omega_0/3$. Each plot (a, b, c, d, e) shows the profile of the transverse electric field $E_\perp = (e/cm_e\omega_0)(E_y^2 + E_z^2)^{1/2}$ for the successive stages of evolution. The coordinate x is normalized to c/ω_0

Neither breaking of the plasma wake-wave behind the shock nor the Raman scattering or modulational instability affect the pulse envelope significantly in the course of its depletion, resulting only in short wavelength perturbations of pulse amplitude. The small amplitude noise behind the pulse (which can be seen in the picture) corresponds to the fields of the wave reflected from the electron density fluctuations in the region of pulse–plasma interaction.

If one is interested in the exact solutions of eqs (2) and (3), stretching them into the main body of the pulse, one can find out that in the cold plasma model without dissipation the decoupling of the plasma wave and the electromagnetic field is broken at some distance from the leading front. Thus, inside the main body of the pulse in the cold plasma model the strong interaction with a plasma seems to be possible. Naturally, from eq. (5) it follows that far from the leading front ($\phi \gg |a|$) the potential ϕ varies as $\phi \simeq (\partial\phi/\partial\xi)_m(\xi - \xi_1) - (\omega_p^2/c^2)(\xi - \xi_1)^2/2$, where $-(\partial\phi/\partial\xi)_m$ corresponds to the maximum value of the longitudinal electric field of the excited nonlinear plasma wave. At the distance $\Delta\xi \simeq -2(c^2/\omega_p^2)(\partial\phi/\partial\xi)_m$ from the leading front the potential ϕ becomes of the order of 1 and eqs (4) and (5) become coupled again. There can be pointed out a discrete row of parameters (pairs of V and ω) for which a and ϕ behind this region can vanish simultaneously as $\xi \to -\infty$ forming the trailing front of a so-called "electric envelope soliton" [7] of relativistically strong amplitude. Our analytical results are valid for a description of the soliton, which was previously (in Ref. [7]) studied numerically.

In fact, even small electron temperatures result in trapping of plasma electrons [9]. The trapping can also be initiated by breaking of the excited nonlinear plasma wave. Trapped particles form the additional space charge inside the pulse, so that the potential ϕ, after reaching its maximum value $\phi_{max} \gg 1$, decreases but not down to values comparable with 1. As a result, eqs (4) and (5) remain decoupled everywhere behind the leading front. So our initial assumption that everywhere inside the main body of the pulse the interaction is suppressed seems to be true. Thus, the considered solution above should be regarded primarily as the kink type solution for the pulse leading front.

4. Conclusions

The shape of the considered stationary laser front appears to be similar to the shape of the usual hydrodynamic shock wave [10] although the nature of these phenomena are fairly different. For example, the decreasing oscillations in our solutions for the pulse amplitude behind the shock-like leading edge appears as a result of relativistic increase in electron mass and has nothing in common with the usual collisional damping.

It is interesting to look upon the stationary structure from the point of view of the energy exchange. The energy of the electromagnetic radiation with extremely high efficiency is transferred to the plasma electrons on the stationary front. As a result of the interaction, the frequency of radiation decreases and so does its group velocity. After taking part in the interaction, the portion of radiation lags behind the front. At the same time new portions of radiation are supplied to the leading edge region. Thus, counterstreaming flows of electromagnetic radiation provide the energy exchange between the region of strong interaction and the main part of the pulse. This effect can be easily understood if one considers the front of the pulse as a mirror moving at a relativistic speed. Let us consider the down-frequency conversion as Doppler shift in the process of reflection from the moving mirror. Then using the simple kinematic relations, one can easily obtain the relation between the frequencies of incident and reflected radiation: $\omega_0 \equiv \omega_+(\phi \to \infty) = \omega(1 + \beta) = \omega_-(\phi \to \infty)(1 + \beta)/(1 - \beta)$, which as well can be obtained directly from eq. (11).

Acknowledgements

One of us (H.A.S.) would like to thank Prof. L. M. Kovrizhnikh for making this collaboration and the funding for this visit possible.

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