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Parametric excitation of a coupled upper hybrid–acoustic wave and an extraordinary wave in a piezoelectric semiconductor plasma

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Abstract. In the present work we have considered a parametric interaction in a piezoelectric semiconductor plasma. The three waves involved in the parametric excitation are the pump wave, a coupled upper hybrid–acoustic wave and an extraordinary wave. An analytical expression for the growth of the unstable mode is derived and an expression for the threshold electric field for the onset of the instability is also derived. We have numerically investigated the dependence of the threshold electric field for different values of the background parameters for a typical piezoelectric semiconductor (InSb) and have given a numerical estimation of the growth rate.

1. Introduction

Parametric interactions have been studied extensively in solids and plasmas for quite some time. The early developments of parametric interaction in solids have been reviewed by Flytzanis (1975). More recently, owing to the applications of parametric processes in optoelectronic devices, parametric amplifiers, non-linear phase conjugation, etc, these processes have been investigated in piezoelectric semiconductors. Work in this direction has received impetus not only from the developments in solid-state physics but also from corresponding developments in the field of laser physics.

In a number of papers Sen and co-workers (Guha and Sen 1979a, b, Guha *et al* 1979, Sen 1979, Sen and Sen 1984, 1985, Aghamkar *et al* 1988, Aghamkar and Sen 1989) have theoretically considered such questions as the excitation of acoustic and acousto-helicon waves in magneto-active semiconductor plasmas, the possibility of occurrence of stimulated Brillouin scattering and consequent estimation gain constants, second-order optical susceptibility and parametric dispersion. Similar calculations have also been made for stimulated Raman scattering in doped piezoelectric semiconductors.

Salimullah (1987) has considered the dependence of a parametric instability on the pump frequency when the carrier drift velocity is comparable to the acoustic velocity in such crystals. Ghosh and Agrawal (1982a, b) and Ghosh and Khan (1986) have studied the excitation of acousto-helicon waves and the subsequent parametric amplification of acoustic waves in piezoelectric semiconducting crystals under the influence of an external magnetic field. More recently, Sanghvi and Ghosh (1992) have investigated the phenomenon of parametric conversion of an electron plasma wave into a circularly polarized electromagnetic wave in an n-type piezoelectric semiconductor.

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In the present paper we analytically investigate the parametric interaction of an electromagnetic pump wave with the extraordinary mode and a coupled upper hybrid–acoustic wave in a doped piezoelectric semiconducting crystal. Semiconductors like GaAs and InSb may be heavily doped with impurities to form an excess free-electron concentration such that the electron plasma frequency is nearly equal to the pump frequency. The semiconductor is irradiated with a spatially uniform laser beam. The crystal sample is subjected to a magnetostatic field B_0 that is applied along the z axis, which in turn is normal to the propagation vectors of the extraordinary wave (scattered wave) and the coupled upper hybrid–acoustic wave (slow mode) and the pump wave. The scattered mode interacts with the pump wave, thereby generating the slow mode, and the scattered mode is generated by the beating of the pump wave with a low-frequency wave (coupled upper hybrid–acoustic mode). In all such three-wave interaction processes, the phase matching conditions

$$\omega^{\mathrm{P}} = \omega^{\mathrm{T}} + \omega^{\mathrm{S}} \qquad k^{\mathrm{P}} = k^{\mathrm{T}} + k^{\mathrm{S}}$$

are satisfied. Here ω^{P}, ω^{T} and ω^{S} are the frequency associated with pump wave, the scattered wave and the slow mode respectively, and k^{P}, k^{T} and k^{S} are the corresponding wavenumbers. We assume that the pump is spatially uniform; thus, the pump wavenumber k^{P} is taken to be zero.

We note here that, since the extraordinary wave is a high-frequency wave, it does not couple with the lattice; but the upper hybrid wave, which is a relatively low-frequency wave, can couple via the piezoelectric coupling constant with acoustic oscillations of the lattice, thereby producing a coupled upper hybrid–acoustic wave. The pump wave is a uniform laser beam with frequency ω^{P} larger than the slow-mode frequency and hence is a travelling electromagnetic wave.

The modes at frequency $\omega^{\mathrm{P}} + \omega^{\mathrm{S}}$ are known as anti-Stokes modes, while modes at $\omega^{\mathrm{P}} - \omega^{\mathrm{S}}$ are Stokes modes. In the present analysis we consider exchange only between the pump field at frequency ω^{P} and the Stokes mode. This interaction satisfies the conditions

$$\omega^{\mathrm{P}} - \omega^{\mathrm{S}} = \omega^{\mathrm{T}} \qquad - k^{\mathrm{S}} = k^{\mathrm{T}}.$$

The non-linearity that has been taken into account is in the non-linear current density. We have investigated analytically the threshold condition for the onset of the parametric instability as well as the growth rate of the unstable mode well above the threshold value of the electric field amplitude of the pump.

In section 2 we give a theoretical formulation of the problem. In section 3 the threshold electric field for the onset of the instability and the growth rate of the instability along with our conclusion are given.

2. Theoretical formulation

We use the hydrodynamic model of a homogeneous n-type semiconductor plasma satisfying the condition $k^{\mathrm{S}}\lambda \ll 1$, where k^{S} is the coupled upper hybrid–acoustic wavenumber and λ is the electron mean free path. The spatially uniform laser beam is applied parallel to the wavevector k (along the x axis) and the ambient magnetic field is perpendicular to the direction of wave propagation (along the z axis). The equations needed for our analysis are given below:

$$\rho\frac{\partial^2 u}{\partial t^2} - c_e\frac{\partial^2 u}{\partial x^2} = -\beta\frac{\partial E}{\partial x} \tag{1}$$

$$\frac{\partial E}{\partial x} = \frac{e}{\varepsilon}n - \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} \tag{2}$$

$$\frac{\partial v}{\partial t} + (v\cdot\nabla)v = -\frac{e}{m}[E + (v\times B)] - \nu v - v_T^2\frac{\nabla n}{n_0} \tag{3}$$

$$\frac{\partial n}{\partial t} + \nabla\cdot(nv) = 0 \tag{4}$$

$$\nabla\times\nabla\times E = \mu_0 e\frac{\partial}{\partial t}(nv) - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}. \tag{5}$$

Equation (1) is the equation of motion for the piezoelectric lattice and it couples the mechanical displacement with the electric field; here c_e is the elastic constant, ρ the mass density and β the piezoelectric constant. Equation (2) is the modified Poisson's equation, and equation (3) is the electron equation of motion. Equation (4) is the equation of continuity, and equation (5) is the general wave equation.

We divide all the physical quantities into components associated with the fluctuations due to the pump, the scattered extraordinary mode and the slow electrostatic wave in the following manner:

$$\begin{aligned} n &= n_0 + n^P + n^T + n^S \\ v &= v^P + v^T + v^S \\ E &= E^P + E^T + E^S \\ B &= B_0\hat{z} + B^T \\ u &= u^S \end{aligned} \tag{6}$$

where the superscripts P, T and S designate the corresponding quantities associated with the pump wave, the extraordinary mode and slow mode (coupled upper hybrid–acoustic wave) and the subscript 0 refers to the background quantities.

We assume that all fluctuating quantities associated with the three waves involved in the parametric interaction vary as $\exp[i(kx - \omega t)] + cc$, where k and ω are the wavenumbers and frequencies of the three waves. Now by solving equations (1)–(4) we can derive the fluctuating velocities, number densities and electric fields associated with the parametric process that we are considering.

For the pump wave we obtain for the fluctuating velocity components the following expressions:

$$v_x^P = -i\frac{e}{m}\frac{\omega^P + i\nu}{(\omega^P + i\nu)^2 - \omega_c^2}E_x^P \tag{7}$$

$$v_y^P = \frac{e}{m}\frac{\omega_c}{(\omega^P + i\nu)^2 - \omega_c^2}E_x^P \tag{8}$$

where $\omega_c = eB_0/m$ is the cyclotron frequency. We note here that we have neglected the spatial dependence of the pump, as it does not play any significant role in non-linear phenomena.

Similarly for the scattered mode we obtain for the density and velocity component fluctuations the following expressions:

$$\frac{n^{\mathrm{T}}}{n_0} = \frac{k^{\mathrm{T}}}{\omega^{\mathrm{T}}} v_x^{\mathrm{T}} + \frac{k^{\mathrm{T}}}{\omega^{\mathrm{T}}} \frac{n^{\mathrm{S}}}{n_0} v_x^{\mathrm{P}} \tag{9}$$

$$v_x^{\mathrm{T}} = \delta(1+\eta)\left[-\mathrm{i}E_x^{\mathrm{T}} - \frac{\omega_c}{\omega^{\mathrm{T}}+\mathrm{i}\nu} E_y^{\mathrm{T}} + k^{\mathrm{T2}} v_{\mathrm{T}}^2 v_x^{\mathrm{P}} \frac{n^{\mathrm{S}*}}{n_0} + \mathrm{i}\frac{\omega_c}{\omega^{\mathrm{T}}+\mathrm{i}\nu} k^{\mathrm{S}} v_x^{\mathrm{P}} v_y^{\mathrm{S}*} - k^{\mathrm{S}} v_x^{\mathrm{P}} v_x^{\mathrm{S}*}\right] \tag{10}$$

$$v_y^{\mathrm{T}} = \delta\left[(1+\eta)\frac{\omega_c}{\omega^{\mathrm{T}}+\mathrm{i}\nu} E_x^{\mathrm{T}} - \mathrm{i}\left(1+\eta\frac{\omega_c^2}{(\omega^{\mathrm{T}}+\mathrm{i}\nu)^2}\right) E_y^{\mathrm{T}}\right] + \frac{\mathrm{i}\omega_c}{\omega^{\mathrm{T}}+\mathrm{i}\nu}\delta(1+\eta)\left(k^{\mathrm{S2}} v_{\mathrm{T}}^2 v_x^{\mathrm{P}} \frac{n^{\mathrm{S}*}}{n_0} + \frac{\mathrm{i}\omega_c}{\omega^{\mathrm{T}}+\mathrm{i}\nu} k^{\mathrm{S}} v_x^{\mathrm{P}} v_y^{\mathrm{S}*} - k^{\mathrm{S}} v_x^{\mathrm{P}} v_x^{\mathrm{S}*}\right) + \frac{k^{\mathrm{S}} v_x^{\mathrm{P}} v_y^{\mathrm{S}*}}{\omega^{\mathrm{T}}+\mathrm{i}\nu} \tag{11}$$

where, to avoid over-use of parentheses, we have written $k^{\mathrm{T2}} \equiv (k^{\mathrm{T}})^2$, v_{T} is the thermal velocity and $*$ denotes the complex conjugate.

In the above set of expressions

$$\delta = \frac{e}{m}\frac{\alpha^{-1}}{\omega^{\mathrm{T}}+\mathrm{i}\nu} \qquad \alpha^{-1} = \left(1 - \frac{\omega_c^2}{(\omega^{\mathrm{T}}+\mathrm{i}\nu)^2}\right)^{-1}$$
$$\eta = \frac{k^{\mathrm{T2}} v_{\mathrm{T}}^2}{\gamma\omega^{\mathrm{T}}} \qquad \gamma = \alpha(\omega^{\mathrm{T}}+\mathrm{i}\nu) - \frac{k^{\mathrm{T2}} v_{\mathrm{T}}^2}{\omega^{\mathrm{T}}} \tag{12}$$

and the non-linear terms are produced due to the interactions of the slow mode and the pump wave at the frequency and wavenumber ω^{T} and k^{T} respectively.

From the equation of motion of lattice ions and the Poisson equation, we get the following expression for E^{S}:

$$E^{\mathrm{S}} = -\frac{\mathrm{i}en_0}{k^{\mathrm{S}}\varepsilon}\frac{\rho\omega^{\mathrm{S2}} - c_e k^{\mathrm{S2}}}{\rho\omega^{\mathrm{S2}} - c_e k^{\mathrm{S2}} - \beta^2 k^{\mathrm{S2}}/\varepsilon}\frac{n^{\mathrm{S}}}{n_0}. \tag{13}$$

From the equation of motion for the slow mode we have the following expression for v_x^{S}:

$$v_x^{\mathrm{S}} = \frac{\omega^{\mathrm{S}}+\mathrm{i}\nu}{(\omega^{\mathrm{S}}+\mathrm{i}\nu)^2 - \omega_c^2}\left[\left(k^{\mathrm{S}} v_{\mathrm{T}}^2 - \frac{\omega_{\mathrm{pe}}^2(\rho\omega^{\mathrm{S2}} - c_e k^{\mathrm{S2}})}{k^{\mathrm{S}}(\rho\omega^{\mathrm{S2}} - c_e k^{\mathrm{S2}} - \beta^2 k^{\mathrm{S2}}/\varepsilon)}\right)\frac{n^{\mathrm{S}}}{n_0} + \frac{\omega_c}{\omega^{\mathrm{S}}+\mathrm{i}\nu}\left(\mathrm{i}k^{\mathrm{T}} v_x^{\mathrm{P}} v_y^{\mathrm{T}*} + \frac{\omega_c}{B_0} v_x^{\mathrm{P}} B_z^{\mathrm{T}*}\right) - \left(k^{\mathrm{T}} v_x^{\mathrm{P}} v_x^{\mathrm{T}*} + \mathrm{i}\frac{\omega_c}{B_0} v_y^{\mathrm{P}} B_z^{\mathrm{T}*}\right)\right]. \tag{14}$$

From the continuity equation for the slow mode and using results for v^{S} and v^{P}, we obtain the following expression for the fluctuating number density due to propagation of the coupled upper hybrid–acoustic wave and the interaction of the pump wave with the excited extraordinary mode:

$$\frac{n^{S*}}{n_0} = \frac{\mathrm{i}e/mk^S k^T \delta a^*}{Q}(-\mathrm{i}L_1 E_x^T + L_2 E_y^T)E_x^{P*} \tag{15}$$

where Q is given by

$$Q = \omega^S[(\omega^S - \mathrm{i}\nu)^2 - \omega_c^2] - (\omega^S - \mathrm{i}\nu)\left(k^{S2}v_T^2 + \frac{\omega_{Pe}^2(\rho\omega^{S2} - c_e k^{S2})}{\rho\omega^{S2} - c_e k^{S2} - \beta^2 k^{S2}/\varepsilon}\right)$$

and

$$a^* = \frac{\omega^P - \mathrm{i}\nu}{(\omega^P - \mathrm{i}\nu)^2 - \omega_c^2}$$

$$L_1 = (1+\eta)\left(\frac{\omega_c^2}{\omega^T} - \frac{k^T(\omega^{S2} - \omega_c^2)}{k^S\omega^T} - \omega^S\right)$$

$$L_2 = \frac{\omega_c^2}{\varepsilon B_0 \omega^T} - \frac{\eta\omega_c^3}{\omega^{T2}} - \omega_c + \frac{\omega_c^2\omega^S}{B_0\delta\omega^T\omega^P} + \frac{(1+\eta)\omega_c\omega^S}{\omega^T} + (1+\eta)\frac{(\omega^{S2} - \omega_c^2)\omega_c k^T}{\omega^{T2}k^S}. \tag{16}$$

The resonant non-linear interaction of the pump wave with the low-frequency wave gives rise to a non-linear current, which becomes the source of the extraordinary wave.

In the wave equation (5) we note that current density j consists of two parts, namely the linear current and the non-linear current, i.e.

$$j = j^L + j^{NL}$$

where the non-linear current density is given by

$$j^{NL} = -ev^P n^{S*}/n_0 \tag{17}$$

and upon using equations (7), (8) and (15) one obtains the components of j as

$$j_x^{NL} = -\frac{e^3/m^2 k^S k^T \delta |a|^2}{Q}(-\mathrm{i}L_1 E_x^T + L_2 E_y^T)|E_x^P|^2 \tag{18}$$

$$j_y^{NL} = -\frac{\mathrm{i}e^3/m^2 k^S k^T \delta\omega_c |a|^2}{\omega^P Q}(L_1 E_x^T + \mathrm{i}L_2 E_y^T)|E_x^P|^2. \tag{19}$$

Substituting equations (18) and (19) in equation (5) we get the following non-linear dispersion relation:

$$\begin{aligned}
&\left\{\left(\alpha\omega^{T2} - \frac{\omega_{Pe}^2\omega^T(1+\eta)}{\omega^T + \mathrm{i}\nu}\right)\left[\alpha(\omega^{T2} - c^2k^{T2}) - \frac{\omega_{Pe}^2\omega^T}{\omega^T + \mathrm{i}\nu}\left(1 + \eta\frac{\omega_c^2}{(\omega^T + \mathrm{i}\nu)^2}\right)\right] - \left(\frac{\omega_{Pe}^2\omega_c\omega^T(1+\eta)}{(\omega^T + \mathrm{i}\nu)^2}\right)^2\right\} \\
&\times\left(\omega^S[(\omega^S - \mathrm{i}\nu)^2 - \omega_c^2] - (\omega^S - \mathrm{i}\nu)\frac{k^{S2}v_T^2 + \omega_{Pe}^2(\rho\omega^{S2} - c_e k^{S2})}{\rho\omega^{S2} - c_e k^{S2} - \beta^2 k^{S2}/\varepsilon}\right) \\
&= \frac{e^2}{m^2}\omega_{Pe}^2|a|^2 k^S k^T |E^P|^2 \\
&\times\left\{\left[\alpha(\omega^{T2} - c^2k^{T2}) - \omega_{Pe}^2\left(1 + \eta\frac{\omega_c^2}{\omega^{T2}}\right) + \frac{\omega_c^2\omega_{Pe}^2}{\omega^P\omega^T}(1+\eta)\right]L_1 \right. \\
&\left. - \left([\alpha\omega^{T2} - \omega_{Pe}^2(1+\eta)]\frac{\omega_c}{\omega^P} + \frac{\omega_{Pe}^2\omega_c(1+\eta)}{\omega^P}\right)L_2\right\}
\end{aligned} \tag{20}$$

We note here that, in the absence of the pump wave, the extraordinary mode and the coupled upper hybrid–acoustic wave decouple from one another and we get the linear dispersion relations of the two modes, i.e. the extraordinary mode and the slow mode.

Let the value of ω^{S} and $\omega^{\mathrm{T}} = \omega^{\mathrm{P}} - \omega^{\mathrm{S}}$ for which both the factors on the left-hand side of equation (20) vanish be $\omega_{\mathrm{r}}^{\mathrm{S}}$. Assume $\omega^{S} = \omega_{\mathrm{r}}^{S} + \mathrm{i}\gamma_0$ where γ_0 (which is the damping rate) is taken as $\gamma_0 \ll \omega_{\mathrm{r}}^{\mathrm{S}}$. Equating the imaginary parts of equation (20) one gets

$$\gamma_0 = [(A_1\nu^2 + A_2|E^{\mathrm{P}}|^2)/A_3]^{1/2} \tag{21}$$

where

$$\begin{aligned}
A_1 \simeq \Bigg[&(\alpha_{\mathrm{r}}\omega^{\mathrm{P}2} - \omega_{\mathrm{Pe}}^2)\left(\frac{2\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}3}}(\omega^{\mathrm{P}2} - c^2k^{\mathrm{T}2}) + \frac{\omega_{\mathrm{Pe}}^2}{\omega^{\mathrm{P}}}\right) \\
&+ \left(\frac{2\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}}} + \frac{\omega_{\mathrm{Pe}}^2}{\omega^{\mathrm{P}}}\right)[\alpha_{\mathrm{r}}(\omega^{\mathrm{P}2} - c^2k^{\mathrm{T}2}) - \omega_{\mathrm{Pe}}^2]\Bigg] \\
&\times [2\omega^{\mathrm{S}2}(\rho\omega^{\mathrm{S}2} - c_{\mathrm{e}}k^{\mathrm{S}2} - \beta^2k^{\mathrm{S}2}/\varepsilon) - \omega_{\mathrm{Pe}}^2(\rho\omega^{\mathrm{S}2} - c_{\mathrm{e}}k^{\mathrm{S}2})] \\
A_2 \simeq \frac{e^2}{m^2}&k^{\mathrm{S}}k^{\mathrm{T}}\omega_{\mathrm{Pe}}^2\frac{\omega^{\mathrm{P}2}}{\omega^{\mathrm{P}4} - 2\omega_{\mathrm{c}}^2\omega^{\mathrm{P}2} + \omega_{\mathrm{c}}^4}(\rho\omega^{\mathrm{S}2} - c_{\mathrm{e}}k^{\mathrm{S}2} - \beta^2k^{\mathrm{S}2}/\varepsilon) \\
&\times \left\{\left(\alpha_{\mathrm{r}}(\omega^{\mathrm{P}2} - c^2k^{\mathrm{T}2}) + \frac{\omega_{\mathrm{Pe}}^2\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}2}} - \omega_{\mathrm{Pe}}^2\right)\left[\frac{\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}}}\left(1 + \frac{k^{\mathrm{T}}}{k^{\mathrm{S}}}\right) - \omega^{S}\right]\right. \\
&\left. - \alpha_{\mathrm{r}}\omega^{\mathrm{P}}\left(\alpha_{\mathrm{r}}\omega_{\mathrm{c}}^2 - \frac{\omega_{\mathrm{c}}^4k^{\mathrm{T}}}{\omega^{\mathrm{P}2}k^{\mathrm{S}}} - \omega_{\mathrm{c}}^2\right)\right\} \\
A_3 \simeq 2\Bigg[&(\alpha_{\mathrm{r}}\omega^{\mathrm{P}2} - \omega_{\mathrm{Pe}}^2)\left(\alpha_{\mathrm{r}}\omega^{\mathrm{P}} + \frac{\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}3}}(\omega^{\mathrm{P}2} - c^2k^{\mathrm{T}2})\right) \\
&+ \left(\alpha_{\mathrm{r}}\omega^{\mathrm{P}} + \frac{\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}}}\right)[\alpha_{\mathrm{r}}(\omega^{\mathrm{P}2} - c^2k^{\mathrm{T}2}) - \omega_{\mathrm{Pe}}^2] + \frac{\omega_{\mathrm{Pe}}^4\omega_{\mathrm{c}}^2}{\omega^{\mathrm{P}3}}\Bigg] \\
&\times [\omega_{\mathrm{Pe}}^2(\rho\omega^{\mathrm{S}2} - c_{\mathrm{e}}k^{\mathrm{S}2}) + 2\rho\omega^{\mathrm{S}2}(\omega_{\mathrm{Pe}}^2 - \omega_{\mathrm{c}}^2) - \omega_{\mathrm{c}}^2(\rho\omega^{\mathrm{S}2} - c_{\mathrm{e}}k^{\mathrm{S}2} - \beta^2k^{\mathrm{S}2}/\varepsilon)].
\end{aligned} \tag{22}$$

In the above expressions for A_1, A_2 and A_3 we have neglected the temperature correction terms η and have used the fact that $\omega^{\mathrm{S}} < \omega^{\mathrm{P}}, \omega^{\mathrm{T}}$. This simplification is made in order to avoid writing down algebraically cumbersome expressions, since the numerical values that we will use subsequently are for such a case. We note that α_{r} is the real part of α.

In the presence of a laser field, one can obtain the threshold value of the electric field necessary for the onset of instability by putting $\gamma_0 = 0$ in equation (21):

$$E_{\mathrm{th}}^{\mathrm{P}} = |(A_1\nu^2/A_2)|^{1/2}. \tag{23}$$

3. Results and discussion

In this section we numerically investigate the threshold electric field for the onset of the parametric instability. As can be seen from expression (23) along with the expressions for A_1 and A_2 given by equations (22), the threshold electric field E_{th}^{P} is a complicated function of the different background parameters, e.g. number density, magnetic field, temperature, propagation frequency and wavenumbers.

As a typical case the numerical investigation has been made for an InSb crystal (see e.g. Aghamkar and Sen 1990). We have plotted in figure 1 the threshold electric field against the different values of the magnetic field for two different values of the doping concentration, i.e. $n_0 = 2.44 \times 10^{24}$ m^{-3} and $n_0 = 7 \times 10^{23}$ m^{-3}. We see that in both cases we can achieve $(E_{th}^{P})_{min}$ of almost the same order for B_0 as low as 11 T and 6 T respectively. For $n_0 = 7 \times 10^{23}$ m^{-3} a second minimum is obtained around $B_0 \simeq 13$ T. For $B_0 \simeq 14$ T when $\omega_c = \omega^T$ a singularity appears for both values of the doping concentration. These results are qualitatively similar to the case considered by Aghamkar and Sen (1990).

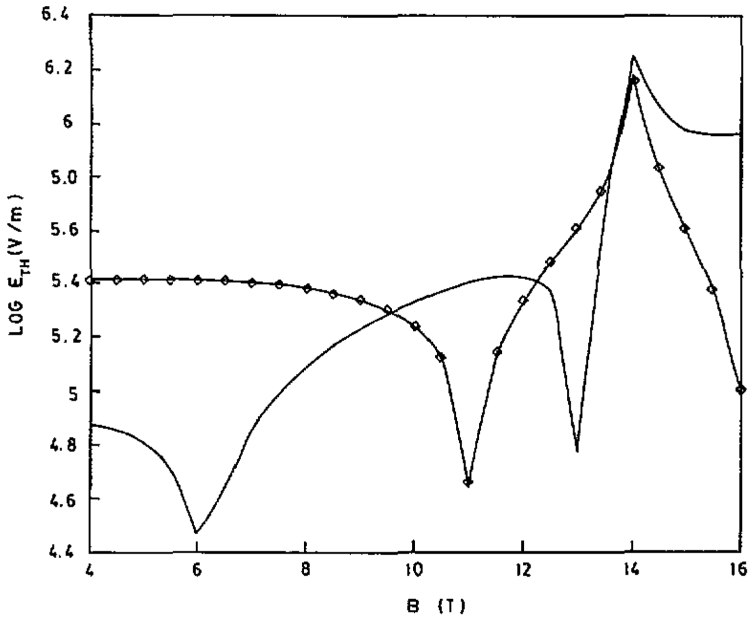

Figure 1. Threshold pump electric field E_{th} (V m^{-1}) versus magnetic field for $n_0 = 2.44 \times 10^{24}$ m^{-3} (——) and $n_0 = 7 \times 10^{23}$ m^{-3} (-◇-◇-).

In figure 2 we have plotted the threshold electric field against different values of ω^P (normalized to the electron plasma frequency) for two different values of the magnetostatic field strength, i.e. $B_0 = 11.3$ T and $B_0 = 8$ T, for a fixed doping concentration given by $n_0 = 2.44 \times 10^{24}$ m^{-3}. Again we see that low values of the E_{th}^{P} are achieved, specially in the case of the larger value of the magnetic field strength. This result is again qualitatively similar to the one investigated by Aghamkar and Sen (1990).

We note here that in the numerical investigation the following numerical values of the InSb crystal have been used (Sen and Sen 1985) at a temperature of 77 K: $m = 0.014m_0$,

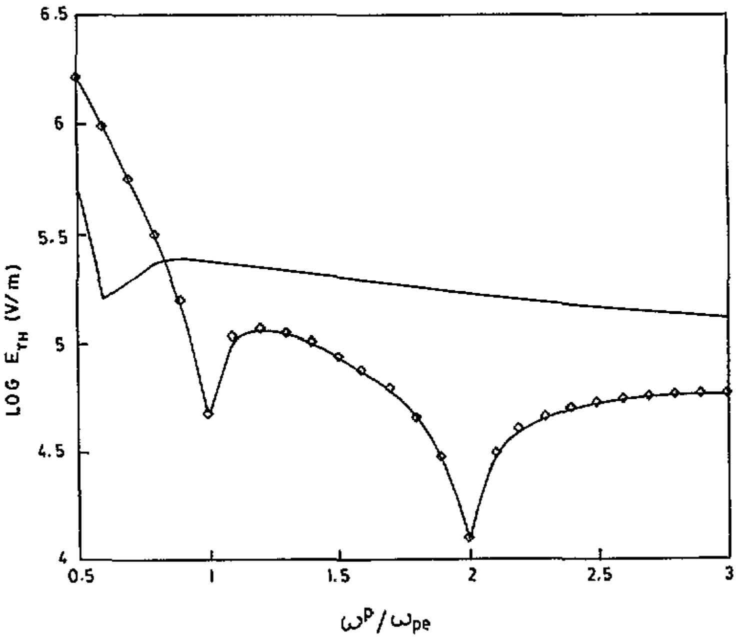

Figure 2. Threshold pump electric field E_{th} (V m^{-1}) versus normalized pump frequency ω^P for $B_0 = 11.3$ T (——) and $B_0 = 8$ T (-◇-◇-) at $n_0 = 2.44 \times 10^{24}$ m^{-3}.

$\varepsilon = 15.8$, $\beta = 0.054$ cm^{-2}, $\rho = 5.8 \times 10^3$ kg m^{-3}, $k = 5 \times 10^7$ m^{-1}, $c_S = 4 \times 10^3$ m s^{-1}, $\omega^T = 1.778 \times 10^{14}$ s^{-1}, $\omega^S = 2 \times 10^{11}$ s^{-1} and $\nu = 3 \times 10^{11}$ s^{-1}. The pump frequency that has been used corresponds to a 10.6 μm wavelength CO_2 laser.

We have also estimated the typical growth rate of the unstable mode for the onset of the decay process. For example for $E_0 \sim 10^6$ V m^{-1} and $\omega^P = 1.78 \times 10^{14}$ s^{-1}, $\omega_c = 0.9\omega^P$ and $\omega_{Pe} = \omega^P$, we find by using the parameters given above and using equations (21) and (22) that the growth rate of the unstable mode is given by $\gamma_0 = 2.07 \times 10^9$ s^{-1}. We once again see that the growth rate falls within a similar range as that obtained in the work of Aghamkar and Sen (1990).

Finally we would like to mention that the present work is already being extended to investigate the chaotic evolution of the instability found in this work.

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