phys. stat. sol. (b) **221**, 693 (2000)

Subject classification: 73.61.–r; 74.90.+n

Nonlinear Density Wave Propagation in Layered Superconducting Plasmas

R. ALI and H. A. SHAH

Centre for Solid State Physics, Quaid-e-Azam Campus, University of the Punjab, Lahore, Pakistan

(Received November 11, 1999; in revised form May 26, 2000)

In the present work the propagation of nonlinear density waves in a superconducting layered structure has been investigated. It is seen that the nonlinear Schrödinger (NLS) equation governs the propagation of density waves in a superconducting plasma. It is shown that in general the NLS equation in this case is modulationally unstable. By using the Kronig-Penney model to depict the layered structure we derive a nonlinear dispersion relation relating the nonlinear analog of the Bloch wave number to the propagation frequency. This nonlinear dispersion relation is numerically investigated.

1. Introduction

Recently Bunch and Grow [1] have investigated the linear propagation of density waves in a superconducting plasma by using the London theory [2] and the two fluid Gorter-Casimir [3] model to describe a superconducting plasma. Ali and Shah [4] extended this work to explore the propagation of density waves in a medium consisting of periodically alternating layers of a superconducting plasma. The London model as well as the Gorter-Casimir and the Ginzburg-Landau [5] models are phenomenological models in superconductivity theory and are based on Maxwell's equations and general laws of physics. The use of such models is justified by the fact that they provide useful insights into observed high temperature superconductivity (HTS) phenomena. The shortcoming of such phenomenological theories is that these are too general to provide an actually rigorous explanation of the mechanism of HTS. Bunch and Grow [1] have noted the applications of the phenomenological theories in devices using superconductors and have enumerated some of the advantages of using superconductors in active devices and have stressed the possible importance of travelling wave behavior of superconducting electrons in these devices.

With the advances of new technology and experimental techniques in solid state physics and material science, the fabrication of more and more artificial materials with layered structures having special properties is taking place. Such materials have included metal–semiconductor or semiconductor–insulator periodic layers etc. The theoretical work of Baynham and Boardman [6, 7] is considered a watershed for the further description of theoretical work in the area of periodic multilayer structures. Since then a lot of theoretical work has appeared investigating both linear and nonlinear properties of wave propagation in periodic multilayer media (e.g. [8, 9]).

Microelectronic applications require HTS multilayers containing different non-conducting, semiconducting, conducting and superconducting multilayers. It is with such applications in view that the properties of multilayered superconductors have been investigated. For example, investigations of vortex motion in layered superconductors have been made by Iye et al. [10]; resistivity studies have been carried out by Raffy et al. [11] and Hao et al. [12], and interface studies by Aarnink (see [13] and the references therein).

In this work we have attempted to explore the nonlinear wave propagation properties of density waves in a periodically layered superconducting plasma (each layer is say a YBCO superconductor described by the London and two fluid model [1]). We employ a standard perturbation and scaling technique to derive the nonlinear Schrödinger (NLS) equation which governs the propagation of density waves in the superconducting medium. The properties of the nonlinear wave for a layered structure consisting of alternating superconductor layers are investigated using the Kronig-Penney model. As in the work of Bunch and Grow [1] and Ali and Shah [4] here, too, we make use of the London model along with the two fluid model to describe the superconducting plasma which can be thought to be composed of both normal and superfluid electrons. The two fluid model takes losses into account, since one fluid is related to the superconducting electrons and the other to the semiconducting electrons. This approach is further justified since high T_c ceramics exhibit metallic properties along the superconducting plane (*ab*-plane) and semiconductor characteristics along the *c*-plane. Bunch and Grow [1] have noted that this approach may have applications in the use of superconductors in travelling wave devices. The advantages of such a device are, that no electron focusing structure is needed, no cathode is necessary. This makes possible the fabrication of millimetre and infrared devices since HTS are compatible with such fabrication (as opposed to conventional electron devices which are difficult to size down at high frequencies) along with the fact that these would be moderate power devices which are of high quality with low noise. The microwave and infrared properties are of importance in superconductors because of the existence of the energy gap, which implies that photons of energy less than the energy gap are not absorbed. For superconductors the frequencies of interest are those which lie below the electron plasma frequency, that is, the microwave and infrared frequency range.

The layout of the paper is as follows. In Section 2 we give a general mathematical formulation of the problem and derive the nonlinear Schrödinger (NLS) equation, which governs the propagation of nonlinear density waves within a single layer of the superconducting medium. The NLS has a known soliton solution, which is used in a self-consistent manner to relate the different parameters entering into the system. We then investigate the modulational instability of the NLS equation.

In Section 3 we introduce the boundary conditions of the Kronig-Penney model and derive an expression for the nonlinear dispersion relation for the propagation of density waves through a periodically layered superconducting medium. The nonlinear dispersion relation relates the nonlinear analog of the Bloch wave vector to the propagation frequency of the soliton. In Section 4 we give a numerical analysis of the results obtained in the previous section.

2. Nonlinear Schrödinger Equation

For the propagation of charge density waves in the superconducting medium, the following set of equations is used [1]:

$$\nabla \times E = -\mu_\alpha \frac{\partial H}{\partial t}\,, \tag{1}$$

$$\nabla \times H = J_\alpha + \varepsilon_\alpha \frac{\partial E}{\partial t}\,, \tag{2}$$

$$\nabla \cdot E = e\frac{n_\alpha}{\varepsilon_\alpha}\,, \tag{3}$$

$$\nabla \cdot H = 0\,, \tag{4}$$

$$\frac{\partial}{\partial t}\left(\frac{m_\alpha}{e^2 n_{s\alpha}} J_{s\alpha}\right) = E\,, \tag{5}$$

$$\nabla \times \left(\frac{m_\alpha}{e^2\, n_{s\alpha}} J_{s\alpha}\right) = -\mu_\alpha H\,, \tag{6}$$

$$J_\alpha = e n_{s\alpha} v_{s\alpha} + b_\alpha \sigma_\alpha E\,, \tag{7}$$

$$m_\alpha \frac{\mathrm{d}v_{s\alpha}}{\mathrm{d}t} = eE\,, \tag{8}$$

$$n_{s\alpha} = (1 - b_\alpha)\, n_\alpha\,. \tag{9}$$

Equations (1) to (4) are the Maxwell's equations, eqs. (5) and (6) are the London equations. Equation (7) is the expression for the total current density containing both the superconducting current and normal resistive current which depends on the conductivity σ_α. Equation (8) is the equation of motion of the superconducting electrons. Equation (9) gives the relation between the superconducting electrons $n_{s\alpha}$ and the total number of electrons n_α, the parameter b is the fraction which gives the number density of the normal-state electrons. The subscript α ($\alpha = 1, 2$) in the above set of equations denotes the layers of the alternating periodic medium. A slow electromagnetic wave [1] propagates close to the surface of the layered superconducting medium, which interacts with the superconducting electrons in each layer, this in turn sets up the density wave in each layer of the medium. The layers have thicknesses d_1 and d_2 and number densities n_1 and n_2, respectively, both of which have a superconducting component n_s and a normal state component n_n.

In order to derive the NLS equation we divide the fluctuating quantities into a low frequency part and a high frequency part (having superscript l and h, respectively) in the following manner:

$$\begin{aligned} j &= j_0 + j^{\mathrm{l}} + j^{\mathrm{h}}\,, \\ n &= n_0 + n^{\mathrm{l}} + n^{\mathrm{h}}\,, \\ E &= E^{\mathrm{h}}\,. \end{aligned} \tag{10}$$

We note that in the above set of equations E only has a high frequency fluctuation term, this enables us to neglect the ion (lattice) motion. Quantities with subscript 0 are the background quantities. We shall only deal with high frequency terms except in the equation of motion (8) where the low frequency v^{l} term occurs in the convective derivative and this contributes to the nonlinearity. Further in eqs. (1) to (9) all high frequency terms are eliminated in favour of J^{h} and considering one-dimensional propagation in the z-direction, we obtain the following equation:

$$\begin{aligned} &\frac{\partial^2}{\partial t^2}\left[\frac{\partial j^{\mathrm{h}}}{\partial t} + v_0(1-b)\frac{\partial j^{\mathrm{h}}}{\partial z}\right] - \frac{b\sigma}{\varepsilon}\frac{\partial}{\partial t}\left[-\frac{\partial j^{\mathrm{h}}}{\partial t} + v_0(1-b)\frac{\partial j^{\mathrm{h}}}{\partial z}\right] \\ &+ v_0\frac{\partial^2}{\partial z\,\partial t}\left[\frac{\partial j^{\mathrm{h}}}{\partial t} + v_0(1-b)\frac{\partial j^{\mathrm{h}}}{\partial z}\right] - \frac{b\sigma v_0}{\varepsilon}\frac{\partial}{\partial z}\left[-\frac{\partial j^{\mathrm{h}}}{\partial t} + v_0(1-b)\frac{\partial j^{\mathrm{h}}}{\partial z}\right] \\ &+ e n_{s0}\frac{\partial^3}{\partial z\,\partial t^2}\left(v^l v^h\right) - \omega_{\mathrm{ps}}^2\left[-\frac{\partial j^{\mathrm{h}}}{\partial t} + v_0(1-b)\frac{\partial j^{\mathrm{h}}}{\partial z}\right] = 0\,. \end{aligned} \tag{11}$$

In order to evaluate the term v^{l} we use the low frequency components of the set of eqs. (1) to (9) bearing in mind that the average over the square of the high frequency terms yields a low frequency term [14]. We obtain the following expression for v^{l}:

$$v^{l} = -\frac{1}{2}\frac{[\omega - kv_0(1-b)]^2}{v_0 e^2 n_{s0}^2 \omega^2}\left\langle (j^{\mathrm{h}})^2 \right\rangle . \tag{12}$$

Using equations (11) and (12) and the following solution for J^{h}:

$$\frac{j^{\mathrm{h}}}{n_0 e v_0} = \varphi(z,t)\,\mathrm{e}^{-\mathrm{i}(kz-\omega t)} + \text{c.c.}\,, \tag{13}$$

we obtain the NLS equation

$$\mathrm{i}\frac{\partial\varphi}{\partial t} + \mathrm{i}v_g\frac{\partial\varphi}{\partial z} + P\frac{\partial^2\varphi}{\partial z^2} + Q\left|\varphi^2\right|\varphi + \mathrm{i}R\varphi = 0\,, \tag{14}$$

where

$$v_g = \frac{-\omega^2 v_0(2-b) + 2\omega k v_0^2(1-b) + \omega_{\mathrm{ps}}^2(1-b)\,v_0}{-3\omega^2 + 2\omega k v_0(2-b) - k^2 v_0^2(1-b) + \omega_{\mathrm{ps}}^2}$$

is the group velocity of the density wave (which can be obtained from the real part of the linear dispersion relation of the density wave) and

$$P = \frac{-\omega v_0^2(1-b)}{-3\omega^2 + 2\omega k v_0(2-b) - k^2 v_0^2(1-b) + \omega_{\mathrm{ps}}^2}\,,$$

$$Q = \frac{k[\omega - kv_0(1-b)]^3\,v_0}{2\omega\left(-3\omega^2 + 2\omega k v_0(2-b) - k^2 v_0^2(1-b) + \omega_{\mathrm{ps}}^2\right)}\,,$$

$$R = \frac{b\sigma[-\omega^2 + bv_0\omega k + k^2 v_0^2(1-b)]}{\varepsilon\left(-3\omega^2 + 2\omega k v_0(2-b) - k^2 v_0^2(1-b) + \omega_{\mathrm{ps}}^2\right)}\,.$$

We note that in obtaining the above we have put equal to zero the real part of the linear dispersion relation for density waves which is given by [1, 4]

$$-\omega^3 + k\omega^2 v_0(2-b) - \omega k^2 v_0^2(1-b) + \omega\omega_{\mathrm{ps}}^2 + v_0(1-b)\,k\omega_{\mathrm{ps}^2} = 0\,. \tag{15}$$

The scaling procedure that we have used in obtaining eq. (14) is the following [15]:

$$\frac{\partial\varphi}{\partial t} \sim \frac{\varphi}{\tau}\,, \qquad \frac{\partial\varphi}{\partial z} \sim \frac{\varphi}{v_g\tau} \quad \text{and} \quad v^{\mathrm{l}} \sim v_{\mathrm{g}}\varphi^2\,,$$

where τ is a time scale and $1/\tau < \omega$, where ω is the density wave propagation frequency (linear).

Equation (14) can be transformed into the standard NLS equation by making the transformation

$$\varphi = \varphi'\,\mathrm{e}^{-Rt}\,, \tag{16}$$

thus the NLS equation (14) has a known soliton solution given by

$$\varphi = \varphi_0\,\mathrm{sech}\,(k^* z - \omega^* t)\,\mathrm{e}^{\mathrm{i}\delta\Omega t}\,\mathrm{e}^{-Rt}\,. \tag{17}$$

Substituting (17) into (16) we can self-consistently evaluate the quantities in the solution of the NLS equation, and these are given by

$$
\begin{aligned}
&-\delta\Omega + Pk^{*2} = 0\,,\\
&-2Pk^{*2} + Q\varphi_0^2 = 0\,,\\
&\omega^* - v_g k^* = 0\,.
\end{aligned}
\tag{18}
$$

Here φ_0 is the amplitude of the NLS equation and $\delta\Omega$, k^* and ω are the nonlinear frequency shift, wave number and the frequency, respectively. The solution given by eq. (18) makes a soliton solution when $PQ > 0$ (which corresponds to modulational instability) and for the case of $PQ < 0$ (which corresponds to modulational stability), it results in a dark soliton as solution.

3. Periodic Boundary Conditions

Since we are investigating the propagation of charge density wave solitons in a superconducting layered medium consisting of two alternating layers of thickness d_1 and d_2, respectively, these two layers repeat periodically – therefore the soliton solution should carry a subscript i (where $i = 1, 2$) denoting different layers:

$$\varphi = \varphi_{0i}\,\mathrm{sech}\left(k_i^* z - \omega_i^* t\right) \mathrm{e}^{\mathrm{i}\delta\Omega_i t}\, \mathrm{e}^{-Rt}\,. \tag{19}$$

We now introduce the boundary conditions, which are used in the standard treatment of layered media having a Kronig-Penney structure, and wave propagation across the layers is considered [6, 7]. Soliton solution for the current densities of the two layers is connected to one another at the boundary of the two layers in the following way:

$$\varphi_1|_{z=d_1} = \varphi_2|_{z=d_1}\,, \tag{20}$$

$$\left.\frac{\partial\varphi_1}{\partial z}\right|_{z=d_1} = \left.\frac{\partial\varphi_2}{\partial z}\right|_{z=d_1}\,, \tag{21}$$

$$\varphi_1|_{z=0} = \mathrm{e}^{i\bar{K}d}\,\varphi_2|_{z=d}\,, \tag{22}$$

$$\left.\frac{\partial\varphi_1}{\partial z}\right|_{z=0} = \mathrm{e}^{\mathrm{i}\bar{K}d}\left.\frac{\partial\varphi_2}{\partial z}\right|_{z=d}\,, \tag{23}$$

where $d_1 + d_2 = d$, and $\bar{K}$ is the nonlinear analog of the Bloch wave number. We now substitute the solution for φ (given by eq. (19)) into the set of eqs. (20) to (23). Following [16] and skipping the details of the rather messy algebra for the nonlinear analog of the Bloch wave number, we directly arrive at the final expression

$$W_1 \cos^2 \bar{K}d + W_2 \cos \bar{K}d + W_3 = 0\,, \tag{24}$$

where W_1, W_2 and W_3 are given by

$$
\begin{aligned}
W_1 &= X_1 C_1^2 C_2^2 + X_2 C_1^2 C_2 S_2 + X_3 C_1^2 S_2^2\,,\\
W_2 &= X_1 A_2 + X_2 D_2 + X_3 B_2\,,\\
W_3 &= X_1 A_1 + X_2 D_1 + X_3 B_1\,,
\end{aligned}
\tag{25}
$$

and A_1, A_2, B_1, B_2 and D_1, D_2 are given by

$$\begin{aligned}
A_1 &= C_{11}^2C_1^2C_{22}^2C_2^2 + C_{11}^2C_1^2S_{22}^2S_2^2 + S_{11}^2S_1^2C_{22}^2C_2^2 + S_{11}^2S_1^2S_{22}^2S_2^2 \\
&\quad - 2C_{11}^2C_1^2C_{22}C_2S_{22}S_2 - 2C_{11}C_1C_{22}^2C_2^2S_{11}S_1 - 2C_{11}C_1S_{22}^2S_2^2S_{11}S_1 \\
&\quad - 2S_{11}^2S_1^2C_{22}C_2S_{22}S_2 + 4C_{11}C_1C_{22}C_2S_{11}S_1S_{22}S_2\,, \\
A_2 &= -C_{11}C_{22}C_1^2C_2^2 + 2C_1^2C_2C_{11}S_{22}S_2 + 2C_1C_2^2S_{11}S_1C_{22} - 2C_1C_2S_{11}S_1S_{22}S_2\,, \\
B_1 &= -2C_{11}^2C_1^2S_{22}C_2C_{22}S_2 - 2C_{11}C_1S_{22}^2C_2^2S_{11}S_1 - 2C_{11}C_1C_{22}^2S_2^2S_{11}S_1 \\
&\quad - 2S_{11}^2S_1^2S_{22}C_2C_{22}S_2 + 4C_{11}C_1S_{22}C_2S_{11}S_1C_{22}S_2 + C_{11}^2C_1^2S_{22}^2C_2^2 \\
&\quad + C_{11}^2C_1^2C_{22}^2S_2^2 + S_{11}^2S_1^2S_{22}^2C_2^2 + S_{11}^2S_1^2C_{22}^2S_2^2\,, \\
B_2 &= 2C_1^2S_2C_{11}S_{22}C_2 - 2C_1^2S_2^2C_{11}C_{22} - 2C_1S_2S_{11}S_1S_{22}C_2 + 2C_1S_2^2S_{11}S_1C_{22}\,, \\
D_1 &= -C_{11}^2C_1^2C_{22}C_2^2S_{22} + C_{11}^2C_1^2C_{22}^2C_2S_2 + 2C_{11}C_1C_{22}C_2^2S_{11}S_1S_{22} \\
&\quad - 2C_{11}C_1C_{22}^2C_2S_{11}S_1S_2 + C_{11}^2C_1^2S_{22}^2S_2C_2 - C_{11}^2C_1^2S_2^2S_{22}C_{22} \\
&\quad - 2C_{11}C_1S_{22}^2S_2S_{11}S_1C_2 + 2C_{11}C_1S_{22}S_2^2S_{11}S_1C_{22} - S_{11}^2S_1^2C_2^2C_{22}S_{22} \\
&\quad + S_{11}^2S_1^2C_{22}^2C_2S_2 + S_{11}^2S_1^2S_{22}^2S_2C_2 - S_{11}^2S_1^2S_{22}S_2^2C_{22}\,, \\
D_2 &= -2C_1^2C_2C_{11}C_{22}S_2 + C_1^2C_2^2C_{11}S_{22} - C_1C_2^2S_{11}S_1S_{22} + 2C_1C_2S_{11}S_1C_{22}S_2 \\
&\quad + C_{11}C_1^2S_{22}S_2^2 - S_{11}S_1S_{22}S_2^2C_1\,;
\end{aligned} \tag{26}$$

X_1, X_2 and X_3 are given by

$$\begin{aligned}
X_1 &= S_1C_2^2C_{11}C_1S_{22} - S_1^2C_2^2S_{11}S_{22} + S_1^2C_2S_{11}C_{22}S_2 - C_1^2S_2S_{11}C_{22}C_2 \\
&\quad + C_1^2S_2^2S_{11}S_{22} - C_1S_2^2C_{11}S_1S_{22}\,, \\
X_2 &= -S_1^2C_2^2S_{11}C_{22} + 2S_1^2S_2S_{11}S_{22}C_2 - S_1^2S_2^2S_{11}C_{22} + C_1^2C_2^2S_{11}C_{22} \\
&\quad - 2C_1^2S_2S_{11}S_{22}C_2 + C_1^2S_2^2S_{11}C_{22}\,, \\
X_3 &= S_1S_2^2C_{11}S_{22}C_1 + S_1^2S_2S_{11}C_{22}C_2 - S_1^2S_2^2S_{11}S_{22} + C_1^2C_2^2S_{11}S_{22} \\
&\quad - C_1^2C_2S_{11}C_{22}S_2 - C_1C_2^2C_{11}S_1S_{22}\,,
\end{aligned} \tag{27}$$

where S_1, S_2, C_1, C_2, S_{11}, S_{22}, C_{11}, C_{22} and S_{21}, C_{21} are given by

$$\begin{aligned}
S_1 &= \sinh \Theta_1\,, & S_{11} &= \sinh k_1^* d_1\,, & S_{21} &= \sinh k_2^* d_1\,, \\
S_2 &= \sinh \Theta_2\,, & S_{22} &= \sinh k_2^* d_2\,, & C_{21} &= \cosh k_2^* d_1\,, \\
C_1 &= \cosh \Theta_1\,, & C_{11} &= \cosh k_1^* d_1\,, \\
C_2 &= \cosh \Theta_2\,, & C_{22} &= \cosh k_2^* d_2\,,
\end{aligned} \tag{28}$$

with $\Theta_{\mathrm{i}} = \omega_{\mathrm{i}}^* t$, $i = 1$, 2. The terms S_{21} and C_{21} do not appear explicitly in expressions (24) to (27) as these have been eliminated and expressed in terms of hyperbolic functions S_{ii} and C_{ii}.

Equation (24) is the nonlinear dispersion relation for density wave solitons propagating across a superconducting medium consisting of two alternating layers. We note here that eq. (24) is quadratic in $\cos(\bar{K}d)$, whereas in the linear case [4, 6, 7] the dispersion relation was linear in $\cos(\bar{K}d)$. Therefore the charge density wave solitons have two modes of propagation corresponding to the two solutions of eq. (24). The result (eq. (24)) is qualitatively similar to that given in our earlier work [16], but is presented here for the sake of completeness since this is used in the numerical analysis given in the following section.

4. Numerical Analysis and Conclusions

Now we numerically investigate eq. (24) by taking some numerical values associated with typical layered superconducting plasmas. We attempt to establish a relationship between the nonlinear analog of the Bloch wave number $\bar{K}$ and the propagation frequency ω :

$$\cos \bar{K}d = \frac{-W_2 \pm \sqrt{W_2^2 - 4W_1W_3}}{2W_1} \tag{29}$$

with W_1, W_2 and W_3 defined by expressions (25) to (28). We note that real propagating roots will be obtained only when

$$\left|\cos \bar{K}d\right| \leq 1\,. \tag{30}$$

Numerical values of the linear wave number $k_{1,2}$ (real) is calculated from the linear dispersion relation and we note nonlinear frequency ω is taken as $\omega^* \ll \omega$. Therefore $k_{1,2}$ is then obtained through eq. (15).

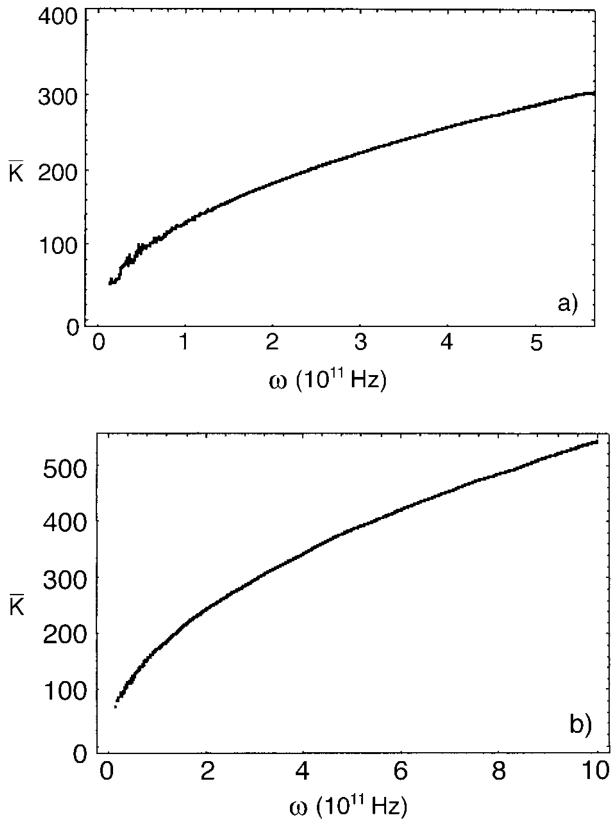

Fig. 1. Bloch wave vector $\bar{K}$ versus frequency ω for a) the upper and b) the lower sign. Numerical values: $n_1 = 1.5 \times 10^{26}$ m^{-3}, $n_2 = 10^{26}$ m^{-3}; $v_{s1} = v_{s2} = 10^5$ m/s; $d_1 = 10^{-5}$ m, $d_2 = 10^{-6}$ m; $\Theta_1 = \Theta_2 = 1$; $\varepsilon_1 = 3$, $\varepsilon_2 = 4$; $b_1 = b_2 = 0.25$; $\sigma_1 = \sigma_2 = 10^7\ \Omega^{-1}$m^{-1}

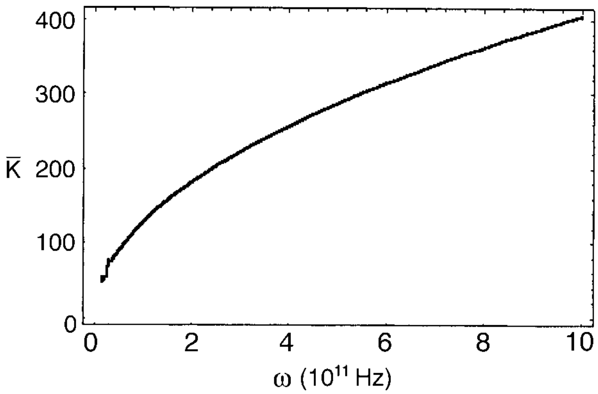

Fig. 2. Bloch wave vector $\bar{K}$ versus frequency ω for the upper sign with conductivities $\sigma_1 = \sigma_2 = 10^5\ \Omega^{-1}\ \mathrm{m}^{-1}$; other values remain the same as in Fig. 1

Figures 1a and b show the dependence of the nonlinear Bloch wave number $\bar{K}$ on the propagation frequency ω for upper and lower roots, respectively. The numerical values used for the thicknesses d_1, d_2, the dielectric constants ε_1, ε_2, the number densities n_1, n_2, the effective masses m_1, m_2, and the values of Θ_1, Θ_2 are given in the caption to Fig. 1, where the propagation frequency ω is taken from 10^9 to 10^{12} Hz. Comparisons of Fig. 1a and b show that in each case there is a small propagation gap in the region around 10^9 Hz and then there is continuous propagation band right up to 10^{12} Hz. We see that for the mode with the upper sign the nonlinear Bloch wave vector has smaller value as compared to lower sign. Further we note that the propagation region shifts to the right to the higher frequency range for the lower sign.

As we decrease the value of conductivity, Fig. 2 (with upper sign), the frequency increases, thus both propagation bands and gaps shift to the right, the lower sign shows the same trend as described above. This is qualitatively similar to the case investigated by Ali and Shah [16], Baynham and Boardman [6]. In Fig. 3 we increase the number of normal-state electrons b_2 in the second layer and we see that the value of Bloch wave vector decreases as compared to the results in Fig. 2. Thus our investigations show that nonlinear density waves can be described by the NLS equation which has soliton solutions. The band gap structure in the propagation characteristics is maintained when a Kronig-Penney model is used to depict a sandwich structure.

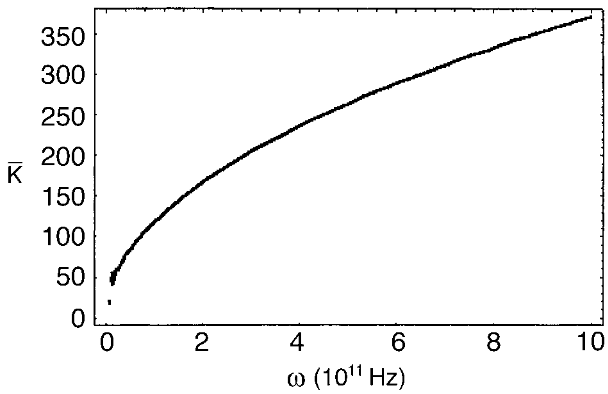

Fig. 3. Bloch wave vector $\bar{K}$ versus frequency ω for the upper sign with conductivities $\sigma_1 = \sigma_2 = 10^5\ \Omega^{-1}\ \mathrm{m}^{-1}$ and fraction of normal-state electrons $b_1 = 0.25$, $b_2 = 0.75$; other values remain the same as in Fig. 1

Acknowledgement This work was conducted under the Pakistan Science Foundation research grant No. PSF/Res/P-PU/Phys(99).

References

[1] K. J. Bunch and R. W. Grow, Internat. J. Infrared Millim. Waves **18**, 57 (1997).
[2] F. and H. London, The Electromagnetic Equations of Superconductors, Proc. Royal Soc. (London) **149**, 71 (1935).
[3] M. Tinkham, Introduction to Superconductivity, McGraw-Hill, New York 1975.
[4] R. Ali and H. A. Shah, J. Phys.: Condensed Matter **12**, 5857 (2000).
[5] V. L. Ginzburg and L. D. Landau, Soviet Phys. – JEPT **5**, 1442 (1957).
[6] A. C. Baynham and A. D. Boardman, J. Phys. A **1**, 363 (1968).
[7] A. C. Baynham and A. D. Boardman, J. Phys. C **2**, 619 (1969).
[8] M. S. Kushwaha and P. Halevi, Phys. Rev. B **35**, 3879 (1987).
[9] H. A. Shah, I. U. R. Durrani, and T. Abdullah, Phys. Rev. B **47**, 1980 (1993).
[10] Y. Iye, I. Oguro, T. Tamegai, W. R. Datars, N. Motohira, and K. Kitazawa, Physica C **199**, 154 (1992).
[11] H. Raffy, S. Labdi, O. Laborde, and P. Monceau, Phys. Rev. Lett. **66**, 2515 (1991).
[12] Z. Hao, C-R. Hu, and C. S. Ting, Phys. Rev. B **51**, 9387 (1995).
[13] W. A. M. Aarnink, Ph.D. thesis, Univ. of Twente, Netherlands, 1992.
[14] R. O. Dendy, Plasma Physics, Calaredon Press, Oxford 1990.
[15] C. R. Ovenden, H. A. Shah, and S. J. Schwarts, J. Geophys. Res. **88**, 6095 (1983).
[16] R. Ali and H. A. Shah, J. Phys.: Condensed Matter **9**, 7583 (1997).