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Some electrostatic modes based on non-Maxwellian distribution functions

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A comparative study of fundamental modes such as Langmuir waves, dust ion acoustic waves, and dust-acoustic waves using non-Maxwellian distribution functions is presented. The real frequency and the growth rate of the modes are calculated by using kappa and generalized (r,q) distribution functions and results are compared with those of Maxwellian distribution. It is noted that in the limit (i) $r=0$, $q\rightarrow\infty$ for generalized (r,q) distributions and (ii) $\kappa\rightarrow\infty$ for kappa distributions, the non-Maxwellian functions reduce to Maxwellian.
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I. INTRODUCTION

Basic electrostatic modes such as Langmuir waves, ion-acoustic waves and dust-acoustic waves in plasmas have been widely studied on the basis of the fluid and the Vlasov models. These studies had assumed Maxwellian velocity distribution for the plasma species. However, with more and more empirical data becoming available, it is realized that the Maxwellian is not a realistic distribution under all circumstances. Other distributions such as kappa,[1] or generalized (r,q) distribution (which is the subject of the present work) fit better, for example, with the data available for space plasmas. Thus it has now become imperative to give attention to such velocity distributions for plasma studies in both space and laboratory.

As is well known, the Maxwellian distribution is applicable to a system in thermodynamic equilibrium. However, a realistic system may be far away from this state since the plasma may be subject to a whole variety of different effects. For example, a spatial variation of physical quantities such as density, temperature, and the intensity of magnetic field may be involved in laboratory as well as in astrophysical plasmas. More importantly, background turbulence, which can be considered to be a quasisteady state, can contribute to the appearance of distribution functions which are different from Maxwellian.[1] Plasmas may also be subject to the influence of external force fields, which would create flows of particles. Consequently, in the natural space environment the distribution functions have been generally observed to have non-Maxwellian high-energy tails or have flat tops with pronounced shoulders. To model such space plasmas, generalized Lorentzian-type distributions have been found to be more useful as compared to the Maxwellian distribution functions.[2,3] More than fifty years ago, it was suggested that such distributions result from wave particles interaction.[4] Now, the same is regarded as almost an established fact, due to the arguments given above.

A three-dimensional kappa distribution[2] is given by

$$f_\kappa=\frac{1}{\pi^{3/2}\theta_\perp^2\theta_\parallel}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right]^{-\kappa-1}, \tag{1}$$

where κ is the spectral index; the thermal speed θ is related to the particle temperature T by

$$\theta_\parallel^2=\left(\frac{2\kappa-3}{\kappa}\right)v_{T_\parallel}^2;\qquad \theta_\perp^2=\left(\frac{2\kappa-3}{\kappa}\right)v_{T_\perp}^2$$

with

$$v_{T_{\parallel,\perp}}^2=\frac{T_{\parallel,\perp}}{m}$$

when $\kappa>\frac{3}{2}$; Γ is the gamma function; and f_κ has been normalized so that $\int f_\kappa d^3v=1$.

In space plasma many particles possess high-energy tails with approximate power-law distributions in velocity space.[2] Such type of particles can be better modeled by a generalized Lorentzian or kappa distribution (containing the spectral index κ), than by the Maxwellian distribution. Important features of kappa distribution are that, first, at high velocities, this distribution obeys an inverse power law and second, in the limit when the spectral index $\kappa\rightarrow\infty$, the distribution approaches the Maxwellian distribution. In this sense, the kappa distribution is a generalization of Maxwellian distribution.

Vasyliunas[5] appears to have been the first to employ the general form of the kappa distribution and to note its relation to the Maxwellian. On the basis of theory and observation, it is noted that space and astrophysical plasmas often contain supra- and superthermal particles and that the kappa distribution offers a useful fit to their ''spectral'' distribution. So, many space plasmas can be modeled more effectively by a superposition of kappa distribution functions than by Maxwellian. The Maxwellian and kappa distributions differ substantially in the presence of the high-energy tail, but the difference become less significant as κ increases.

Several authors[2–4,6,7] employed the kappa distribution to study a number of wave modes in space plasmas and introduced a new function $Z_\kappa^*(\xi)$ with $\xi=\kappa+iy$, called the modified plasma dispersion function. This modified dispersion function is based on the generalized Lorentzian (kappa) particle distribution function analogous to the plasma dispersion function $Z(\xi)$ based on Maxwellian distribution. They evaluated the rate of Landau damping for electron plasma waves, ion acoustic waves, and electromagnetic (R-mode and L-mode) waves and noted that the presence of a high-energy tail leads to an increase in the damping rate of the electron plasma wave compared to that of Maxwellian plasma. This increase in the high-energy tail (low values of κ) confines weakly damped Langmuir oscillations to a very restricted frequency range above the electron plasma frequency. They, however, noted that the dispersive properties of Langmuir waves remain unaffected, if the ion term is included in the dispersion relation. This ion component plays an important role in the damping rate of the ion acoustic wave calculated with the generalized kappa distribution function. Consequently, the damping rate decreases.

In the present paper we introduce a three-dimensional generalized (r,q) distribution. This distribution function is of a more general form than the above-mentioned kappa distribution and hence better suited to model plasmas exhibiting characteristics which cannot be described by the kappa distribution functions. This distribution function is given by

$$f_{(r,q)}=\left(\frac{3}{4\pi\Psi_\perp^2\Psi_\parallel}\right)\times\left(\frac{\Gamma(q)}{(q-1)^{3/(2+2r)}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\left[1+\frac{1}{q-1}\left(\frac{v_\parallel^2}{\Psi_\parallel^2}+\frac{v_\perp^2}{\Psi_\perp^2}\right)^{r+1}\right]^{-q},$$

where q and r are the spectral indices, the thermal speed Ψ is related to the particle thermal velocity by v_T in the following manner:

$$\Psi_\parallel^2=\left(\frac{3(q-1)^{-1/(r+1)}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(\frac{3}{2+2r}\right)}{\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)}\right)v_{T_\parallel}^2$$

and

$$\Psi_\perp^2=\left(\frac{3(q-1)^{-1/(r+1)}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(\frac{3}{2+2r}\right)}{\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)}\right)v_{T_\perp}^2.$$

Here the specific forms of $\Psi_{\parallel,\perp}$ have been obtained in a manner similar to that used for obtaining $\theta_{\parallel,\perp}$ in the kappa distribution function.[1] We also note that the spectral indices r, q, satisfy the constraints $q>1$ and $q(1+r)>\frac{5}{2}$, which emerge from the normalization and the definition of temperature for a distribution function Γ is the gamma function and $f_{(r,q)}$ has been normalized such that $\int f_{(r,q)}d^3v=1$.

The above distribution is a generalized form of the kappa distribution function and reduces to kappa distribution function if $r=0$ and $q=\kappa+1$, and to a Maxwellian if $q\to\infty$ and $r=0$. If we increase the value of r keeping q fixed, then the contribution of high-energy particles reduces but the shoulders in the distribution function increase. The same also occurs if we fix the value of r and increase the value of q.

We adopt this distribution function because it gives a better data fit to the empirical data, especially when there are shoulders in the profile of the distribution function along with a high-energy tail. Employing kappa and generalized (r,q) distributions with real valued spectral indices κ and (r,q), we determine the dispersion relations for some electrostatic modes in dusty plasmas. The real and imaginary parts of the dielectric function $D(k,\omega)$ for these modes are calculated and then compared with those of the Maxwellian distribution.[8,9]

II. GENERAL FORM OF THE DISPERSION RELATION OF ELECTROSTATIC MODES

We consider an unmagnetized, collisionless multicomponent dusty plasma consisting of electrons, singly ionized positive ions, and negatively charged dust grains, which is characterized by equilibrium number density $n_{\alpha 0}$, temperature T_α, and mass m_α where $\alpha=i$, e, d denotes the species namely ions, electrons, and dust grains, respectively. The charge neutrality condition is $n_{i0}=n_{e0}+Z_{d0}n_{d0}$ where Z_{d0} is the equilibrium dust charge state.[10]

To calculate the perturbed distribution function, we use the linearized Vlasov–Poisson coupled system of equations and obtain a general expression for the dispersion function[9] $D(k,\omega)$ which can be written in the following manner in cylindrical coordinates:

$$D(k,\omega)=D_r+iD_i=1-\sum_\alpha\frac{2\pi\omega_{p\alpha}^2}{k^2}\left(1+i\omega_i\frac{\partial}{\partial\omega_r}\right)\times\left[\int_{-\infty}^{+\infty}\frac{(\partial f_{\alpha 0}/\partial v_\parallel)}{(v_\parallel-\omega_r/k)}dv_\parallel\int_0^\infty v_\perp dv_\perp\right]. \quad (2)$$

Here we have used $\omega=\omega_r+\iota\omega_\iota$, with $\omega_\iota\ll\omega_r$.

A. Electrostatic waves in Maxwellian plasmas

For Maxwellian distribution

$$f_0=\frac{1}{(2\pi)^{3/2}v_{T_\perp}^2v_{T_\parallel}}\exp\left[-\left(\frac{v_\parallel^2}{2v_{T_\parallel}^2}+\frac{v_\perp^2}{2v_{T_\perp}^2}\right)\right]$$

the dispersion relation becomes

$$1-\sum_{\alpha}\frac{\omega_{p\alpha}^{2}}{\sqrt{2\pi}k^{2}v_{T_{\parallel}}^{3}}\left(1+i\omega_{i}\frac{\partial}{\partial\omega_{r}}\right)\left[\oint\frac{v_{\parallel}}{(\omega_{r}/k-v_{\parallel})}\times\exp\left(-\frac{v_{\parallel}^{2}}{2v_{T_{\parallel}}^{2}}\right)dv_{\parallel}+i\pi v_{\parallel}\exp\left(-\frac{v_{\parallel}^{2}}{2v_{T_{\parallel}}^{2}}\right)\Bigg|_{v_{\parallel}=\omega_{r}/k}\right]=0. \tag{3}$$

The real and imaginary parts of $D(\mathbf{k},\omega)$ are in general given by

$$D_{r}=1-\frac{\omega_{p\alpha}^{2}}{\sqrt{2\pi}k^{2}v_{T_{\parallel}}^{3}}\int_{-\infty}^{+\infty}\frac{v_{\parallel}}{(\omega_{r}/k-v_{\parallel})}\exp\left(-\frac{v_{\parallel}^{2}}{2v_{T_{\parallel}}^{2}}\right)dv_{\parallel}, \tag{4}$$

$$D_{i}=-\left(\sqrt{\frac{\pi}{2}}\frac{\omega_{p\alpha}^{2}}{k^{2}v_{T_{\parallel}}^{3}}\right)\left[v_{\parallel}\exp\left(-\frac{v_{\parallel}^{2}}{2v_{T_{\parallel}}^{2}}\right)\right]\Bigg|_{v_{\parallel}=\omega_{r}/k} \tag{5}$$

which gives the growth (or damping) rate

$$\omega_{i}=-\frac{D_{i}(\mathbf{k},\omega_{r})}{\partial D_{r}(\mathbf{k},\omega_{r})/\partial\omega_{r}}. \tag{6}$$

1. Langmuir wave or electron wave

For the longitudinal (Langmuir) waves propagating in an unmagnetized, collisionless multicomponent dusty plasma with $\omega/k \gg (v_{Te_{\parallel}})$, so that the real part of the dispersion relation and the growth rate are obtained as[11–13]

$$\omega_{r}^{2}=\omega_{pe}^{2}[1+3k^{2}\lambda_{De}^{2}] \tag{7}$$

and

$$\omega_{i}=-\sqrt{\frac{\pi}{8}}\left(\frac{\omega_{r}}{k^{3}\lambda_{De}^{3}}\right)\exp\left[-\left(\frac{1}{2k^{2}\lambda_{De}^{2}}+\frac{3}{2}\right)\right]. \tag{8}$$

We note here that the specific effect of the dust does not appear in these waves, as these are high-frequency modes and the dust and the ions only contribute the neutralizing background. However, we have included this mode in our discussion for the sake of completeness.

2. Dust ion acoustic wave

The condition for dust ion acoustic wave is $(v_{Td_{\parallel}})$, $(v_{Ti_{\parallel}})<\omega/k<(v_{Te_{\parallel}})$. Applying this constraint yields the dispersion relation[11]

$$\frac{\omega_{r}^{2}}{k^{2}}=\frac{C_{DIA}^{2}}{(1+k^{2}\lambda_{De}^{2})}, \tag{9}$$

where C_{DIA} is the dust ion acoustic speed given by

$$C_{DIA}^{2}=\omega_{pi}^{2}\lambda_{De}^{2}=\frac{C_{s}^{2}}{1-\eta Z_{d}},$$

where $C_{s}^{2}=T_{e\parallel}/m_{i}$ is the ion sound speed, $\eta=n_{d0}/n_{i0}$ is the concentration of the negative dust, and Z_{d} is the dust charge state.

The growth rate for this wave is[11]

$$\omega_{i}=-\sqrt{\frac{\pi}{8}}\left(\frac{\omega_{r}}{(1+k^{2}\lambda_{De}^{2})^{3/2}(1-Z_{d}\eta)^{3/2}}\right)\times\left\{\left(\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{3/2}\exp\left(-\frac{T_{e\parallel}/T_{i\parallel}}{2(1+k^{2}\lambda_{De}^{2})(1-Z_{d}\eta)}\right)+(1-Z_{d}\eta)\sqrt{\frac{m_{e}}{m_{i}}}\right\}. \tag{10}$$

3. Dust acoustic waves

For dust acoustic wave $(v_{Td_{\parallel}})<\omega/k<(v_{Ti_{\parallel}})$, $(v_{Te_{\parallel}})$ so that the real and imaginary parts of $D(\mathbf{k},\omega)$ give us the dispersion relation and the growth rates:[11]

$$\frac{\omega_{r}^{2}}{k^{2}}=\frac{C_{DA}^{2}}{(1+k^{2}\lambda_{Deff}^{2})}, \tag{11}$$

where C_{DA} is the dust acoustic speed given by

$$C_{DA}^{2}=\frac{(\eta Z_{d}^{2})(T_{i\parallel}/m_{d})}{\left[1+\frac{T_{i\parallel}}{T_{e\parallel}}(1-\eta Z_{d})\right]},$$

where $\lambda_{Deff}^{-2}=(\lambda_{Di}^{-2}+\lambda_{De}^{-2})$ is the effective Debye length, $\lambda_{D\alpha}=(KT_{\alpha}/4\pi n_{\alpha 0}e^{2})^{1/2}$ is the Debye length of α species, and $\eta=n_{d0}/n_{i0}$ is the concentration of the negative dust.

The growth rate is given by the following expression:

$$\omega_{i}=-\sqrt{\frac{\pi}{8}}\left(\frac{\omega_{r}}{(1+k^{2}\lambda_{Deff}^{2})^{3/2}\left[1+\frac{T_{i\parallel}}{T_{e\parallel}}(1-\eta Z_{d})\right]^{3/2}}\right)\times\left\{\left(\eta Z_{d}^{2}\frac{T_{i\parallel}}{T_{d\parallel}}\right)^{3/2}\times\exp-\left[\frac{(\eta Z_{d}^{2})(T_{i\parallel}/T_{d\parallel})}{2(1+k^{2}\lambda_{Deff}^{2})[1+T_{i\parallel}/T_{d\parallel}(1-\eta Z_{d})]}\right]+\sqrt{\eta Z_{d}^{2}\frac{m_{i}}{m_{d}}}+(1-\eta Z_{d})\left(\frac{T_{i\parallel}}{T_{e\parallel}}\right)^{3/2}\sqrt{\eta Z_{d}^{2}\frac{m_{e}}{m_{d}}}\right\}. \tag{12}$$

B. Electrostatic dusty modes using non-Maxwellian distribution

In the following, we proceed to calculate the dispersion relation and the growth rates for kappa and the generalized (r,q) distributions.

1. Kappa distribution

Using the kappa distribution in Eq. (2), we obtain

$$1-\sum_{\alpha}\frac{2\omega_{p\alpha}^2}{\sqrt{\pi}k^2\theta_\parallel^3}\left(\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\right)\left(1+i\omega_i\frac{\partial}{\partial\omega_r}\right)$$
$$\times\left[\oint\frac{v_\parallel}{(\omega_r/k-v_\parallel)}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}\right)^{-\kappa-1}dv_\parallel\right.$$
$$\left.+i\pi v_\parallel\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}\right)^{-\kappa-1}\Bigg|_{v_\parallel=\omega_r/k}\right]=0. \qquad (13)$$

Here the integration with respect to $v_\perp$ have been performed by using the following integral:

$$\int_0^\infty v_\perp\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2}dv_\perp$$
$$=\frac{\kappa\theta_\perp^2}{2(\kappa+1)}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}\right)^{-\kappa-1}.$$

The real and imaginary parts of $D(\mathbf{k},\omega)$ then become

$$D_r=1-\frac{2\omega_{p\alpha}^2}{\sqrt{\pi}k^2\theta_\parallel^3}\left(\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\right)$$
$$\times\oint\frac{v_\parallel}{(\omega_r/k-v_\parallel)}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}\right)^{-\kappa-1}dv_\parallel \qquad (14)$$

and

$$D_i=-\left(\frac{2\sqrt{\pi}\omega_{p\alpha}^2}{k^2\theta_\parallel^3}\right)\left(\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\right)$$
$$\times\left[v_\parallel\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}\right)^{-\kappa-1}\right]\Bigg|_{v_\parallel=\omega_r/k}. \qquad (15)$$

Using Eqs. (13)–(15), we obtain the dispersion relations and the growth rates for the Langmuir wave, dust ion acoustic wave (DIA) and dust acoustic wave (DA) as follows.

a. Langmuir wave. For the high-frequency mode condition $\omega/k \gg v_{te}$, the velocity integral in Eq. (13),

$$I=-\int_{-\infty}^{+\infty}\frac{v_\parallel}{(v_\parallel-\omega_r/k)}\left(1+\frac{v_\parallel^2}{\kappa\theta_{\parallel e}^2}\right)^{-\kappa-1}dv_\parallel,$$

can be written in the following manner by expanding the denominator:

$$I=-\int_{-\infty}^{+\infty}\left(1+\frac{v_\parallel^2}{\kappa\theta_{\parallel e}^2}\right)^{-\kappa-1}$$
$$\times\left(\frac{v_\parallel}{\omega_r/k}+\frac{v_\parallel^2}{\omega_r^2/k^2}+\frac{v_\parallel^3}{\omega_r^3/k^3}\cdots\right)dv_\parallel.$$

Noting that

$$\int_{-\infty}^{+\infty}\left(1+\frac{v_\parallel^2}{\kappa\theta_{\parallel e}^2}\right)^{-\kappa-1}dv_\parallel=\sqrt{\pi}\theta_{\parallel e}\left(\frac{\kappa^{1/2}\Gamma\left(\kappa+\frac{1}{2}\right)}{\Gamma(\kappa+1)}\right),$$

$$\int_{-\infty}^{+\infty}v_\parallel\left(1+\frac{v_\parallel^2}{\kappa\theta_{\parallel e}^2}\right)^{-\kappa-1}dv_\parallel=0,$$

and

$$\int_{-\infty}^{+\infty}v_\parallel^2\left(1+\frac{v_\parallel^2}{\kappa\theta_{\parallel e}^2}\right)^{-\kappa-1}dv_\parallel=\frac{\sqrt{\pi}\theta_{\parallel e}^3}{2}\left(\frac{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}{\Gamma(\kappa+1)}\right),$$

we get

$$I=-\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{\sqrt{\pi}\theta_{\parallel e}^3}{2}\right)\left(\frac{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}{\Gamma(\kappa+1)}\right)$$
$$\times\left[1+\frac{3\theta_{\parallel e}^2}{2}\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{\kappa}{\kappa-\frac{3}{2}}\right)+\cdots\right]. \qquad (16)$$

Thus the real part of $D(\mathbf{k},\omega)$ gives the dispersion relation

$$1-\frac{\omega_{pe}^2}{k^2}\left[\frac{k^2}{\omega_r^2}\left\{1+\left(\frac{3\theta_{\parallel e}^2}{2}\right)\left(\frac{k^2}{\omega_r^2}\right)+\cdots\right\}\right]=0 \qquad (17)$$

which, for long wavelengths ($k^2\lambda_{De}^2 \gg 1$) reduces to

$$\omega_r^2=\omega_{pe}^2\left[1+\frac{3}{2}k^2\lambda_{De}^2\right]. \qquad (18)$$

The growth rate ω_i turns out to be

$$\omega_i=-\sqrt{\frac{\pi}{8}}\left(\frac{\omega_r}{k^3\lambda_{De}^3}\right)\left(\frac{\Gamma(\kappa+1)}{\left(\kappa-\frac{3}{2}\right)^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\right)$$
$$\times\left[1+\frac{1}{(2\kappa-3)k^2\lambda_{De}^2}+\frac{3}{(2\kappa-3)}\right]^{-\kappa-1}. \qquad (19)$$

We note that the real part of the dispersion relation given by Eq. (18) does not differ from the expression obtained by using Maxwellian. However, the growth rate ω_i in Eq. (19) differs significantly from the corresponding expression obtained from a Maxwellian distribution function. The numerical analysis presented later will highlight the differences in ω_i.

b. Dust ion acoustic wave. For dust ion acoustic wave, satisfying the condition $(v_{Td_\parallel})$, $(v_{Ti_\parallel})<\omega/k<(v_{Te_\parallel})$, the velocity integral for ions becomes

$$I_1=-\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{\sqrt{\pi}\theta_{\parallel i}^3}{2}\right)\left(\frac{\kappa^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}{\Gamma(\kappa+1)}\right)\times\left[1+\frac{3\theta_{\parallel i}^2}{2}\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{\kappa}{\kappa-\frac{3}{2}}\right)+\cdots\right]$$

and for electrons

$$I_2=\sqrt{\pi}\theta_{\parallel e}\left(\frac{\kappa^{1/2}\Gamma\left(\kappa+\frac{1}{2}\right)}{\Gamma(\kappa+1)}\right)\left[1-\left(\frac{2}{\theta_{\parallel e}^2}\right)\left(\frac{\omega_r^2}{k^2}\right)\left(\frac{\kappa+\frac{1}{2}}{\kappa}\right)\right]$$

so that, using appropriate expressions, we obtain

$$\frac{\omega_r^2}{k^2}=\frac{C_{DIA}^2}{\left(k^2\lambda_{De}^2+\frac{(\kappa-1/2)}{(\kappa-3/2)}\right)}, \tag{20}$$

where C_{DIA} is the dust ion acoustic speed given by $C_{DIA}^2=\omega_{pi}^2\lambda_{De}^2=C_s^2/(1-\eta Z_d)$ where $C_s^2=T_{e\parallel}/m_i$ is the ion sound speed and $\eta=n_{d0}/n_{i0}$ is the concentration of the negative dust.

The imaginary part of the dielectric constant which gives the growth rate of the mode is given by

$$\omega_i=-\sqrt{\frac{\pi}{8}}\left(\frac{\omega_r}{\left(k^2\lambda_{De}^2+\frac{(\kappa-1/2)}{(\kappa-3/2)}\right)^{3/2}(1-Z_d\eta)^{3/2}}\right)\times\left(\frac{\Gamma(\kappa+1)}{\left(\kappa-\frac{3}{2}\right)^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\right)\left\{\left(\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{3/2}\times\left[1+\frac{T_{e\parallel}/T_{i\parallel}}{(2\kappa-3)\left(k^2\lambda_{De}^2+\frac{(\kappa-1/2)}{(\kappa-3/2)}\right)(1-Z_d\eta)}\right]^{-\kappa-1}+(1-Z_d\eta)\sqrt{\frac{m_e}{m_i}}\right\}. \tag{21}$$

c. Dust acoustic waves. For dust acoustic wave the phase velocity constraint is $(v_{Td_\parallel})<\omega/k<(v_{Ti_\parallel})$, $(v_{Te_\parallel})$ so that the real and imaginary parts can be obtained by solving the velocity integral in Eq. (13) for the electrons, ions, and the dust species.

Thus the dispersion relation can be written as

$$\frac{\omega_r^2}{k^2}=\frac{C_{DA}^2}{\left(k^2\lambda_{Deff}^2+\frac{(\kappa-1/2)}{(\kappa-3/2)}\right)}. \tag{22}$$

The expression for ω_i is obtained as

$$\omega_i=-\sqrt{\frac{\pi}{8}}\left(\frac{\omega_r}{\left(k^2\lambda_{Deff}^2+\frac{(\kappa-1/2)}{(\kappa-3/2)}\right)^{3/2}\left[1+\frac{T_{i\parallel}}{T_{e\parallel}}(1-\eta Z_d)\right]^{3/2}}\right)\left(\frac{\Gamma(\kappa+1)}{(\kappa-3/2)^{3/2}\Gamma\left(\kappa-\frac{1}{2}\right)}\right)\times\left\{(\eta Z_d^2T_{i\parallel}/T_{d\parallel})^{3/2}\left[1+\frac{(\eta Z_d^2)(T_{i\parallel}/T_{d\parallel})}{(2\kappa-3)(k^2\lambda_{Deff}^2+(\kappa-1/2)/(\kappa-3/2))[1+T_{i\parallel}/Te(1-\eta Z_{d_0})]}\right]^{-\kappa-1}+\sqrt{\eta Z_d^2\frac{m_i}{m_d}}+(1-\eta Z_d)\left(\frac{T_{i\parallel}}{T_{e\parallel}}\right)^{3/2}\sqrt{\eta Z_d^2\frac{m_e}{m_d}}\right\}. \tag{23}$$

The comparison of the results derived from kappa distribution function with those of Maxwellian, reveals that the real parts and growth rates differ by some factors involving kappa and some gamma functions. Using the asymptotic form for the gamma function, it is not difficult to show that in the limit $\kappa\to\infty$, Eqs. (19)–(23) reduce to those for a Maxwellian plasma.

2. Generalized (r,q) distribution

In the present section we use the generalized (r,q) distribution function to evaluate the real and imaginary parts of the frequency for all the modes that have been considered in the preceeding section.

For the generalized (r,q) distribution, the dispersion relation becomes

$$1-\sum_\alpha\frac{3\omega_{p\alpha}^2}{2k^2\Psi_\parallel^3}\left(\frac{\Gamma(q)}{(q-1)^{-q+3/2(1+r)}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\right)\left(1+i\omega_i\frac{\partial}{\partial\omega_r}\right)\left[\oint\frac{v_\parallel}{(\omega_r/k-v_\parallel)}\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-q(r+1)}\times\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-r-1}\right]^{-q}dv_\parallel+i\pi v_\parallel\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-r-1}\right]^{-q}\Bigg|_{v_\parallel=\omega_r/k}\right]=0, \tag{24}$$

where the integration with respect to $v_\perp$ has been performed using the integral

$$\int_0^\infty v_\perp \left(\frac{v_\parallel^2}{\Psi_\parallel^2}+\frac{v_\perp^2}{\Psi_\perp^2}\right)^r \left[1+\frac{1}{(q-1)}\left(\frac{v_\parallel^2}{\Psi_\parallel^2}+\frac{v_\perp^2}{\Psi_\perp^2}\right)^{r+1}\right]^{-q-1} d v_\perp = \left(\frac{\left(\frac{1}{q-1}\right)^{-q-1}\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-q(r+1)}}{q(r+1)}\right)\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-r-1}\right]^{-q}.$$

The real and imaginary parts of $D(\mathbf{k},\omega)$ are

$$D_r = 1-\left(\frac{3\omega_{p\alpha}^2}{2k^2\Psi_\parallel^3}\right)\left(\frac{\Gamma(q)}{(q-1)^{-q+3/2(1+r)}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\right)\oint \frac{v_\parallel}{(\omega_r/k-v_\parallel)}\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-q(r+1)} \times\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-r-1}\right]^{-q} d v_\parallel \tag{25}$$

and

$$D_i = -\left(\frac{3\pi\omega_{p\alpha}^2}{2k^2\Psi_\parallel^3}\right)\left(\frac{\Gamma(q)}{(q-1)^{-q+3/2(1+r)}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\right) \times\left\{v_\parallel\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_\parallel^2}\right)^{-r-1}\right]^{-q}\right\}\Bigg|_{v_\parallel=\omega_r/k} \tag{26}$$

with

$$\omega_i = -\frac{D_i(\mathbf{k},\omega_r)}{\partial D_r(\mathbf{k},\omega_r)/\partial\omega_r}. \tag{27}$$

In the following, we determine the dispersion relations and growth rates for various modes.

a. Langmuir wave. Using the phase velocity constraint $\omega/k \gg (v_{Te_\parallel})^{1/2}$, the integral in Eq. (24) can be expanded,

$$\begin{aligned} I_3 &= -\oint \frac{v_\parallel}{(\omega_r/k-v_\parallel)}\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-r-1}\right]^{-q} d v_\parallel \\ &= -\int_{-\infty}^{+\infty}\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-r-1}\right]^{-q}\left(\frac{v_\parallel}{\omega_r/k}+\frac{v_\parallel^2}{\omega_r^2/k^2}+\frac{v_\parallel^3}{\omega_r^3/k^3}\cdots\right) d v_\parallel . \end{aligned} \tag{28}$$

Using the following integrals:

$$\int_{-\infty}^{+\infty} v_\parallel\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-r-1}\right]^{-q} d v_\parallel = 0,$$

$$\int_{-\infty}^{+\infty} v_\parallel^2\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-r-1}\right]^{-q} d v_\parallel = \frac{2}{3}\left(\frac{1}{\Psi_{\parallel e}^2}\right)^{-3/2}\left(\frac{\Gamma\left(\frac{5+2r}{2+2r}\right)\Gamma\left(\frac{-3+2q(r+1)}{2+2r}\right)}{(q-1)^{q-3/2(r+1)}\Gamma(q)}\right),$$

and

$$\int_{-\infty}^{+\infty} v_\parallel^4\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-q(r+1)}\left[1+(q-1)\left(\frac{v_\parallel^2}{\Psi_{\parallel e}^2}\right)^{-r-1}\right]^{-q} d v_\parallel = \frac{2}{5}\left(\frac{1}{\Psi_{\parallel e}^2}\right)^{-5/2}\left(\frac{\Gamma\left(\frac{7+2r}{2+2r}\right)\Gamma\left(\frac{-5+2q(r+1)}{2+2r}\right)}{(q-1)^{q-5/2(r+1)}\Gamma(q)}\right),$$

we obtain

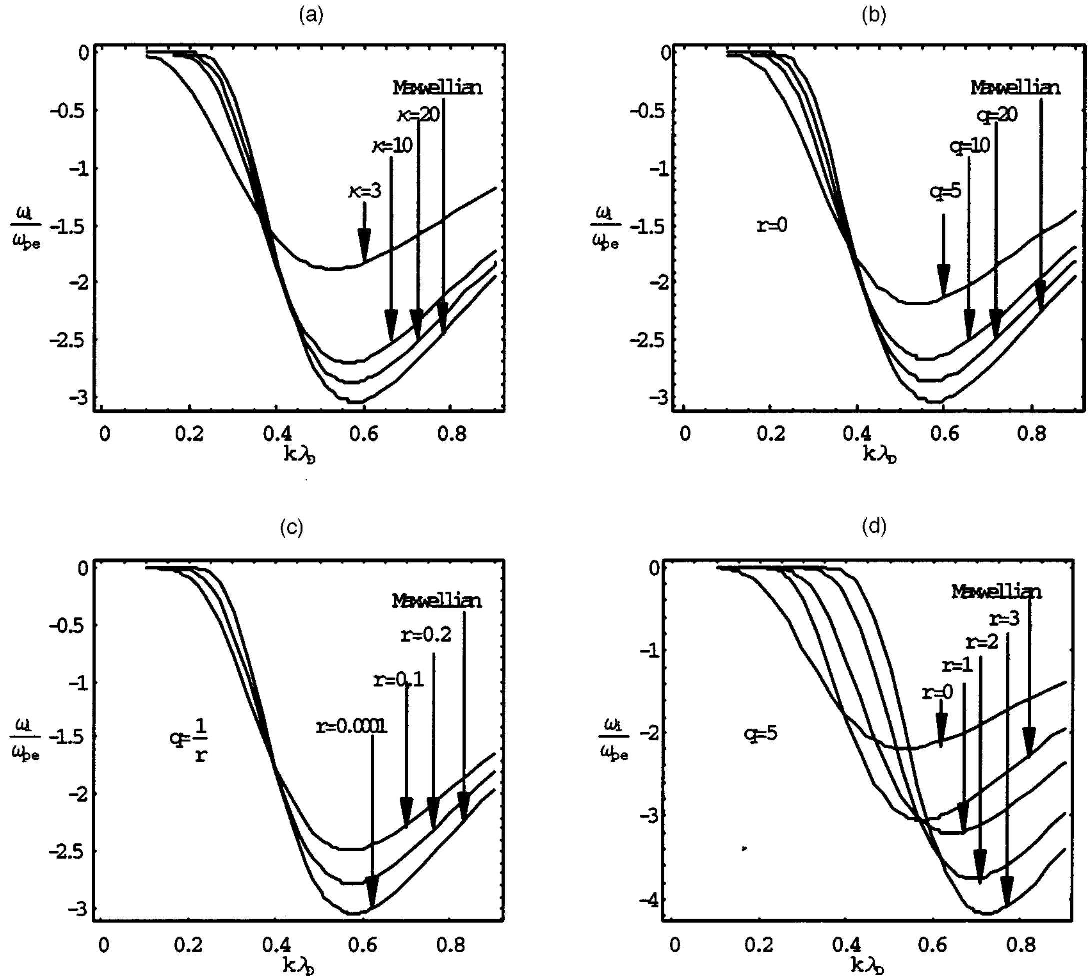

FIG. 1. Growth rates of Langmuir wave showing the comparison of Maxwellian with non-Maxwellian distributions for different values of spectral indices κ or q and r.

$$I_3=-\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{2\Psi_{\|e}^3}{3}\right)\left(\frac{\Gamma\left(\frac{5+2r}{2+2r}\right)\Gamma\left(\frac{-3+2q(r+1)}{2+2r}\right)}{(q-1)^{q-3/2(r+1)}\Gamma(q)}\right)\times\left[1+\frac{3\Psi_{\|e}^2}{2}\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{1}{q-1}\right)^{-1/(r+1)}\times\left(\frac{\Gamma\left(q-\frac{5}{2+2r}\right)\Gamma\left(1+\frac{5}{2+2r}\right)}{\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)+\cdots\right]. \quad (29)$$

Thus the real part of $D(\mathbf{k},\omega)$ gives the dispersion relation

$$\omega_r=\omega_{pe}\left[1+\frac{3}{2}k^2\lambda_{De}^2\right]. \quad (30)$$

The growth rate ω_i turns out to be

$$\omega_i=-C\left(\frac{3\pi}{4}\right)\left(\frac{\omega_r}{k^3\lambda_{De}^3}\right)\times\left\{1+\frac{1}{q-1}\left[\left(\frac{1}{k^2\lambda_{De}^2}+3\right)B\right]^{r+1}\right\}^{-q}, \quad (31)$$

where

$$B=\frac{\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)}{3(q-1)^{-(1/r+1)}\Gamma\left(\frac{3}{2+2r}\right)\Gamma\left(q-\frac{3}{2+2r}\right)},$$

$$C=\left(\frac{\Gamma(q)}{\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\left(\frac{\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)}{3\Gamma\left(\frac{3}{2+2r}\right)\Gamma\left(q-\frac{3}{2+2r}\right)}\right)^{3/2}.$$

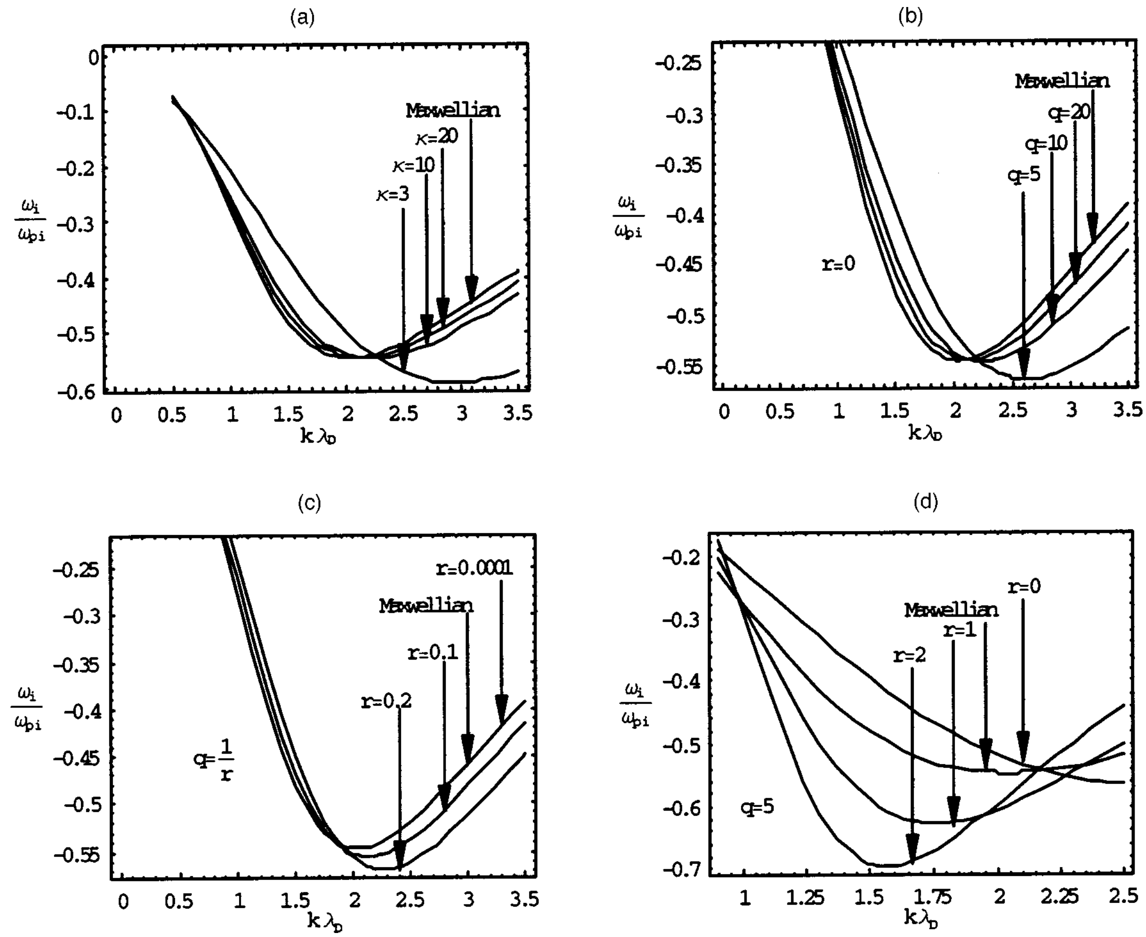

FIG. 2. Growth rates of dust ion acoustic wave showing the comparison of Maxwellian with non-Maxwellian distributions for different values of spectral indices κ or q and r.

b. Dust ion acoustic wave. The velocity integral in Eq. (24) for ions is

$$I_4=-\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{2\Psi_{\parallel i}^3}{3}\right)\left(\frac{\Gamma\left(\frac{5+2r}{2+2r}\right)\Gamma\left(\frac{-3+2q(r+1)}{2+2r}\right)}{(q-1)^{q-3/2(r+1)}\Gamma(q)}\right)$$
$$\times\left[1+\frac{3\Psi_{\parallel i}^2}{2}\left(\frac{k^2}{\omega_r^2}\right)\left(\frac{1}{q-1}\right)^{-1/(r+1)}\right.$$
$$\left.\times\left(\frac{\Gamma\left(q-\frac{5}{2+2r}\right)\Gamma\left(1+\frac{5}{2+2r}\right)}{\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\right] \tag{32}$$

and for electrons

$$I_5=2\left(\frac{\Gamma\left(\frac{3+2r}{2+2r}\right)\Gamma\left(\frac{-1+2q(r+1)}{2+2r}\right)}{(q-1)^{q-1/2(r+1)}\Gamma(q)}\right)\Psi_{\parallel e}. \tag{33}$$

These integrals yield dispersion relation

$$\frac{\omega_r^2}{k^2}=\frac{C_{DIA}^2}{(1-\eta Z_d)(k^2\lambda_{De}^2+A)}, \tag{34}$$

where C_{DIA} has been defined earlier and A is given by

$$A=\frac{\Gamma\left(q-\frac{1}{2+2r}\right)\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)\Gamma\left(1+\frac{1}{2+2r}\right)}{\Gamma\left(\frac{3}{2+2r}\right)\Gamma\left(q-\frac{3}{2+2r}\right)^2\Gamma\left(1+\frac{3}{2+2r}\right)}.$$

From the imaginary part of the dielectric constant we obtain ω_i for the dust ion acoustic wave

$$\omega_i=-C\left(\frac{3\pi}{4}\right)\left(\frac{|\omega_r|}{[(k^2\lambda_{De}^2+A)(1-Z_d\eta)]^{3/2}}\right)\times\left[\left(\frac{T_{e\parallel}}{T_{i\parallel}}\right)^{3/2}\right.$$
$$\times\left[1+\frac{1}{q-1}\left(\frac{(T_{e\parallel}/T_{i\parallel})(B)}{(k^2\lambda_{De}^2+A)(1-Z_d\eta)}\right)^{r+1}\right]^{-q}$$
$$\left.+(1-Z_d\eta)\sqrt{\frac{m_e}{m_i}}\right]. \tag{35}$$

c. Dust acoustic waves Calculating the velocity integrals in Eq. (24) for dust, ions, and electrons according to the condition, we obtain the dispersion relation

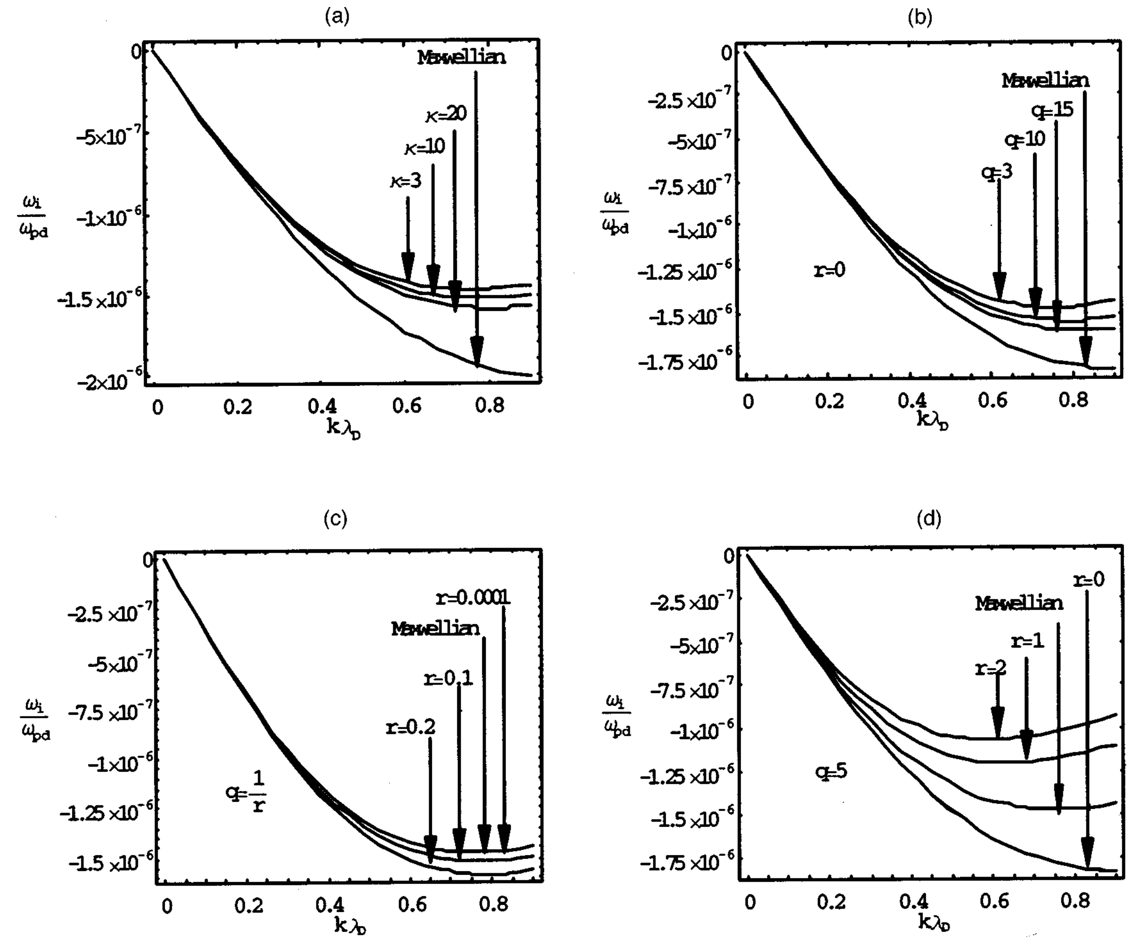

FIG. 3. Growth rates of dust acoustic wave showing the comparison of Maxwellian with non-Maxwellian distributions for different values of spectral indices κ or q and r.

$$\frac{\omega_r^2}{k^2}=\frac{Z_d\eta C_{DA}^2}{(k^2\lambda_{Deff}^2+A)\left[1+\frac{T_{i\parallel}}{T_{e\parallel}}(1-\eta Z_{d_0})\right]}. \tag{36}$$

The value of D_i can be obtained by substituting the values of $\partial f_{\alpha 0}/\partial v|_{v=\omega_r/k}$ and assuming C_{DA}^2/v_{ti}^2 and C_{DA}^2/v_{te}^2 negligibly small. As a result we obtain ω_i,

$$\omega_i=-\left(\frac{3\pi}{4}C\right)\left(\frac{\omega_r}{\left[(k^2\lambda_{Deff}^2+A)\left(1+\frac{T_{i\parallel}}{T_{e\parallel}}(1-Z_d\eta)\right)\right]^{3/2}}\right.$$
$$\times\left\{[\eta Z_d^2(T_{i\parallel}/T_{d\parallel})]^{3/2}\left(1+\frac{1}{q-1}\left\{(\eta Z_d^2)(T_{i\parallel}/T_{d\parallel})\right.\right.\right.$$
$$\left.\left.\left.\times(B)/(k^2\lambda_{Deff}^2+A)\left[1+\frac{T_{i\parallel}}{T_{e\parallel}}(1-\eta Z_{d0})\right]\right\}^{r+1}\right)^{-q}$$
$$\left.+\sqrt{\eta Z_d^2(m_i/m_d)}+(1-\eta Z_d)\left(\frac{T_{i\parallel}}{T_{e\parallel}}\right)^{3/2}\sqrt{\eta Z_d^2(m_e/m_d)}\right\}.$$

It may be noted that, here too as in the κ case, both the real parts and growth rates for (r,q) distribution differ from the results of Maxwellian plasma through the appearance of the gamma functions which in turn depends upon q and r. For $r=0$, $q\rightarrow\infty$, (r,q) distribution approaches to Maxwellian. However, for other values of r the dispersion function differs from the Maxwellian.

III. NUMERICAL RESULTS AND DISCUSSION

We have derived in computable form, expressions for electrostatic waves propagating in an unmagnetized dusty plasma modeled by generalized kappa and (r,q) particle distribution functions. To illustrate the behavior of the distribution functions, we present in Figs. 1–3, graphs of growth rates of different electrostatic waves obtained by using Maxwellian, kappa and (r,q) distribution functions. For our numerical analysis we have used the following plasma parameters:[14] $T_{e\parallel}=1ev$, $T_{i\parallel}=0.11ev$, $T_{d\parallel}=0.1ev$, $n_i=10^9\ \mathrm{cm}^{-3}$, $n_e=9.8\times10^8\ \mathrm{cm}^{-3}$, $n_d=10^4\ \mathrm{cm}^{-3}$, $m_d=10^8 m_p$, $Z_d=10$.

We have evaluated the rate of Landau damping for Langmuir, dust ion acoustic, and dust acoustic waves in a generalized Lorentzian plasma containing a high-energy tail modeled with a spectral index κ. The presence of a high-energy tail (for low value of κ) leads to a decrease in the damping rate of Langmuir wave compared to that of Maxwellian plasma. In the case of ion acoustic and dust acoustic waves (calculated with κ distribution function) in which ions and dust particles, respectively, play an important role, the damping rate also decreases. It can be seen from the series of graphs [Figs. 1(a), 2(a), and 3(a)] how the growth rates reduce to Maxwellian, when κ increases. On increasing the value of κ, the contribution of high-energy particles reduces and consequently the damping decreases.

For the case of a generalized (r,q) distribution function we have carried out a similar analysis as above and have compared the results to those obtained for a Maxwellian and a kappa distribution function. We note that the parameters r and q in general represent the flat part and the high-energy tail of the distribution function, respectively. It is evident from graphs shown in Figs. 1(a), 2(b), and 3(b) that for $r=0$ and large values of q, the (r,q) distribution exhibits the same behavior as that of kappa distribution. But, if we fix the value of q and increase the value of r, then the contribution of high-energy particles reduces and the shoulders become more prominent in the distribution function. When the growth rates of the various wave modes are compared we see from Figs. 1(d), 2(d), and 3(d) that in all the three cases (i.e., Langmuir wave, DIA and DA waves) damping increases as compared to Maxwellian or a κ plasma as the value of the spectral index r increases for a fixed value of q. However, with increasing q and a fixed value of r the damping decreases. This is as expected because as r increases the tail in the distribution function begins to vanish and the particles participating in the wave particle interaction become such that those giving energy to the wave become fewer than those receiving energy from the wave. (For different values of the spectral indices r and q the plots of the distribution function have been presented elsewhere.[15])

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