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Effects of positron concentration, ion temperature, and plasma β value on linear and nonlinear two-dimensional magnetosonic waves in electron–positron–ion plasmas

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Magnetosonic waves are intensively studied due to their importance in space plasmas and also in fusion plasmas where they are used in particle acceleration and heating experiments. This work considers magnetosonic waves propagating obliquely at an angle θ to an external magnetic field in an electron–positron–ion plasma, using the effective one-fluid magnetohydrodynamic model. Two separate modes (fast and slow) for the waves are discussed in the linear approximation, and the Kadomstev–Petviashvilli soliton equation is derived by using reductive perturbation scheme for these modes in the nonlinear regime. It is observed that for both the modes the angle θ, positron concentration, ion temperature, and plasma β-value affect the propagation properties of solitary waves and behave differently from the simple electron–ion plasmas. Likewise, current density, electric field, and magnetic field for these waves are investigated, for their dependence on the above mentioned parameters. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1814115]

I. INTRODUCTION

In contrast to the usual plasma consisting of electrons and positive ions, it has been observed that nonlinear waves in plasmas having an additional component of positrons behave differently.[1] Electron–positron–ion plasmas appear in the early universe,[2–4] in the active galactic nuclei,[5] pulsar magnetospheres,[6] and also in the solar atmosphere[7] and in fact most of the astrophysical plasmas usually consist of ions, in addition to electrons and positrons, and it is pertinent to study the behavior of nonlinear wave motions in an electron–positron–ion plasma. When positrons are introduced in the plasma, the response of the plasma to disturbances changes drastically.

Recently, there has been a great deal of interest in studying linear as well as nonlinear wave motions in electron–positron plasmas.[8–14] The nonlinear studies have been focused on the nonlinear self-consistent structures,[8,9] such as envelope solitons, vortices, etc. In Ref. 15 the solitary wave solutions propagating perpendicular to the magnetic field were discussed. Effects of particle reflection (by the magnetic field) have been investigated both theoretically and numerically in Refs. 16 and 17. Korteweg–de Vries (KdV) equation for magnetosonic waves and modified KdV equation for the Alfvén waves were also discussed.[18,19]

Nonlinear waves propagating in electron–positron–ion plasmas have also attracted a great deal of attention in examining the nonlinear structures.[20–24] Energy transfer in a shock wave in an electron–positron–ion $(e\text{-}p\text{-}i)$ plasma was studied using particle simulation in Ref. 20, in which ions were assumed to be a minority population. The effect of the ion temperature on large amplitude ion-acoustic waves in an electron–positron–ion plasma was studied theoretically by Nejoh,[21] where it was shown that the ion temperature increases the maximum Mach number and decreases the amplitude of the ion-acoustic waves. The region of existence of soliton structures sensitively depends on the ion temperature and the positron density and temperature.

Ion-acoustic waves propagating obliquely with respect to an external magnetic field in a homogeneous magnetized electron–positron–ion plasma were studied by Mahmood *et al.*,[22] here the authors found that the amplitude of the solitary structure increases with the percentage presence of positrons.

Berezhiani *et al.*[23] have investigated envelope solitons of electromagnetic waves in three-component electron–positron–ion plasmas, and it was shown that electromagnetic radiation of arbitrary amplitude in presence of heavy ions, in contrast to the case of pure *e*-*p* plasma, may be localized with the generation of a humped ambipolar potential in the plasma, i.e., the driving field intensity creates intense soliton in the plasma with the generation of double hump ambipolar potentials. With increase of the value of n_{po}/n_{eo}, they found a tendency that a single hump soliton becomes a double humped one. In their investigation they neglected the ion dynamics.

Ion-acoustic solitons in electron–positron–ion plasma were also studied by Popel *et al.*,[24] where they presented an investigation of the nonlinear ion-acoustic waves in the presence of cold ions and hot electrons and positrons. In this case the ion dynamics were shown to be governed by hydrodynamic equations, whereas the electron and positron fluids follow the Boltzmann distribution. Accordingly, the phase velocity of the oscillation is assumed to be smaller (larger) than the thermal velocity of the electrons and positrons (ions). It was found that the presence of the positron component in such a multispecies plasma can result in reduction of the ion-acoustic soliton amplitudes.

Nonlinear characteristics of magnetosonic waves have been the subject of investigation by many authors[25–29] due to their importance in space plasmas and also in fusion plasmas

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where they are used in particle acceleration and heating experiments. Adlam and Allen[25] and Davis *et al.*[26] found stationary solutions of finite amplitude magnetosonic waves propagating perpendicular to a magnetic field. Gardner and Morikawa[27] showed that these waves propagating perpendicular to a magnetic field can be described by the KdV equation. Magnetosonic waves propagating obliquely to an external magnetic field were also shown to obey the KdV equation.[28,29] De Vito and Pantano[30] have investigated the propagation of two-dimensional nonlinear magnetosonic waves in a cold plasma where only one mode of this wave is excited and it was shown that the propagation characteristics of these waves is governed by the Kadomstev–Petvashvilli (KP) equation. Obliquely propagating nonlinear magnetosonic waves in warm plasma were investigated by Shah and Bruno,[31] it was found that for both slow and fast modes the governing nonlinear evolution equation is the KP equation.

However, the study of obliquely propagating (with respect to an external magnetic field) two-dimensional nonlinear magnetosonic waves using the Kadomstev–Petviashvilli (KP) equation has not yet been studied in an electron–positron–ion plasmas. Hence in this paper, we theoretically and numerically investigate two-dimensional linear and nonlinear propagation of fast and slow modes for magnetosonic waves in electron–positron–ion plasmas. It is shown that propagation of such waves is also governed by the KP soliton equation. The effects of ion temperature, plasma β (the ratio of kinetic energy to magnetic energy) value, and the concentration of positrons modify the magnetosonic wave dynamics both in the linear as well as in the nonlinear regimes. The main goal of the paper is to derive a compact closed set of nonlinear equations [for (e-p-i) hot plasmas], which would describe two-dimensional nonlinear magnetosonic waves and provide a basis for further analysis. The organization of the paper is as follows

In Sec. II the basic set of nonlinear equations and dispersion relation are presented and the nonlinear KP equation is obtained by using reductive perturbation technique. In Sec. III the numerical results of linear and nonlinear magnetosonic waves (MAW) are presented along with brief discussion of these results. Finally in Sec. IV, conclusion of the results is presented.

II. BASIC EQUATIONS AND FORMULATION

Let us consider a magnetized three-component (e-p-i) plasma. We consider a Cartesian coordinate system, where the ambient magnetic field B_o lies in (x,y) plane making a small angle θ with the x axis and propagation is considered in the (x,z) plane. The basic equations that are used in this paper are the effective one-fluid isotropic isothermal magnetohydrodynamic (MHD) equations. An effective one-fluid MHD model for e-p-i plasma can be developed by starting with the usual fluid equations for the electrons, positrons, and ions. For low frequency motion one can neglect the electron and positron inertia term in their corresponding momentum equations and due to the fact that we consider only low frequency waves the displacement current term in Ampère's law is also neglected. To develop equations for effective one-fluid MHD model for e-p-i plasmas we follow the work of Rao[32] and the basic equations for e-p-i plasmas can be written as

$$m_i n_i \frac{d\vec{v}_i}{dt} = n_i e\vec{E} + \frac{n_i e}{c}(\vec{v}_i \times \vec{B}) - \vec{\nabla} p_i, \tag{1}$$

$$0 = n_p e\vec{E} + \frac{n_p e}{c}(\vec{v}_p \times \vec{B}) - \vec{\nabla} p_p, \tag{2}$$

$$0 = -n_e e\vec{E} + \frac{n_e e}{c}(\vec{v}_e \times \vec{B}) - \vec{\nabla} p_e, \tag{3}$$

where Eqs. (1)–(3) are the equations of motion for the ions, positrons, and electrons, respectively, with $m_e = m_p = 0$.

For number density conservation, we use the continuity equations, which are given by

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot (n_s \vec{v}_s) = 0 \tag{4}$$

and the Maxwell equations are given by

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j}, \tag{5}$$

where

$$\vec{j} = \sum_{s=e,p,i} q_s n_s \vec{v}_s,$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}. \tag{6}$$

Here v_s, $n_s(n_{so})$, and p_s are the fluid velocity, perturbed (unperturbed) particle density, and thermal pressure of s species, respectively, where $s=e,p,i$ stands for electrons, positrons, and ions, respectively, $p_s = n_s T_s$, where T_s is the thermal energy, m_i is the ion mass, q is the charge on s, e is the electron charge, c is the speed of light, $\vec{E}$ is the electric field vector, $\vec{B}$ is the magnetic field vector, j is the current density, and $(d/dt) = (\partial/\partial t) + (v \cdot \nabla)$ is the hydrodynamic derivative.

In order to derive the basic governing equations of the MHD model, we substitute for $\vec{v}_e$ in Eq. (3) from Eq. (5) to obtain

$$\vec{E} = -\frac{1}{n_e c}\left(n_i\vec{v}_i + n_p\vec{v}_p - \frac{c}{4\pi}\vec{\nabla} \times \vec{B}\right) \times \vec{B} - \frac{\vec{\nabla} p_e}{n_e e}. \tag{7}$$

Using Eqs. (1), (2), and (7), we obtain the momentum equations for ions and positrons, respectively,

$$m_i n_i \frac{d\vec{v}_i}{dt} = \frac{n_i}{4\pi n_e}(\vec{\nabla} \times \vec{B}) \times \vec{B} + \frac{n_i n_p e}{c n_e}(\vec{v}_i - \vec{v}_p) \times \vec{B} - \frac{n_i}{n_e}\vec{\nabla} p_e - \vec{\nabla} p_i, \tag{8}$$

$$0 = -\vec{\nabla} p_p - \frac{n_p}{n_e}\vec{\nabla} p_e - \frac{n_i n_p e}{c n_e}(\vec{v}_i - \vec{v}_p) \times \vec{B} + \frac{n_p}{4\pi n_e}(\vec{\nabla} \times \vec{B}) \times \vec{B}, \qquad (9)$$

and further by using the quasineutrality condition

$$n_i + n_p \simeq n_e \qquad (10)$$

and adding Eqs. (8) and (9) we obtain the effective one-fluid momentum equation

$$n_i \frac{d\vec{v}_i}{dt} = \frac{n_{io} v_A^2}{B_o^2}(\nabla \times \vec{B}) \times \vec{B} - c_s^2(1+\sigma)\vec{\nabla} n_i - c_s^2(1+\rho)\vec{\nabla} n_p. \qquad (11)$$

On the other hand, eliminating $\vec{E}$ between Eqs. (1) and (6), the magnetic field induction equation takes the form

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v}_i \times \vec{B}) - \frac{B_o}{\Omega_i}\left(\vec{\nabla} \times \frac{d\vec{v}_i}{dt}\right). \qquad (12)$$

Here the subscript zero represents the background quantities, $v_A = B_o/(4\pi n_{io} m_i)^{1/2}$ is the Alfvén velocity, $c_s = (T_e/m_i)^{1/2}$ is the ion sound speed, $\Omega_i = (eB_o/m_i c)$ is the ion gyrofrequency, $\sigma = T_i/T_e$ is the ratio of ion and electron temperature, and $\rho = (Tp/T_e)$ is the ratio of positron and electron temperatures.

From Eq. (9) we obtain

$$\vec{v}_{p\perp} \approx \frac{1}{B^2}\vec{B} \times (\vec{v}_i \times \vec{B}) + \frac{c(n_i + n_p)}{n_i n_p e B^2}(\vec{B} \times \vec{\nabla} p_p) + \frac{c}{n_i e B^2}(\vec{B} \times \vec{\nabla} p_e) - \frac{c}{4\pi e n_i B^2}\vec{B} \times [(\vec{\nabla} \times \vec{B}) \times \vec{B}]. \qquad (13)$$

Neglecting the positron fluid velocity component parallel to the magnetic field[32] and using Eq. (13) in the positron continuity equation (4), we finally obtain

$$\frac{\partial n_p}{\partial t} + \vec{\nabla}_\perp \cdot \left[\frac{n_p}{B^2}\vec{B} \times (\vec{v}_i \times \vec{B}) - \frac{c n_p}{4\pi e n_i B^2}\vec{B} \times [(\vec{\nabla} \times \vec{B}) \times \vec{B}] + \frac{c(n_i + n_p)}{e n_i B^2}(\vec{B} \times \vec{\nabla} p_p) + \frac{c n_p}{n_i e B^2}(\vec{B} \times \vec{\nabla} p_e)\right] = 0. \qquad (14)$$

Equations (11) and (14) together with the ion continuity equation (4) and the induction equation (12) are the basic governing equations of the effective one-fluid model for electron–positron–ion (e-p-i) plasmas.

We need a procedure which, in a systematic fashion, will reduce the above sets of equations to simpler forms so that linear and nonlinear analyses are possible. Such procedures are usually perturbative in nature and the one we use is known as the reductive perturbation method. The main significance of the reductive perturbation technique is that it enables us to look in a natural way for long waves, that is, waves whose wavelengths are long compared to a typical length scale. This technique has been extensively used in plasma physics[33–39] and is mostly applied to small amplitude nonlinear waves (e.g., Refs. 33 and 34). This technique, on a mathematical level, enables us to rescale both space and time variables in the original equations of the system, thereby making it possible to consider long wave length phenomena. This rescaling isolates from the system the relevant equations, which describes how the system reacts on new space and time scales. The reduction process is slightly ill defined in that it rests on experience in knowing how to pick the relevant scales; however, a method on how to pick these scales is given in Ref. 40, where it is used for deriving ion-acoustic soliton. It may be noted that the reductive perturbation technique is a special form of multiple scale expansion technique (e.g., Ref. 38). However in our case we follow Refs. 30 and 31, by expanding the variables in the following manner:

$$\begin{aligned}
n_s &= n_{so} + \epsilon n_{s1} + \epsilon^2 n_{s2} + \cdots,\\
v_{sx} &= \epsilon u_{s1} + \epsilon^2 u_{s2} + \cdots,\\
v_{sy} &= \epsilon v_{s1} + \epsilon^2 v_{s2} + \cdots,\\
v_{sz} &= \epsilon^{3/2} w_{s1} + \epsilon^{5/2} w_{s2} + \cdots,\\
B_x &= B_o \cos\theta,\\
B_y &= B_o \sin\theta + \epsilon B_{y1} + \epsilon^2 B_{y2} + \cdots,\\
B_z &= \epsilon^{3/2} B_{z1} + \epsilon^{5/2} B_{z2} + \cdots.
\end{aligned} \qquad (15)$$

It is noted here that all perturbed quantities are functions of x, z, and t, and ϵ is a small parameter such that $\epsilon < 1$. Stretched variables are introduced in the standard fashion

$$\begin{aligned}
\xi &= \epsilon^{1/2}(x - v_{ph} t),\\
\eta &= \epsilon z,\\
\tau &= \epsilon^{3/2} t,
\end{aligned} \qquad (16)$$

where v_{ph} is the phase velocity and its exact expression is evaluated below. This variable stretching procedure (e.g., Refs. 35 and 39) assumes the possibility of introducing new coordinates and variables such that the slowness of coordinate dependence and smallness of some of the physical variables can be taken out in a uniform way.

Substituting Eqs. (15) and (16) into ion continuity equation of Eq. (4) and in Eqs. (11), (12), and (14), and collecting terms of lowest order in ϵ, i.e., $(\epsilon^{3/2})$ we obtain

$$\begin{aligned}
&-v_A \lambda \frac{\partial n_{i1}}{\partial \xi} + n_{io}\frac{\partial u_{i1}}{\partial \xi} = 0,\\
&\frac{\lambda}{v_A}\frac{\partial u_{i1}}{\partial \xi} = \sin\theta \frac{\partial}{\partial \xi}\frac{B_{y1}}{B_o} + \beta(1+\sigma)\frac{\partial}{\partial \xi}\frac{n_{i1}}{n_{io}} + \beta(1+\rho)\frac{\partial}{\partial \xi}\frac{n_{p1}}{n_{io}},\\
&\frac{\lambda}{v_A}\frac{\partial v_{i1}}{\partial \xi} = -\cos\theta \frac{\partial}{\partial \xi}\frac{B_{y1}}{B_o},\\
&v_A \lambda \frac{\partial}{\partial \xi}\frac{B_{y1}}{B_o} = \sin\theta \frac{\partial u_{i1}}{\partial \xi} - \cos\theta \frac{\partial v_{i1}}{\partial \xi},
\end{aligned} \qquad (17)$$

$$v_A\lambda\frac{\partial}{\partial\xi}\frac{n_{p1}}{n_{po}}=\sin^2\theta\frac{\partial u_{i1}}{\partial\xi}-\sin\theta\cos\theta\frac{\partial v_{i1}}{\partial\xi},$$

where $\beta=c_s^2/v_A^2$ and λ is the normalized phase velocity given by $\lambda=v_{ph}/v_A$, and let $n_{i1}/n_{io}=\phi$, $n_{p1}/n_{po}=n_{p1}$, $n_{po}/n_{eo}=p$, $n_{po}/n_{io}=p/(1-p)$, $B_{y1}/B_o=B_{y1}$, $(B_{z1}/B_o)=B_{z1}$, $w_1=(w_{i1}/v_A)$, $u_1=(u_{i1}/v_A)$, $v_1=(v_{i1}/v_A)$, $\gamma=\beta(1+\sigma)$, $\zeta=\beta(1+\rho)$, and $j_\mu=j_\mu/en_{io}v_A$ (where $\mu=x,y,z$).

Using the set of Eqs. (17), the fluctuating variables u_1, v_1, B_{y1}, and n_{p1} can be expressed in terms of ϕ as

$$u_1=\lambda\phi,$$

$$v_1=-\frac{\lambda\cos\theta}{\sin\theta}\left[\frac{\lambda^2-\gamma-\zeta\left(\frac{p}{1-p}\right)\sin^2\theta}{\lambda^2+\zeta\left(\frac{p}{1-p}\right)\cos^2\theta}\right]\phi,$$

$$B_{y1}=\frac{\lambda^2}{\sin\theta}\left[\frac{\lambda^2-\gamma-\zeta\left(\frac{p}{1-p}\right)\sin^2\theta}{\lambda^2+\zeta\left(\frac{p}{1-p}\right)\cos^2\theta}\right]\phi, \tag{18}$$

$$n_{p1}=\left[\frac{\lambda^2-\gamma\cos^2\theta}{\lambda^2+\zeta\left(\frac{p}{1-p}\right)\cos^2\theta}\right]\phi.$$

Using Eqs. (17) and (18) one can obtain

$$\lambda^2=\frac{1+\gamma+\zeta\left(\frac{p}{1-p}\right)\sin^2\theta}{2}\times\left[1\pm\left(1-\frac{4\gamma\cos^2\theta}{\left[1+\gamma+\zeta\left(\frac{p}{1-p}\right)\sin^2\theta\right]^2}\right)^{1/2}\right]. \tag{19}$$

Equation (19) is the linear dispersion relation for two-dimensional magnetosonic waves propagating obliquely, i.e., making an angle θ with external magnetic field in an electron–positron–ion (e-p-i) plasma. This expression shows that the obliquely propagating low frequency magnetosonic wave depends on the angle θ, ratio of ion to electron temperatures σ, the plasma β (the ratio of kinetic energy to magnetic energy)-value, and the relative positron concentration p. For $p=0$, we get the relation given in Ref. 41 for two component electron–ion plasmas. In expression (19) the upper (positive) sign corresponds to the fast mode and the lower (negative) sign to the slow magnetosonic mode. We note here that at $\theta=0$, Eq. (18) shows apparent divergences for v_1 and B_{y1}. The reasons for this are the following. First at $\theta=0$, the wave decouples into a parallel propagating Alfvén wave ($\lambda^2=1$ or $\omega=kv_A$) and an ion-acoustic wave ($\lambda^2=\gamma$ or $\omega=k\,c_{IA}$); for the former case $v_1(v_{y1})$ and $B_{y1}\neq 0$ but $n_{i1}=n_{e1}=n_{p1}=0$ and for the latter, which is an electrostatic wave, $v_1(v_{y1})=B_{y1}=0$, but the fluctuating number densities are not equal to zero. Thus the set of expressions given by Eq. (18) should not be used to obtain the limiting cases without taking the above into account. However, the linear dispersion relation yields the correct results in the case when $\theta=0$. We further mention that at $\theta=\pi/2$ we obtain a perpendicularly propagating magnetosonic wave with the effect of positron concentration.

The detailed calculations for further analysis of linear dispersion relation given by Eq. (19) are found in the Appendix.

To develop the nonlinear KP soliton equation for both the slow and fast modes of the two-dimensional magnetosonic waves in electron–positron–ion (e-p-i) plasmas, we substitute Eqs. (15) and (16) into the ion continuity equation [Eq. (4)] and in Eqs. (11), (12), and (14), and then collecting terms of order ϵ^2 and $\epsilon^{5/2}$, we have in order ϵ^2

$$\lambda\frac{\partial w_1}{\partial\xi}=\sin\theta\frac{\partial B_{y1}}{\partial\eta}-\cos\theta\frac{\partial B_{z1}}{\partial\xi}+\gamma\frac{\partial\phi}{\partial\eta}+\zeta\left(\frac{p}{1-p}\right)\frac{\partial n_{p1}}{\partial\eta},$$

$$-\lambda\frac{\partial B_{z1}}{\partial\xi}=\cos\theta\frac{\partial w_1}{\partial\xi}+\frac{\lambda v_A}{\Omega_i}\frac{\partial^2 v_1}{\partial\xi^2}. \tag{20}$$

From Eqs. (18) and (19), we get the following two equations for w_1 and B_{z1} in terms of ϕ and λ:

$$\frac{\partial w_1}{\partial\xi}=\frac{\lambda^3}{\lambda^2-\cos^2\theta}\frac{\partial\phi}{\partial\eta}-\frac{\lambda^2 v_A\sin\theta\cos^2\theta}{\Omega_i(\lambda^2-\cos^2\theta)^2}\frac{\partial^2\phi}{\partial\xi^2},$$

$$\frac{\partial B_{z1}}{\partial\xi}=-\frac{\lambda^2\cos\theta}{(\lambda^2-\cos^2\theta)}\frac{\partial\phi_1}{\partial\eta}+\frac{\lambda^3 v_A\cos\theta\sin\theta}{\Omega_i(\lambda^2-\cos^2\theta)^2}\frac{\partial^2\phi}{\partial\xi^2}. \tag{21}$$

From term of order $\epsilon^{5/2}$ we obtain the following set of equations:

$$\lambda\frac{\partial B_{y2}}{\partial\xi}-\sin\theta\frac{\partial u_2}{\partial\xi}+\cos\theta\frac{\partial v_2}{\partial\xi}=f_1,$$

$$-\lambda\frac{\partial n_{i2}}{\partial\xi}+\frac{\partial u_2}{\partial\xi}+\frac{1}{v_A}\frac{\partial\phi}{\partial\tau}+\frac{\partial}{\partial\xi}(u_1\phi)+\frac{\partial w_1}{\partial\eta}=0,$$

$$\lambda v_A\frac{\partial v_2}{\partial\xi}+v_A\cos\theta\frac{\partial B_{y2}}{\partial\xi}-\frac{\partial v_1}{\partial\tau}=0, \tag{22}$$

$$\lambda v_A\frac{\partial u_2}{\partial\xi}-v_A\sin\theta\frac{\partial B_{y2}}{\partial\xi}-v_A\gamma\frac{\partial n_{i2}}{\partial\xi}-v_A\zeta\left(\frac{p}{1-p}\right)\frac{\partial n_{p2}}{\partial\xi}=f_2,$$

$$\lambda\frac{\partial n_{p2}}{\partial\xi}-\sin^2\theta\frac{\partial u_2}{\partial\xi}+\sin\theta\cos\theta\frac{\partial v_2}{\partial\xi}=f_3,$$

where $f_1=1/v_A(\partial B_{y1}/\partial\tau)+\sin\theta(\partial w_1/\partial\eta)+(\partial/\partial\xi)(u_1B_{y1})+(\lambda v_A/\Omega_i)(\partial^2 w_1/\partial\xi^2)-(\lambda v_A/\Omega_i)(\partial^2 u_1/\partial\xi\partial\eta)$, $f_2=(\partial u_1/\partial\tau)+v_AB_{y1}(\partial B_{y1}/\partial\xi)$, $f_3=(1/v_A)(\partial n_{p1}/\partial\tau)+2\sin\theta\cos^2\theta(\partial/\partial\xi)\times(u_1B_{y1})+\cos\theta\,(1-2\cos^2\theta)(\partial/\partial\xi)(v_1B_{y1})+\sin^2\theta\,(\partial/\partial\xi)\times(u_1n_{p1})-\sin\theta\cos\theta(\partial/\partial\xi)(v_1n_{p1})+f_4$, and $f_4=(\partial w_1/\partial\eta)-(v_A/\Omega_i)\cos\theta[\sin\theta(\partial^2B_{z1}/\partial\xi^2)+\cos\theta(\partial^2B_{y1}/\partial\eta\partial\xi)]$.

By eliminating quantities with subscript 2 and terms containing B_{z1} and w_1 from Eq. (22) by using Eqs. (18) and (19) we obtain

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi}{\partial \tau} + l\phi\frac{\partial \phi}{\partial \xi} - q\frac{\partial^3 \phi}{\partial \xi^3}\right] + r\frac{\partial^2 \phi}{\partial \eta^2} = 0. \tag{23}$$

Equation (23) is the KP equation for the propagation of magnetosonic solitons in two dimensions for electron–positron–ion plasmas. KP equation considered the evolution of weakly nonlinear long waves in dispersive media in which the transverse coordinate η is also taken into account. This KP equation is considered to be a two-dimensional KdV equation. In the above Eq. (23) the coefficients l, q, and r are given by

$$l = b/a, \tag{24}$$

$$q = \frac{c}{a}, \tag{25}$$

$$r = d/a, \tag{26}$$

where a, b, c, and d are given by

$$\begin{aligned} a &= \lambda^6(1+\sin^2\theta) \\ &\quad + \lambda^4\left\{\gamma - 2\cos^2\theta + \zeta\left(\frac{p}{1-p}\right)\cos^2\theta\sin^2\theta\right\} \\ &\quad + \lambda^2\left\{(1-2\gamma)\cos^2\theta + \zeta\left(\frac{p}{1-p}\right)(1+\cos^2\theta\sin^2\theta)\right\} \\ &\quad + \left(\gamma\cot^2\theta - \frac{\zeta p}{1-p}\right)\cos^2\theta\sin^2\theta, \end{aligned}$$

$$\begin{aligned} b &= 3\lambda^7 v_A\sin^2\theta + \lambda^5 v_A\left\{\left(\frac{7\zeta p}{1-p} - 2\right)\cos^2\theta\sin^2\theta + 2\gamma\right\} \\ &\quad + \lambda^3 v_A\left\{\frac{2\zeta p}{1-p}(\sin^2\theta - \cos^4\theta)\sin^2\theta \right. \\ &\quad \left. + 2\cos^2\theta(1-\gamma)\right\}, \end{aligned}$$

$$c = \frac{\lambda^5 v_A^3\cos^2\theta\sin^2\theta\{(\zeta+1)p-1\}}{\Omega_i^2(1-p)(\lambda^2-\cos^2\theta)},$$

$$d = \frac{\lambda^5 v_A\left(\lambda^2 + \zeta\left(\frac{p}{1-p}\right)\cos^2\theta\right)(\lambda^2-\cos^2\theta)}{(\lambda^2 - 2\cos^2\theta)}.$$

Following Ref. 42, the solution of Eq. (23) can be written as

$$\phi = \phi_o \sec h^2\alpha, \tag{27}$$

where $\phi_o = -(12q/l)K_\xi^2$ and $\alpha = K_\xi\xi + K_\eta\eta - \Omega\tau$. Here K_ξ and K_η are the nonlinear wave number along x and z axes, respectively, such that $K_\xi^2 + K_\eta^2 = K^2$, and Ω is the nonlinear frequency for the KP soliton. The KP equation and its solution, which we have obtained, are for low frequency magnetosonic waves, which are partially electrostatic and partially electromagnetic and, in the case considered here, incorporate the effect of positrons on the propagation characteristics (which are discussed later). From the point of view of the reductive perturbation method that has been used to obtain the KP equation, we point out that the expansion scheme that has been used is valid only in the long wavelength approximation (see, e.g., Refs. 40 and 43) and the KP equation is valid only for small angle of propagation (since the coordinate η is of a higher order than the coordinate ξ). We further note that since we have considered $T_{e,p} > T_i$, that is, the electrons and positrons are more energetic as compared to the ions there by satisfying the general condition which is necessary for magnetosonic waves, $v_{Ti} \ll (\omega/k_z) \ll v_{Te,p}$ (where v_{Ti} and $v_{Te,p}$ are the thermal velocities of ions and electrons, positrons, respectively).[41] Here this is easily satisfied since both electrons and positrons are taken to be massless as compared to the more massive ions.

The nonlinear dispersion relation is obtained by using Eqs. (23) and (27), we get $\phi_o = -(12q/l)K_\xi^2$ and

$$\Omega K_\xi + 4qK_\xi^4 - rK_\eta^2 = 0. \tag{28}$$

Equation (28) is the nonlinear dispersion relation which relates K_ξ, K_η, and Ω to each other. In the expressions for l, q, and r the upper (positive) sign for λ represents the fast magnetosonic solitons and the lower (negative) sign represents the case of slow magnetosonic solitons with two dimensions in *e-p-i* plasmas. The magnetosonic soliton which makes an angle θ (θ is taken as small due to the way we have used the stretched variables) with the x axis and propagates with a phase velocity, is obtained from Eq. (28) as

$$\frac{\Omega}{K} = \frac{rK_\eta^2}{K_\xi(K_\xi^2+K_\eta^2)^{1/2}} - \frac{4qK_\xi^3}{(K_\xi^2+K_\eta^2)^{1/2}}. \tag{29}$$

Using Eqs. (21) and (27) the w_1 and B_{z1} in terms of ϕ can be calculated as

$$\begin{aligned} w_1 &= \left(\frac{\lambda^3}{(\lambda^2-\cos^2\theta)}\frac{K_\eta}{K_\xi} \right. \\ &\quad \left. + \frac{2\lambda^2 K_\xi v_A\cos^2\theta\sin\theta}{\Omega_i(\lambda^2-\cos^2\theta)^2}\tanh\alpha\right)\phi, \end{aligned} \tag{30}$$

$$\begin{aligned} B_{z1} &= -\left(\frac{\lambda^2\cos\theta}{(\lambda^2-\cos^2\theta)}\frac{K_\eta}{K_\xi} \right. \\ &\quad \left. + \frac{2\lambda^3 v_A K_\xi\cos\theta\sin\theta}{\Omega_i(\lambda^2-\cos^2\theta)^2}\tanh\alpha\right)\phi. \end{aligned} \tag{31}$$

With the application of Eqs. (7), (9), and (19), we find the expressions for the normalized electric field components of the two-dimensional obliquely propagating magnetosonic waves in *e-p-i* plasmas,

$$\begin{aligned} \frac{cE_x}{B_o} &= w_1\sin\theta + \frac{2v_A K_\xi}{\Omega_i}[\beta(1-p)\Pi_1 \\ &\quad + \Pi_2\sin^2\theta]\phi\tanh\alpha, \end{aligned} \tag{32}$$

$$\frac{cE_y}{B_o} = -w_1\cos\theta - \left(\frac{2v_A K_\xi \Pi_2\cos\theta\sin\theta}{\Omega_i}\right)\phi\tanh\alpha, \tag{33}$$

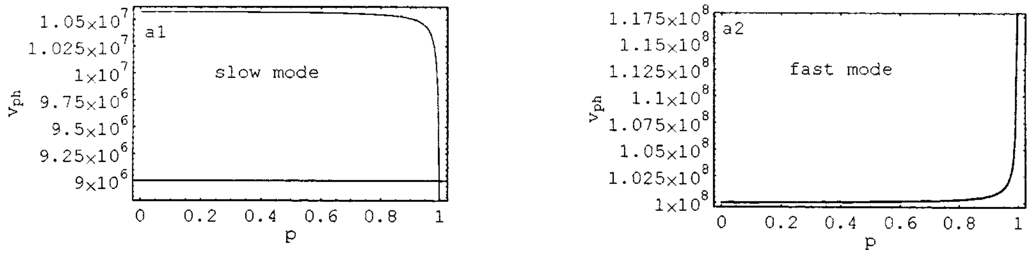

FIG. 1. Linear dependency of phase velocity on $p(=n_{po}/n_{eo})$ for both slow and fast modes of the two-dimensional magnetosonic wave in *e-p-i* plasmas.

$$\frac{cE_z}{B_o}=\frac{-\lambda^3\sin\theta}{(\lambda^2-\cos^2\theta)}\phi, \tag{34}$$

where

$$\Pi_1=\frac{\lambda^2+p(\zeta-\gamma)\cos^2\theta}{(\lambda^2-\lambda^2 p+\zeta p\cos^2\theta)},$$

$$\Pi_2=\left(\frac{\lambda^2}{(\lambda^2-\cos^2\theta)}+p\beta+\frac{p\beta(\zeta+p)(\lambda^2-\gamma\cos^2\theta)}{(\lambda^2-\lambda^2 p+\zeta p\cos^2\theta)}\right).$$

Using Eqs. (5), (18), (27), (30), and (31), we find the expressions for the normalized current density components of the two-dimensional obliquely propagating magnetosonic waves in *e-p-i* plasmas,

$$j_x=\frac{2\lambda^2 v_A K_\eta\sin\theta}{\Omega_i(\lambda^2-\cos^2\theta)}\phi\tanh\alpha, \tag{35}$$

$$j_y=-\frac{2\lambda^2 v_A K_\xi}{\Omega_i}\left[\frac{\cos\theta}{(\lambda^2-\cos^2\theta)}\frac{K_\eta}{K_\xi}\tanh\alpha-\frac{\lambda v_A K_\xi}{\Omega_i}\frac{\cos\theta\sin\theta}{(\lambda^2-\cos^2\theta)^2}(1-3\tanh^2\alpha)\right]\phi, \tag{36}$$

$$j_z=-\frac{2\lambda^2 v_A K_\xi\tanh\alpha}{\Omega_i(\lambda^2-\cos^2\theta)}\phi. \tag{37}$$

III. RESULTS AND DISCUSSION

Equations (19), (27), and (35)–(37) are investigated numerically for linear and nonlinear two-dimensional magnetosonic waves obliquely propagating at an angle θ to an external magnetic field B_o in electron–positron–ion (*e-p-i*) plasmas under the required conditions for the existence of the localized solution. It is assumed in all cases that the electron temperature is equal to the positron temperature, i.e., $\rho=T_e/T_p=1$. The plot of linear phase velocity (ω/k) for both slow and fast modes of magnetosonic waves against $p(=n_{po}/n_{eo})$ is shown in Fig. 1 for $\beta=0.01$, $\sigma=0.2$, and $\theta=15°$. In this figure the phase velocity of slow wave shows a decreasing trend and the fast wave shows an increasing trend, as the positron concentration increases.

The results of Fig. 2 are obtained from the numerical solution of Eq. (27). The dependency of the amplitude of the nonlinear ion density $\phi(p)$ of magnetosonic soliton on the fractional number of $p(=n_{po}/n_{eo})$ for both fast and slow mode is shown in Figs. 2(a1) and 2(a2). It is found that for slow mode of the wave, by increasing p between 0 and 1, the amplitude of soliton increases, however, the results are not valid at $p\approx 1$. While the fast mode decreases in amplitude as the positron concentration increases. This behavior is also evident in Figs. 2(b1) and 2(b2) for $\beta=0.01$, $\sigma=0.2$, $\rho=1$, $K_\xi=0.48\times 10^{-5}\ \mathrm{m}^{-1}$, and $K_\eta=0.12\times 10^{-5}\ \mathrm{m}^{-1}$ by varying the value of p from 0 to 0.4. In Figs. 2(b1) and 2(b2) the solid curve shows the normalized nonlinear ion density hump in the presence of positrons, i.e., $p=0.4$ and the dashed curve represents the density structure in the absence of positron, i.e., $p=0$. A similar behavior of variation of amplitude for slow and fast modes, respectively, are shown in Figs. 2(c1) and 2(c2), and 2(d1) and 2(d2). We have found that for $\beta<1$ both the modes of magnetosonic waves are supersonic $(M_a=v_p/c_s>1)$, where M_a is the Mach number, and for $\beta\geqslant 1$ the fast mode of the waves are supersonic and slow waves are subsonic $(M_a<1)$. The plots of Figs. 2(c1) and 2(c2) and 2(d1) and 2(d2) have been obtained for the following parameters $\beta=0.1$ [for Figs. 2(c1) and 2(c2)], $\sigma=0.2$ [for Figs. 2(d1) and 2(d2)], $\rho=1$, $p=0.4$, $K_\xi=0.48\times 10^{-5}\ \mathrm{m}^{-1}$, and $K_\eta=0.12\times 10^{-5}\ \mathrm{m}^{-1}$. The set of Figs. 3 are the graphical representation of the solution given by the Eq. (35) for x component of current density as a function of α. From Figs. 3(a1) and 3(a2), 3(b1) and 3(b2), 3(c1) and 3(c2) it is clear that the amplitude of x component of current density J_x increases for slow mode and decreases for fast mode for negative values of J_x by varying the values of p, σ, and β, respectively. Set of Figs. 4 are the graphical results of Eq. (36) for y component of current density. The behavior of the plots in Fig. 4 is the same as in the Fig. 3, i.e., for the fast mode for positive values of α the values of the amplitude of J_y initially is negative and becomes positive as α increases. The plots in the set of Fig. 5 show that the amplitude of the z component of current density as a function of α increases for slow mode and decreases for fast mode for positive values of the amplitude. The plasma parameters for Figs. 3–5 are the same as in Fig. 2. Likewise E_x, E_y, E_z, B_z, and B_y follow a similar behavior for fast and slow modes; however, these plots are not shown here.

IV. CONCLUSION

We have theoretically studied linear and nonlinear propagation of obliquely propagating magnetosonic waves in a three component *e-p-i* plasmas and have presented our results graphically. The linear dispersion relations of the two modes have been discussed in detail and it is noted that when magnetosonic waves propagate obliquely to the magnetic

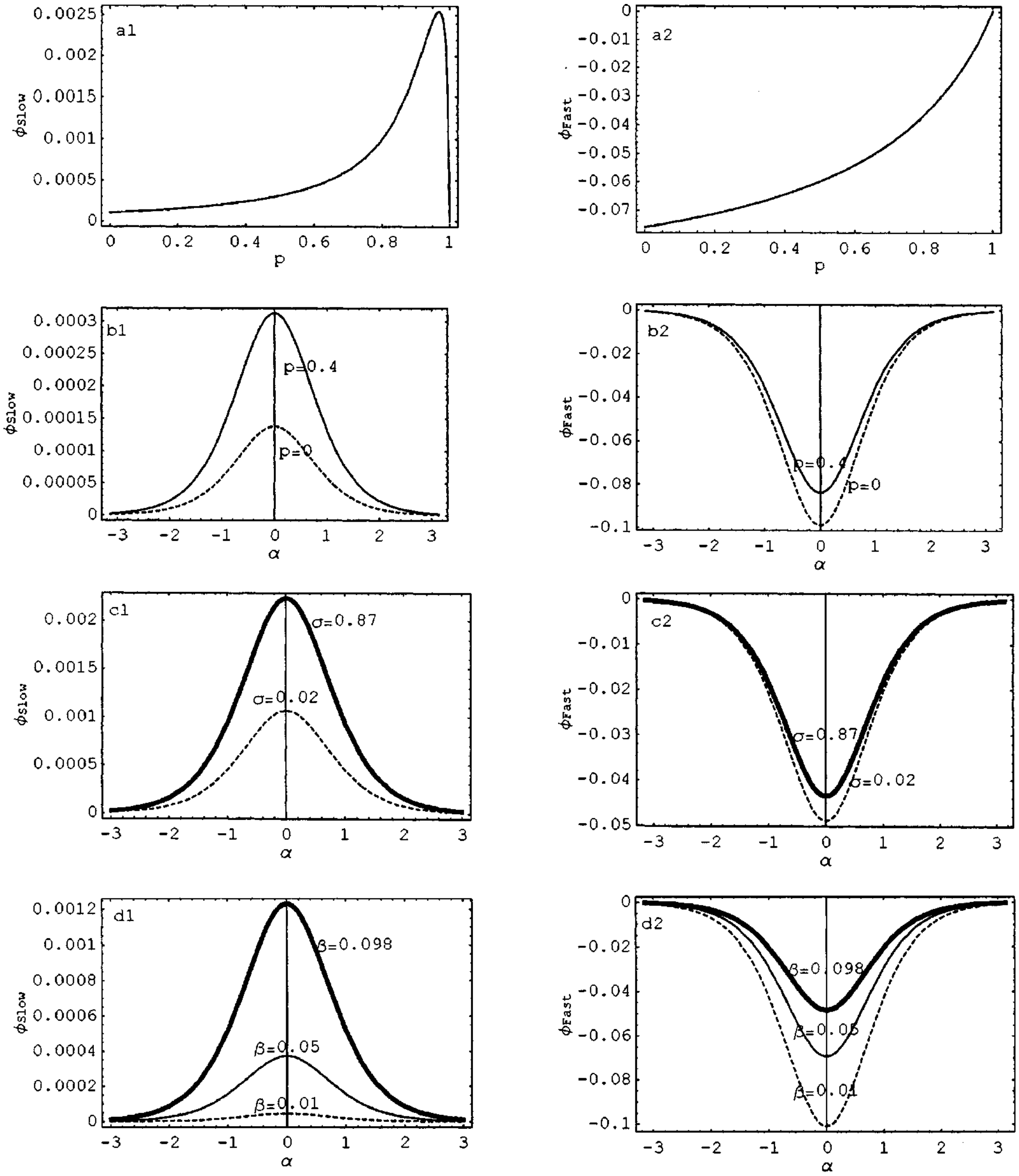

FIG. 2. Dependency of nonlinear normalized ion density as a function of α (a1, a2) on the fractional number p, (b1, b2) on positron concentration, i.e., when $p=0$ and when $p=0.4$, (c1, c2) on the variation of ratio of ion temperature to electron temperature, and (d1, d2) on the value of β-parameter, for both modes of the magnetosonic wave in *e-p-i* plasmas for small angle of wave propagation.

field the phase velocity of the waves becomes appreciably different from the case when purely parallel or perpendicular cases are considered. We assume that the angles at which the waves propagate relative to magnetic field are small so that the dispersion is determined by the fact that the ion Larmor radius is finite and also satisfies the limitations imposed by the reductive perturbation technique. It is also found that for $\beta<1$ both modes of the wave are supersonic ($M_a=v_p/c_s>1$) and for $\beta\geqslant 1$, the fast mode is supersonic and slow one is subsonic ($M_a<1$).

In the nonlinear regime the solitons of these magnetosonic waves are described by the KP equation given by Eq. (23), which are obtained by using reductive perturbation technique. This technique imposes restrictions on the amplitude of the wave, which means that this method can be applied for small amplitude waves and not for arbitrary amplitudes. This technique also restricts us to investigate the long wave length magnetosonic solitons only. According to the perturbation technique we have used, the angle of propagation will be small, because the effect of the perpendicular direction is small and the maximum contribution comes from the longitudinal direction (x axis).[44] It is also noted that for oblique propagation of the magnetosonic waves, if the frequency is not very high and the wavelength is not very short, then in the whole range, except $|\theta-\pi/2|\leqslant(m/m_i)^{1/2}$ (where $m=m_e=m_p$), the contribution of the terms containing m/m_i is negligible, that is, $m/m_i=0$, and that is why under this limitation, for $\theta\sim 0$, no contribution from positron occurs, i.e., $p\sim 0$ otherwise it can give some contribution even at $\theta\sim 0$.

It has been observed that for small angles of propagation, the fast magnetosonic soliton propagates as a negative pulse and the slow soliton as a positive pulse. It has been also found that the amplitude of the solitary structure depends in a complicated fashion on the different plasma pa-

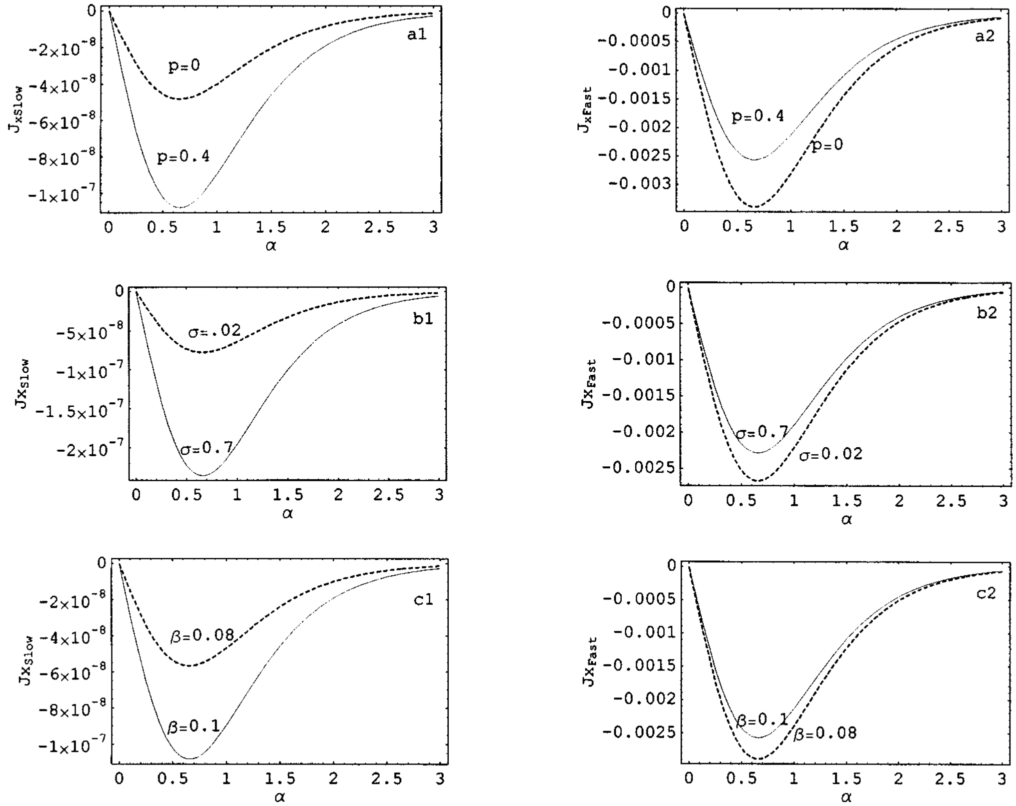

FIG. 3. Effects of the (a1, a2) positron concentration, (b1, b2) ion temperature, and (c1, c2) plasma pressure, on the amplitude of J_x for α for both slow and fast modes of the two-dimensional magnetosonic waves in *e-p-i* plasmas for small angle of wave propagation.

rameters, i.e., the positron concentration, the ratio of ion temperature to electron temperature, and the plasma β-value (which was not investigated in earlier work of Ref. 31). From the graphical representations it is found that by increasing the values of positron concentration, ion temperature, and plasma β-value, for the slow mode, the amplitude increases and the opposite is observed for the fast mode. We also note that for $\beta \sim 0$, we obtain only one mode of the magnetosonic wave.

Nonlinear characteristics of magnetosonic waves in multispecies plasmas may find application in space plasmas and also in fusion plasmas where they are used in particle acceleration and heating experiments.[25–29] Presently we have studied the case of two-dimensional magnetosonic waves by using effective one-fluid model, we think these results should be extended to include relativistic effects in *e-p-i* plasmas which are believed to exist in space and astrophysical plasmas.

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APPENDIX: FURTHER ANALYSIS OF LINEAR DISPERSION RELATION

We shall analyze briefly the dispersion relation given by Eq. (19) in different limits.

1. Parallel propagation ($\theta=0°$)

In this case the dispersion relation (19) is reduced to the following:

$$\lambda^2 = \left[\frac{1+\gamma \pm (1-\gamma)}{2}\right].$$

When $\theta=0$, then the fast mode of magnetosonic waves gives the ion-acoustic waves

$$\lambda^2 = \gamma$$

or

$$\frac{\omega^2}{k^2} = C_{IA}^2 = \frac{T_i + T_e}{m_i},$$

and the slow one gives ion-Alfvén waves

$$\lambda^2 = 1,$$

$$\frac{\omega^2}{k^2} = V_{IA}^2 = \frac{B_o^2}{4\pi m_i n_{io}}.$$

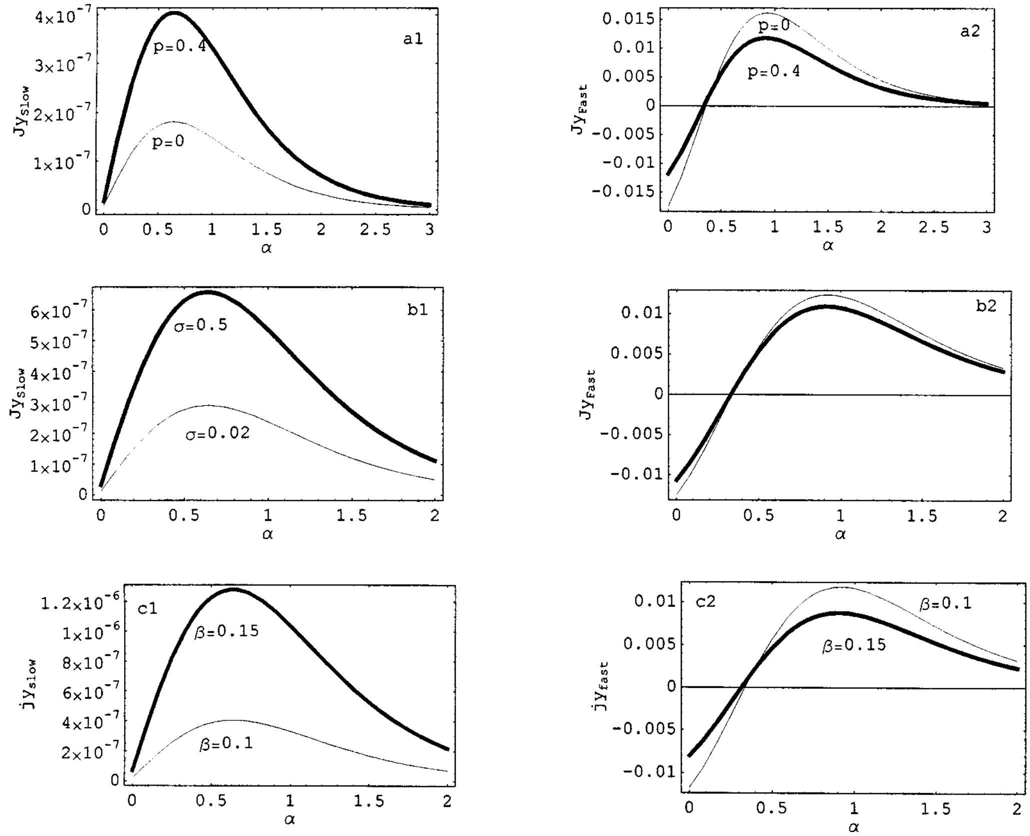

FIG. 4. Variation of normalized nonlinear y component of current density (J_y) for α by varying the value of (a1, a2) the ratio of background positron number density to the electron number density p, i.e., for $p=0$ and $p=0.4$, (b1, b2) the ratio of ion temperature to electron temperature σ, and (c1, c2) plasma β-value.

For $\theta=0$, the positron contribution vanishes; however, if we consider $m_p \neq 0$ then the positron contribution would appear here.

2. Perpendicular propagation (θ=90°)

In this case the dispersion relation (19) implies that

$$\lambda^2 = \left[\frac{1+\gamma+\zeta\left(\frac{p}{1-p}\right)}{2} \pm \frac{1+\gamma+\zeta\left(\frac{p}{1-p}\right)}{2}\right],$$

$$\frac{\omega^2}{k^2} = V_M^2 = \frac{(B_0^2/4\pi) + (n_{eo}T_e + n_{io}T_i + n_{po}T_p)}{m_i n_{io}}.$$

The upper sign in the above equation corresponds to the usual fast magnetosonic wave in the long wavelength range. In this mode, all the three fluids are frozen to the magnetic field, that is, the perturbed quantities satisfy the relations

$$\frac{\delta B}{B_o} = \frac{\delta n_i}{n_{io}} = \frac{\delta n_e}{n_{eo}} = \frac{\delta n_p}{n_{po}}.$$

3. Almost perpendicular propagation ($\theta \sim 90°$)

In this case the dispersion relation (19) is reduced to the following:

$$\lambda^2 \approx \left[\frac{1+\gamma+\zeta\left(\frac{p}{1-p}\right)}{2} \pm \frac{1+\gamma+\zeta\left(\frac{p}{1-p}\right)}{2} \times\left(1 - \frac{2\gamma\cos^2\theta}{\left(1+\gamma+\frac{\zeta p}{1-p}\right)^2}\right)\right].$$

(a) Fast ion-magnetosonic wave:

$$\lambda^2 \approx \left[\frac{\left(1+\gamma+\frac{\zeta p}{1-p}\right)^2 - 2\gamma\cos^2\theta}{\left(1+\gamma+\frac{\zeta p}{1-p}\right)}\right].$$

(b) Slow ion-magnetosonic wave:

$$\lambda^2 \approx \left[\frac{\gamma}{\left(1+\gamma+\frac{\zeta p}{1-p}\right)}\right]\cos^2\theta.$$

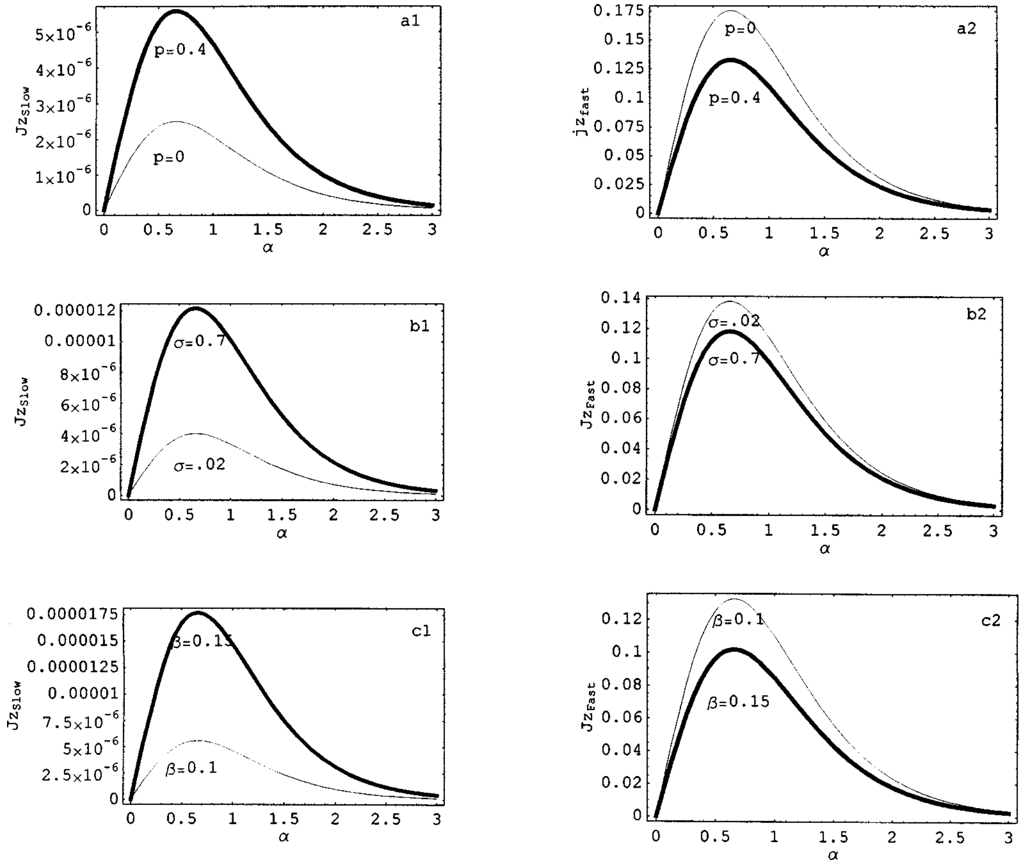

FIG. 5. The effects of (a1, a2) positron concentration $p(=n_{po}/n_{eo})$, (b1, b2) ion temperature $\sigma(=T_i/T_e)$, and (c1, c2) plasma pressure $\beta(=c_s^2/v_a^2)$ on the amplitude of normalized z component current density for α.

4. Low plasma pressure ($\beta = c_s^2 / v_A^2 \ll 1$)

Under this condition the spectrum (19) takes an especially simple form. Then the fast magnetosonic waves become purely transverse ($E \perp k$) and obey

$$\omega_+^2 = k^2 v_A^2.$$

The slow magnetosonic wave become purely longitudinal in low plasma pressure $\beta \ll 1$ and its spectrum takes the form

$$\omega_-^2 = k^2 c_s^2 \cos^2 \theta.$$

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