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Citation: Phys. Plasmas **12**, 072306 (2005); doi: 10.1063/1.1946729

View online: http://dx.doi.org/10.1063/1.1946729

View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v12/i7

Published by the American Institute of Physics.

PHYSICS OF PLASMAS 12, 072306 (2005)

Nonlinear Zakharov–Kuznetsov equation for obliquely propagating two-dimensional ion-acoustic solitary waves in a relativistic, rotating magnetized electron-positron-ion plasma

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(Received 23 November 2004; accepted 10 May 2005; published online 23 June 2005)

The purpose of this work is to investigate the linear and nonlinear properties of the ion-acoustic waves (IAW), propagating obliquely to an external magnetic field in a weakly relativistic, rotating, and magnetized electron-positron-ion plasma. The Zakharov–Kuznetsov equation is derived by employing the reductive perturbation technique for this wave in the nonlinear regime. This equation admits the solitary wave solution. The amplitude and width of this solitary wave have been discussed with the effects of obliqueness, relativity, ion temperature, positron concentration, magnetic field, and rotation of the plasma and it is observed that for IAW these parameters affect the propagation properties of solitary waves and these plasmas behave differently from the simple electron-ion plasmas. Likewise, the current density and electric field of these waves are investigated for their dependence on the above-mentioned parameters. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1946729]

I. INTRODUCTION

In contrast to the usual plasma consisting of electrons and positive ions, it has been observed that the nonlinear waves in plasmas having an additional component of positrons behave differently.[1] The electron-positron-ion plasma has an important role in the understanding of the plasmas in the early universe,[2–4] in the active galactic nuclei,[5] in the pulsar magnetospheres,[6] and in the solar atmosphere.[7,8]

It is well known that when positrons are introduced into electron-ion (*e-i*) plasma the response of the plasma changes significantly. The positrons can be used to probe particle transport in tokamaks and, since they have sufficient lifetime, the two-component (*e-i*) plasma becomes a three-component (*e-i-p*) one.[9,10] During the last decade, *e-p-i* plasma has attracted the attention of several authors.[11–16] They have studied linear and nonlinear wave propagations in *e-p-i* plasmas using different models. The ion-acoustic waves (IAW) in multicomponent plasmas has long been studied and both the linear[17–20] and nonlinear[21–27] dynamics associated with this wave have been investigated.

Relativistic plasmas can be found in many situations, for example, under the influence of high-power laser radiation, plasma particles may attain relativistic speeds.[28,29] A number of nonlinear phenomena occur in relativistic plasmas, and thus relativistic Langmuir and electromagnetic waves have been studied as subjects of laser-plasma interaction[30] and space-plasma phenomena.[31] In the nonlinear regime relativistic effects can significantly affect the wave character.

Plasmas with high-energy ion beams occur in the plasma sheet boundary layer of the Earth's magnetosphere[32] and in the Van Allen radiation belts.[33] Propagation of ion-acoustic waves in a relativistic plasma having streaming ions has been found to be most interesting.[34] The results of Ref. 34 were later rederived by the use of the pseudopotential method,[35] with the assumption that the ion temperature is zero ($T_i=0$). Since the ion temperature is very high in the relativistic plasmas of solar flares,[36] the solar wind,[37] and interplanetary space, the ratio of the ion-to-electron temperature is sometimes more than unity. In such situations both the relativistic effects and the ion temperature appreciably affect the propagation characteristics of the soliton. It is therefore important to consider finite ion temperature. In previous theories, like those discussed in Refs. 34 and 35, the investigations made were restricted to only the one-dimensional flow of the ions and the electrons. Kadomtsev and Petviashivili[38] made the first attempt to model a soliton in a two-dimensional system, later Zakharov and Kuznetsov (ZK)[39] made the first attempt to model a soliton in a three-dimensional system. For a nonrelativistic magnetized plasma with $T_i=0$, they obtained a three-dimensional differential equation, which is known as the ZK equation. However, this ZK equation may also be used for a two-dimensional magnetized system.[40]

The rotating flows of electrically conducting fluid (such as plasma) in the presence of a magnetic field is encountered in cosmic and geophysical fluid dynamics. It can provide an explanation for the observed maintenance and secular variation of the geomagnetic field.[41] It is also important in the solar physics involved in sunspot development, the star cycle, and the structure of rotating magnetic stars.[42] When a star is transformed into a neutron star, the moment of inertia decreases strongly; thus, the conservation of angular momentum causes a high rotation of the star. Under the condition of frozen in force lines, magnetic flux is also conserved; thus, the field varies in proportion to r^{-2} (r is the radius of the star). Therefore; as a rule, neutron stars should rotate quite rapidly and should be strongly magnetized. The nonlinear evolution of the electrostatic wave propagation in this type of highly rotating and strongly magnetized *e-p-i* plasma is the aim of the present work. Nonlinear wave propagation in the

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electron-positron plasma of a pulsar magnetosphere has been investigated using different approaches by various authors.[43,44]

The study of obliquely propagating (with respect to an external magnetic field) two-dimensional, ion-acoustic soliton via the ZK equation has not yet been studied in a rotating, strongly magnetized, and collisionless weakly relativistic *e*-*p*-*i* plasma with finite ion temperature ($T_i \neq 0$). Thus in this paper, we theoretically investigate two-dimensional propagation of ion-acoustic waves in such plasmas and show that the propagation of these waves is governed by the ZK soliton equation. The effects of ion temperature, relativity, the external magnetic field, rotation of the plasma, and the concentration of positrons on the IAW dynamics, both in the linear as well as in the nonlinear regimes are investigated. The organization of the paper is as follows.

In Sec. II the basic set of nonlinear equations and dispersion relation are presented and the nonlinear ZK equation is obtained by using the reductive perturbation technique. The stationary solution of this equation is obtained in Sec. III. In Sec. IV the numerical results of linear and nonlinear IAWs are discussed along with the conclusion of these results.

II. GOVERNING EQUATIONS AND FORMULATION

We consider a two-dimensional, magnetized, rotating, and collisionless weakly relativistic three-component (*e*-*p*-*i*) plasma. The external magnetic field is directed along the x axis, i.e., $\mathbf{B}_0 = B_0\hat{x}$, and the propagation is considered in the (x,y) plane. The electrons and positrons are assumed to be isothermally hot, while the ions are treated as a fluid with finite temperature. The phase velocity of the IAW is much larger than the ion thermal velocity and much less than the electron (positron) thermal velocities, i.e., $v_{ti} \ll \omega/k \ll v_{te}, v_{tp}$ [where $v_{tj} = (T_j/m_j)^{1/2}$ is the thermal speed of jth species while $j = e, p, i$]. Since we consider low-frequency IAWs, we neglect the effect of the electron (positron) inertia. To maintain quasineutrality, the dimensions of the system are assumed to be much larger than the electron Debye length. In the absence of perturbations, we assume that the plasma is in an equilibrium condition with the relativistic ion streaming only in the x direction. The y and z components of the ion velocity are considered to be nonrelativistic. We neglect any transport properties, such as viscosity and heat conduction, etc. Under these conditions the nonlinear dynamics of the low-frequency IAW in a rotating magnetized three-component plasma are governed by the following set of equations: the ion continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0 \tag{1}$$

and the ion momentum equation

$$\frac{d\mathbf{v}_i}{dt} = -\nabla\phi + \omega_{ci}(\mathbf{v}_i \times \hat{x}) - \frac{\sigma}{n_i}\nabla p_i + 2(\mathbf{v}_i \times \Omega). \tag{2}$$

The electrons and positrons in the electrostatic potential perturbation obey the Boltzmann distributions, since they are considered inertialess. These Boltzmann relations for the electrons and positrons are, respectively,

$$n_e = \exp(\phi) \tag{3}$$

and

$$n_p = \exp(-\alpha\phi). \tag{4}$$

The Poisson equation for this system is

$$\nabla^2\phi = \mu e^{\phi} + (1-\mu)e^{-\alpha\phi} - n_i. \tag{5}$$

We have taken $\mathbf{E} = -\nabla\phi$ (where $\mathbf{E}$ is the electric field and ϕ is the electrostatic wave potential normalized by T_i/e), $\mathbf{v}_i$ is the ion fluid velocity normalized by the ion-acoustic speed $c_{si} = (T_e/m_i)^{1/2}$, and n_j is the number density of particle species j normalized by their unperturbed density n_{jo}. The rotation frequency (angular velocity) $\Omega = \Omega_0\hat{x}$ (where $\hat{x}$ is the unit vector along x axis and Ω_0 is the magnitude of rotation frequency) and the ion gyrofrequency $\omega_{ci} = eB_0/m_i c$ (where e is the magnitude of electron charge, m_i is the mass of ion, B_0 is the magnitude of the ambient magnetic field, and c is the speed of light) are normalized by ion plasma frequency $\omega_{pi} = \sqrt{(4\pi n_{i0}e^2/m_i)}$. The space and time coordinates $\mathbf{r}$ and t are normalized, respectively, by Deby length $\lambda_d = \sqrt{T_e/4\pi n_{i0}e^2}$ and the ion plasma period ω_{pi}^{-1}. Also $\mu = 1/1-p$ where $p(=n_{po}/n_{eo})$ is the ratio of positron background density to electron background density, $\sigma(=T_i/T_e)$ is the ratio of ion temperature to electron temperature, and $\alpha(=T_e/T_p)$ is the ratio of electron temperature to positron temperature. Here T_j is the temperature of jth species, where $j(=e,p,i)$ represents an electron, positron, and ion, respectively. The last term in Eq. (2) represents the Coriolis force due to rotation of the plasma with frequency Ω_0.

Equations (1), (2), and (5) in component form in the xy plane can be written as

$$\frac{\partial n_i}{\partial t} + \partial_x(n_i v_{ix}) + \partial_y(n_i v_{iy}) = 0, \tag{6}$$

$$\partial_t(\gamma v_{ix}) + (v_{ix}\partial_x + v_{iy}\partial_y)\gamma v_{ix} + \partial_x\phi + \frac{\sigma}{n_i}\partial_x n_i = 0, \tag{7}$$

$$\partial_t v_{iy} + (v_{ix}\partial_x + v_{iy}\partial_y)v_{iy} + \partial_y\phi + \frac{\sigma}{n_i}\partial_y n_i - \Omega_c v_{iz} = 0, \tag{8}$$

$$\partial_t v_{iz} + (v_{ix}\partial_x + v_{iy}\partial_y)v_{iz} + \Omega_c v_{iy} = 0, \tag{9}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = \mu e^{\phi} + (1-\mu)e^{-\alpha\beta} - n_i, \tag{10}$$

where $\gamma = [1-(v_{ix}^2/c^2)]^{1/2} \simeq (1+v_{ix}^2/2c^2)$ in the weakly relativistic regime, c the speed of light is also normalized with c_{si}, and $\Omega_c = (\omega_{ci} + 2\Omega_0)$.

We apply the reductive perturbation technique (RPT) to Eqs. (6)–(10) to obtain the nonlinear Zakharov–Kuznetsov equation for the two-dimensional, small-amplitude ion-acoustic solitary waves in the weakly relativistic rotating magnetized three-component (*e*-*p*-*i*) plasma. The RPT is a well-known method[45–51] mostly applied to small-amplitude nonlinear waves (see, e.g., Refs. 45 and 46). This technique on the mathematical level rescales both space and time in the original equations of the system in order to introduce space and time variables, which are appropriate for the description of long-wavelength phenomena. This rescaling gives the isolation from the system of relevant equations, which describe how the system reacts on the new space and time scales. We note that the reduction process is slightly ill-defined in that it rests on experience in knowing how to pick the relevant scales. The reductive perturbation theory general principles are based on multiscale expansion (e.g., Ref. 50), which in our case can be written in the following manner:

$$n_i = 1 + \epsilon n_1 + \epsilon^2 n_2 + \cdots,$$

$$v_{ix} = u_0 + \epsilon u_1 + \epsilon^2 u + \cdots,$$

$$v_{iy} = \epsilon^2 v_1 + \epsilon^3 v_2 + \cdots,$$

$$v_{iz} = \epsilon^{3/2} w_1 + \epsilon^{5/2} w_2 + \cdots,$$

$$\phi = \epsilon\phi_1 + \epsilon^2\phi_2 + \cdots. \tag{11}$$

It is noted here that all perturbed quantities are functions of x, y, and t, and ϵ is a small $(0<\epsilon\leqslant 1)$ expansion parameter characterizing the strength of the nonlinearity, such that the stretched variables are introduced in the standard fashion,

$$\xi = \epsilon^{1/2}(l_x x - \lambda t),$$

$$\eta = \epsilon^{1/2} l_y y,$$

$$\tau = \epsilon^{3/2} t, \tag{12}$$

where λ is the normalized phase velocity of the ion-acoustic wave and $l_x(=k_\theta/k)=\cos\theta$, and $l_y(=k_\perp/k)=\sin\theta$, such that $l_x^2+l_y^2=1$, where θ is the angle between propagation vector k and the external magnetic field B_0. This variable stretching procedure (see, e.g., Refs. 47 and 51) assumes the possibility of introducing new coordinates and variables such that the slowness of coordinate dependence and smallness of some of the physical variables can be taken out in a uniform way.

Substituting Eqs. (11) and (12) into Eqs. (6)–(10) and collecting terms of lowest order in ϵ, i.e. ($\sim\epsilon$ and $\epsilon^{3/2}$) we obtain

$$-(\lambda - l_x u_0)\gamma_1\frac{\partial u_1}{\partial\xi} + l_x\frac{\partial\phi_1}{\partial\xi} + \sigma l_x\frac{\partial n_1}{\partial\xi} = 0,$$

$$-(\lambda - u_0 l_x)\frac{\partial n_1}{\partial\xi} + l_x\frac{\partial u_1}{\partial\xi} = 0,$$

$$n_1 - \acute{\alpha}\,\phi_1 = 0,$$

$$l_y\frac{\partial\phi_1}{\partial\eta} + \sigma l_y\frac{\partial n_1}{\partial\eta} - \Omega_c w_1 = 0, \tag{13}$$

where $\gamma_1=(1+3u_0^2/2c^2)$ and $\acute{\alpha}=\mu-\alpha(1-\mu)$. Using the first three equations of the above set of Eq. (13) we obtain

$$-(\lambda - l_x u_0)\gamma_1 u_1 + l_x\phi_1 + \sigma l_x n_1 = 0,$$

$$n_1 = \acute{\alpha}\phi_1,$$

$$u_1 = l_x^{-1}(\lambda - u_0 l_x)\acute{\alpha}\phi_1. \tag{14}$$

The linear phase velocity for ion-acoustic wave can be written as

$$\lambda = u_0 l_x + l_x\left(\frac{1}{\gamma_1\acute{\alpha}} + \frac{\sigma}{\gamma_1}\right)^{1/2}. \tag{15}$$

It can be noted from Eq. (15) that the linear phase velocity is influenced by the relativistic effect, positron concentration, obliqueness, and the ion temperature, but not by the external magnetic field ω_{ci} and rotation of the plasma Ω_0. In the case when $\theta=0$ and positron concentration $p=0$ then we obtain Eq. (11) of Ref. 52 and Eq. (24) of Ref. 36 in electron ion plasmas.

In the next order ($\sim\epsilon^2$) by using Eqs. (9) and (10) we obtain

$$-(\lambda - u_0 l_x)\frac{\partial w_1}{\partial\xi} + \Omega_c v_1 = 0,$$

$$\left(l_x^2\frac{\partial^2}{\partial\xi^2} + l_y^2\frac{\partial^2}{\partial\eta^2}\right)\phi_1 - \left(\acute{\alpha}\,\phi_2 + \tfrac{1}{2}\acute{\beta}\,\phi_1^2 - n_2\right) = 0, \tag{16}$$

where $\acute{\beta}=[\mu+\alpha^2(1-\mu)]/2$, by using Eqs. (14) and (16) we get

$$w_1 = \Omega_c^{-1} l_y(1+\sigma\acute{\alpha})\frac{\partial\phi_1}{\partial\eta}, \tag{17}$$

$$v_1 = \Omega_c^{-2} l_y(\lambda - u_0 l_x)(1+\sigma\acute{\alpha})\frac{\partial^2\phi_1}{\partial\xi\,\partial\eta}. \tag{18}$$

Equation (17) is the $E\times B$ drift along z axis and Eq. (18) is the polarization drift along y axis. These drifts appear in the higher orders also.

From terms of order $\epsilon^{5/2}$ we obtain the following set of equations:

$$(\lambda - u_0 l_x)\gamma_1\frac{\partial u_2}{\partial\xi} - l_x\frac{\partial\phi_2}{\partial\xi} - \sigma l_x\frac{\partial n_2}{\partial\xi} = \gamma_1\frac{\partial u_1}{\partial\tau} - \lambda\gamma_2\frac{\partial u_1^2}{\partial\xi} + u_0\gamma_2 l_x\frac{\partial u_1^2}{\partial\xi} + \gamma_1 l_x u_1\frac{\partial u_1}{\partial\xi} - \sigma l_x n_1\frac{\partial n_1}{\partial\xi},$$

$$(\lambda - u_0 l_x)\frac{\partial n_2}{\partial\xi} - l_x\frac{\partial u_2}{\partial\xi} = \frac{\partial n_1}{\partial\tau} + l_x\frac{\partial}{\partial\xi}(u_1 n_1) + l_y\frac{\partial v_1}{\partial\eta},$$

$$l_y\frac{\partial\phi_2}{\partial\eta} + \sigma l_y\frac{\partial n_2}{\partial\eta} - \Omega_c w_2 = \sigma l_y n_1\frac{\partial n_1}{\partial\eta} - (\lambda - u_0 l_x)\frac{\partial v_1}{\partial\xi}, \tag{19}$$

where $\gamma_2=3u_0/2c^2$. By eliminating quantities with subscript 2 and terms containing v_1 and w_1 from Eqs. (16) and (19) by means of Eqs. (14) and (15) we obtain the nonlinear

Zakharov–Kuznetsov equation for the first-order electrostatic potential as

$$\frac{\partial \phi_1}{\partial \tau} + l\phi_1 \frac{\partial \phi_1}{\partial \xi} + r\frac{\partial^3 \phi_1}{\partial \xi^3} + q\frac{\partial^3 \phi_1}{\partial \xi \, \partial \eta^2} = 0. \tag{20}$$

Equation (20) is the ZK equation for the ion-acoustic soliton in a two-dimensional, magnetized, rotating, and collisionless weakly relativistic three-component (e-p-i) plasma. The ZK equation describes the evolution of weakly nonlinear long waves in dispersive media in which the transverse coordinate η is also taken into account for a strongly magnetized plasma. The ZK equation has retained much of the essential physics including the $E\times B$ drift along z axis and the polarization drift along y axis. In Eq. (20) the coefficients l, r, and q are given by

$$l = \acute{\alpha}(\lambda - u_0 l_x)\left[\frac{3}{2} - \frac{\gamma_2}{\gamma_1}\left(\frac{\lambda - u_0 l_x}{l_x}\right) - \frac{l_x^2}{2\acute{\alpha}^2\gamma_1}\frac{(2\acute{\beta} + \acute{\alpha}^3\sigma)}{(\lambda - u_0 l_x)}\right], \tag{21}$$

$$r = \frac{l_x^4}{2\gamma_1 \acute{\alpha}^2(\lambda - u_0 l_x)}, \tag{22}$$

$$q = \frac{l_x^2 l_y^2}{2\gamma_1 \acute{\alpha}^2(\lambda - u_0 l_x)}\left[1 + \frac{\acute{\alpha}\gamma_1(1 + \acute{\alpha}\,\sigma)(\lambda - u_0 l_x)^2}{l_x^2 \Omega_c^2}\right]. \tag{23}$$

The ZK equation (Ref. 39) is one of the better studied canonical two-dimensional extensions of the Korteweg–de Vries equation the other being the Kadomtsev–Petviashvilli (KP) equation (Ref. 38). In contrast to the KP equation, which is valid only in isotropic situations, the ZK equation is valid in anisotropic settings, which is exactly the case of rotating fluids where the differential longitudinal dependence of the rotation rate causes anisotropy between the meridional and the longitudinal directions. We also note that for the nonlinear mode of the electrostatic wave (such as ion-acoustic wave) the KP equation is valid in an unmagnetized and unrotating plasma, whereas the ZK equation is valid for magnetized (Ref. 40) and rotating plasma. Moreover, in contrast to the KP equation, the ZK equation supports stable lump solitary waves. This makes the ZK equation a very attractive model equation for the study of vortex soliton in plasmas and fluid physics.[53] During the last two decades, the ZK equation has attracted the attention of several authors.[54–57] They have studied this equation for different models in different areas of physics.

III. SOLUTION OF THE ZK EQUATION

The steady-state solution of the ZK equation [Eq. (20)] is obtained by transforming the independent variables ξ, η, and τ into a new coordinate χ, and then by following Ref. 4 the solution of Eq. (20) is

$$\phi_1 = \phi_m \sec h^2\chi, \tag{24}$$

where $\chi=(\xi+\eta-v_0\tau)/\Delta$, here v_0 is a constant velocity normalized to c_{si}, Δ is the width of soliton, and ϕ_m is the amplitude of soliton. Both Δ and ϕ_m (normalized to λ_d) are given by

$$\phi_m = \left(\frac{3v_0}{l}\right), \quad \Delta = \sqrt{\frac{4(r+q)}{v_0}}. \tag{25}$$

By using Eqs. (14) and (24) we can find the solution for n_1 and u_1 as

$$n_1 = n_m \sec h^2\chi$$

$$u_1 = u_m \sec h^2\chi, \tag{26}$$

where n_m and u_m are, respectively, the peak soliton ion density and peak soliton x component of ion velocity and are given by

$$n_m = \acute{\alpha}\ \phi_m,$$

$$u_m = \frac{\acute{\alpha}(\lambda - u_0 l_x)}{l_x}\phi_m. \tag{27}$$

The soliton energy ε_s can be calculated by using the integral

$$\varepsilon_s = \int_{-\infty}^{\infty} u_1^2(\chi)d\chi. \tag{28}$$

By substituting Eq. (26) into Eq. (28) and after integration, we obtain

$$\varepsilon_s = \tfrac{4}{3}u_m^2\Delta. \tag{29}$$

With the application of Eqs. (17), (18), and (24) w_1 (the $E\times B$ drift along z axis) and v_1 (the polarization drift along y axis) can be calculated and yields

$$w_1 = \left[\frac{-3\ v_0^{3/2}(1+\acute{\alpha}\ \sigma)l_y}{\Omega_c l\sqrt{(r+q)}}\tanh\chi\right]\sec h^2\chi, \tag{30}$$

$$v_1 = \left[-\frac{2\phi_m}{\Delta^2}\frac{l_y(1+\acute{\alpha}\ \sigma)(\lambda_0 - u_0 l_x)(3\sec h^2\chi - 2)}{\Omega_c^2}\right] \sec h^2\chi. \tag{31}$$

By using the relation $\mathbf{E}=-\boldsymbol{\nabla}\phi$ and Eq. (24) we find the expressions for the normalized electric field components of the two-dimensional obliquely propagating IAW in e-p-i plasmas as

$$E_x = \left[\frac{3\ v_0^{3/2} l_x \tanh\chi}{l\sqrt{(r+q)}}\right]\sec h^2\chi, \tag{32}$$

$$E_y = \left[\frac{3\ v_0^{3/2} l_y \tanh\chi}{l\sqrt{(r+q)}}\right]\sec h^2\chi. \tag{33}$$

Using Eqs. (11), (12), and (24) we find the expressions for the normalized current density components of the two-dimensional obliquely propagating ion-acoustic waves in e-p-i plasmas as (with $\sim\epsilon^2$)

$$j_x = \left[\frac{3\ \lambda\ v_0^2 l_x(3\sec h^2\chi - 2)}{2l(r+q)c}\right]\sec h^2\chi, \tag{34}$$

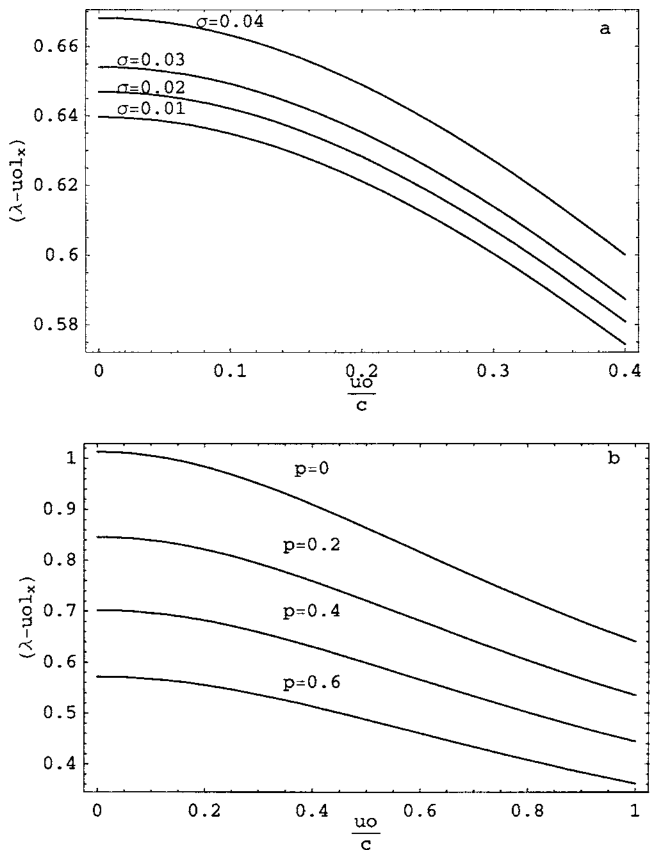

FIG. 1. Linear dependency of phase velocity $(\lambda - u_0 lx)$ on the relativistic streaming factor u_0/c for different values of (a) σ and (b) positron concentration p each marked on the curve.

$$j_y = \left[\frac{3\ \lambda\ v_0^2 l_y (3 \sec h^2 \chi - 2)}{2l(r+q)c} \right] \sec h^2 \chi. \tag{35}$$

It is seen from Eqs. (24)–(35) that all the quantities are significantly affected by the streaming relativistic factor, the finite ion temperature, positron concentration, magnetic field, and the rotation of the plasma, which are also evident from the graphical results in the following section.

IV. RESULTS AND DISCUSSION

The graphical results are presented under the required conditions for the existence of the localized solution. It is assumed in all cases that the electron temperature is equal to the positron temperature, i.e., $\alpha = T_e/T_p = 1$. The dependence of linear phase velocity of two-dimensional (2D) IAW on the relativistic streaming effect u_0/c, the ratio of the ion-to-electron temperatures σ, and positron concentration p is shown in Fig. 1. The phase velocity decreases against u_0/c for all the values of σ and p, but for a fixed value of u_0/c, the phase velocity increases as σ increases and decreases for the higher values of p. The plots of Fig. 2 are the graphical results of the amplitude of electrostatic potential of soliton ϕ_m against the relativistic streaming factor u_0/c for different values of (a) σ, (b) p, and (c) θ. It is evident that the amplitude increases with u_0/c for all values of σ, p, and θ; however, at a fixed value of u_0/c it decreases with increasing values of σ and p and increases with higher values of θ. The variations of width Δ versus u_0/c and obliqueness θ are shown, respectively, in the plots of Figs. 3 and 4. From Fig. 3 it is clear that the width of the soliton decreases with u_0/c for all values of σ and p, and increases for decreasing values of σ and p for a fixed value of u_0/c. Figure 4 shows how the width of the soliton changes with obliqueness θ with variation of (a) rotation Ω_0 and (b) the external magnetic field ω_{ci}. In these two cases it is observed that width Δ increases with θ for its lower range but decreases for its higher value. The variation of width in this set of figures has been shown for an arbitrary value of θ. The graphical representations of the effects of rotation Ω_0 on the amplitude of current density j_x and j_y are shown in the plots of Fig. 5. It is shown that the

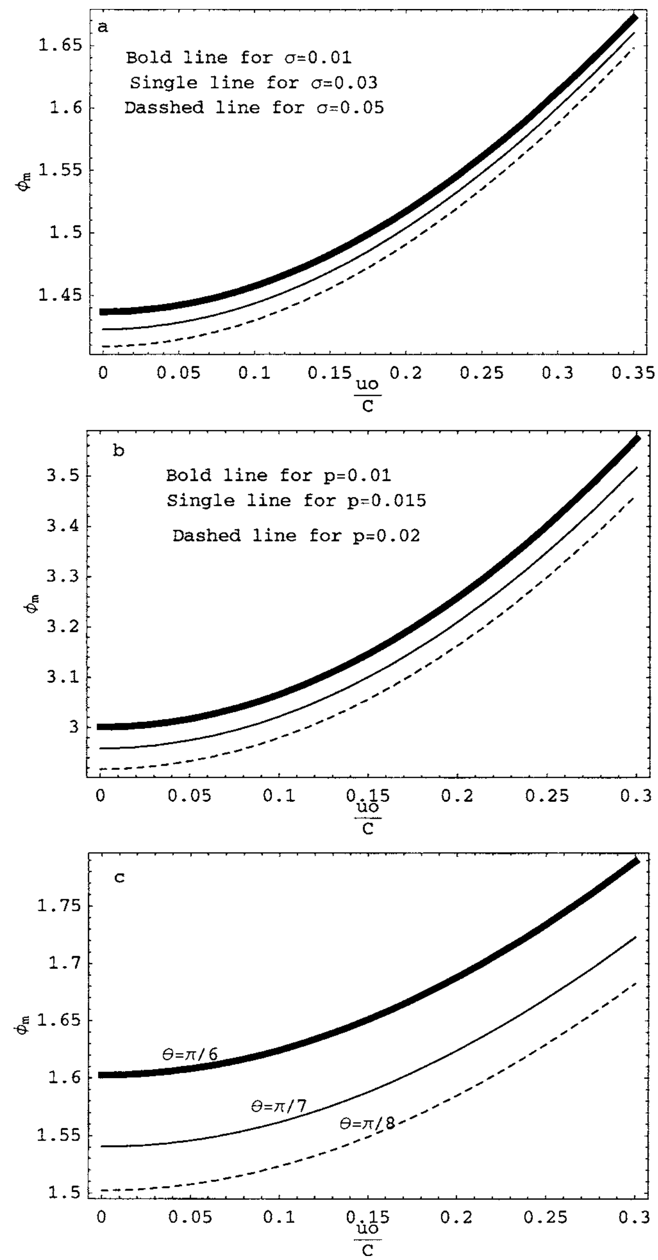

FIG. 2. The amplitude of soliton potential ϕ_m against the relativistic streaming factor u_0/c with the variations of (a) ion-to-electron temperature ratio σ, (b) ratio of positron back ground density to electron back ground density p, and (c) obliqueness θ.

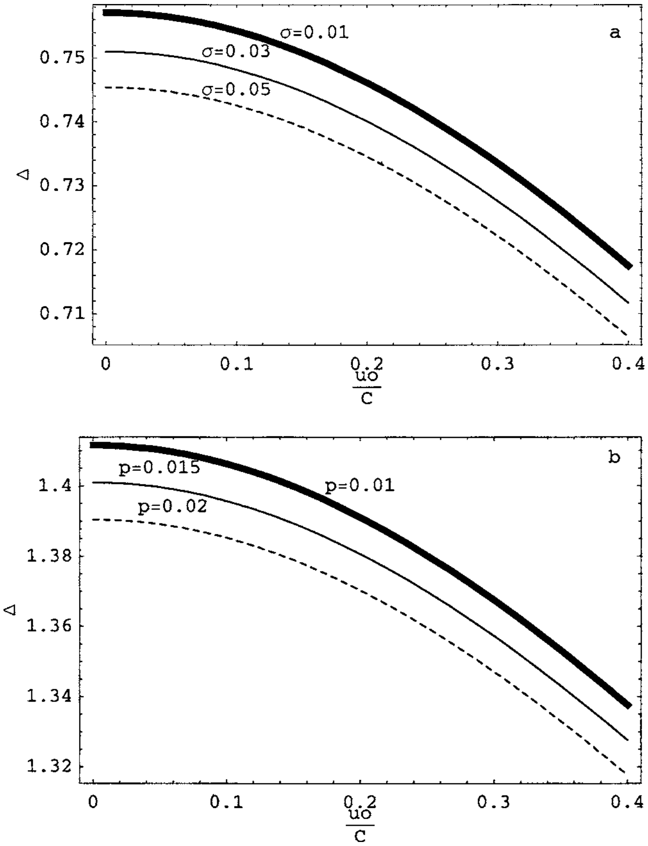

FIG. 3. Variation of the width of soliton Δ with relativistic streaming factor u_0/c by varying the parameters (a) ratio of ion temperature to electron temperature σ and (b) positron concentration p.

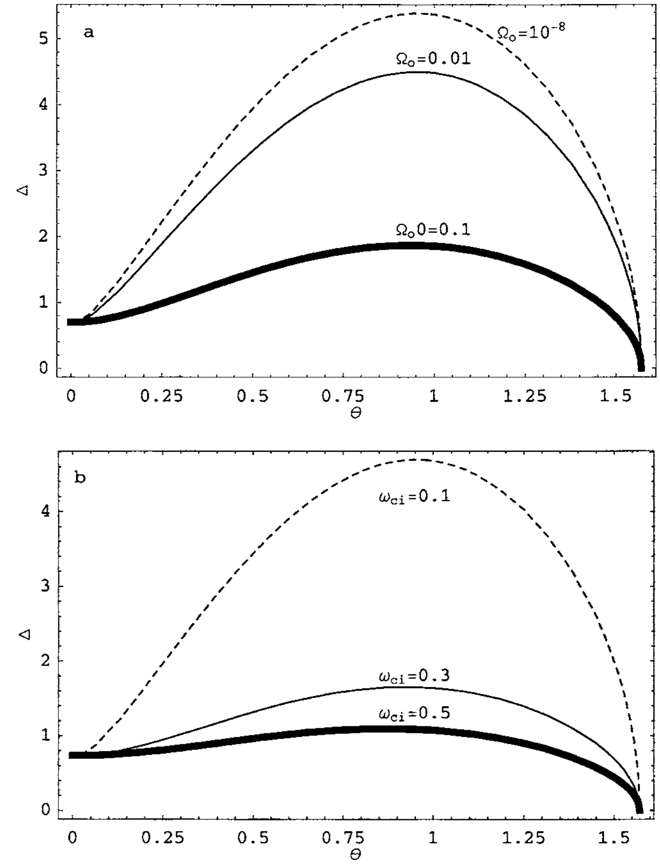

FIG. 4. Variation of the width of soliton Δ with obliqueness θ by varying the parameters (a) rotational frequency Ω_0 and (b) cyclotron frequency ω_{ci}.

amplitudes of j_x and j_y increase with increasing values of rotation Ω_0.

The numerics of the physical parameters used for all figures are $v_0=1$, $p=0.4$, $\sigma=0.01$, $\theta=\pi/12$, $\alpha=T_e/T_p=1$, $\beta(=u_0/c)=0.2$, $\omega_{ci}=1$, and $\Omega_0=0.001$.

The properties of linear and nonlinear IAW with effects of different independent parameters in relativistic rotating magnetized e-p-i plasma have been studied. Since propagation of the wave depends strongly on the medium (plasma), the stability and instability (damping or growing) of the wave depend on the different parameters used in the plasma. Since soliton formation is due to either some instabilities or nonlinearities in the medium, the amplitude, maximum amplitude, and width of the soliton are strongly dependent on the parameters used and this is also clear from Figs. 2–4. Some salient features of the results of this work may be summarized and concluded as follows.

(1) We have shown that in the linear study of IAW, phase velocity is inversely related to positron concentration and the relativistic ion streaming factor, while it is directly proportional to the ion temperature and cosine of the angle.

(2) It has been found that for given values of the parameters in the system, ϕ_m is positive, which shows that the ion-acoustic wave admits only the positive potential (rarefactive solitary wave).

(3) The effects of ion temperature and positron concentration modify the height and width of this wave. It is found that for increasing values of ion temperature and positron concentration the height of soliton decreases with increasing u_0/c, while width decreases with decreasing u_0/c.

(4) It is shown that the obliqueness θ and the external magnetic field ω_{ci} modify the stability of the soliton and also change the amplitude and width of these solitary wave dynamics. It is obvious from relations (19) and (25) that amplitude is inversely related to cos θ. From relations [Eqs. (19) and (25)] and graphical results (Fig. 4) it is obvious that width increases with θ for its lower range and decreases for higher values of θ. It should be pointed out that for large angles the amplitude becomes ∞ and width goes to zero. It is likely that for large angles the assumption that the waves are electrostatic is no longer a valid one and in that case we should look for fully electromagnetic structures of the wave. It means there is some restriction imposed on the angle θ which, according to our perturbation scheme, means that the angle should be small. The maxima for width was calculated and the angle for which the width becomes maximum in Fig. 4 is 54.67° and at this value $\partial^2\Delta/\partial\theta^2 \leqslant 0$. Thus $\theta_c=54.67°$ is called the critical angle below which the electrostatic nature of this wave is dominant and for $\theta > \theta_c$ the nature of the wave become more electromagnetic.

(6) Rotation of the plasma (around the axis of external mag-

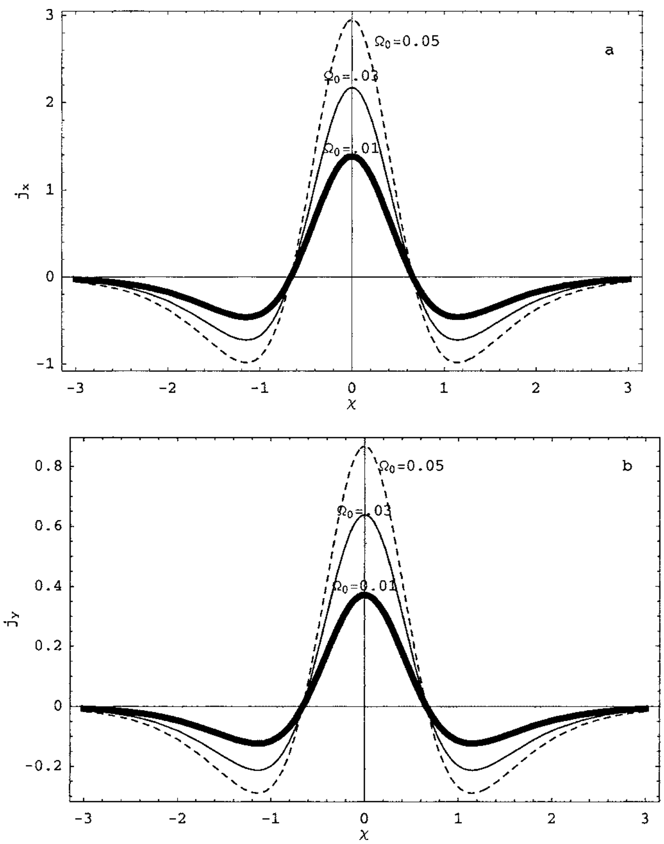

FIG. 5. The effects of rotation Ω_0 on (a) x component of current density j_x and (b) y component of current density j_y for the two-dimensional IAWs in e-p-i rotating strongly magnetized plasmas.

netic field) considerably affects the nature of the nonlinear structure of IAW. The magnitude of rotational frequency and the external magnetic field have no direct effect on the amplitude of the soliton; however, they do have a direct effect on the width of these solitary waves. It is shown that as we increase both the rotation and cyclotron frequencies the width of these solitary waves decreases, and this means that the coupling effect of rotation and external magnetic field makes the solitary structures more spiky. Further, due to rotation the energy of soliton decreases, and as a result, the amplitude of soliton u_m also decreases. A decreasing soliton amplitude may, therefore, be attributed to the decreasing soliton energy, which may be possible due to the reflection of ions and positron from the electrostatic field generated inside the plasma. Another reason for the decrease of soliton amplitude may be due to the wave particle exchange mechanism, the calculations of which are possible from kinetic theory and beyond the scope of the present work.

(7) The parameters (ion temperature, positron, relativistic streaming factor, rotation, and external magnetic) also affect the current density significantly. It is shown that for rotation and external magnetic field the amplitude of current density increases. From relations (31) and (32) it is obvious that when $3\,\mathrm{sec}\,h^2\chi > 2$ the current density will be positive otherwise it is negative. In Fig. 5 it is clear that for $\chi \sim (0-\pm 0.65)$, $3\,\mathrm{sec}\,h^2\chi > 2$, which make j_x and j_y positive and for $\chi > \pm 0.65$, $3\,\mathrm{sec}\,h^2\chi < 2$ the current densities become negative. Physically it means that, in the first case, the field is concentrated on ions and positrons and the electrons are depleted from that particular region and, subsequently, the currents due to ions and positrons are dominant, as compared to the current due to electrons and vice versa in the latter case.

In the end, we conclude that we have studied the IAW propagating obliquely to the external magnetic field in a weakly relativistic rotating e-p-i plasma. The system consists of electrons and positrons as Boltzmannian particles, and ions provide the dynamics of the system. The ZK-soliton equation is derived for this wave by employing the RPT. It is noted that the finite ion temperature, positron concentration, obliqueness of the propagation direction, relativistic ion streaming term, and the magnitude of external magnetic field affect significantly the nature of the solitary structure. In particular, it is shown that due to the rotation of the plasma (around the axis of external magnetic field) the width of the soliton becomes narrow. Thus, such a 2D IAW with finite amplitude and narrow width generated in a rotating plasma could illustrate the motion of high-energy protons, which are believed to be present in the Van Allen radiation belts[33] and in pulsar magnetospheres.[44,45] Also, the rotating flows of plasma in the presence of a magnetic field are believed to exist in cosmic plasmas and in the solar atmosphere.[43] In this work we have given some insights about a strongly magnetized physical system with rotational and relativistic particle flow in two dimensions. Further studies of different kinds of solitons and other related nonlinear phenomena in such a system are subjects for our future investigations.

ACKNOWLEDGMENTS

One of the authors (M.A.) thanks the Higher Education Commission (HEC) and the PAEC for financial support of this work. They are also indebted to Professor N. L. Tsintsadze for useful discussions. They also thank the referee for his useful comments.

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