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Effects of inhomogeneity on the Shukla–Nambu–Salimullah potential in a magnetized plasma

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Abstract

Detailed properties of the electrostatic Shukla–Nambu–Salimullah potential in an inhomogeneous magnetoplasma in the presence of ion streaming due to diamagnetic drift as in a laboratory discharge plasma have been examined analytically. The potential becomes a sensitive function of the external static magnetic field, the scalelength of inhomogeneity, and the diamagnetic ion streaming velocity. For a decreasing ion density gradient, there is a limit of existence of this static modified shielding potential.

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1. Introduction

The most fundamental property of a plasma is the existence of a static electrostatic shielding potential known as the Debye–Hückel potential. However, an external homogeneous magnetic field is usually applied to confine and control the properties of the plasma, or the plasma as in space situation is present in ambient magnetic field. In magnetized plasmas, there exists two electrostatic potentials viz., the almost spherically symmetric Debye–Hückel potential and the strongly anisotropic potential known as the Shukla–Nambu–Salimullah (SNS) potential [1]. It may be mentioned here that the SNS potential originates from the ion polarization drift in magnetized plasmas in contrast to the Debye–Hückel potential which arises from the charge quasi-neutrality. The existence of this new electrostatic (SNS) potential in magnetized plasmas was first pointed out in several papers [2–8]. Recently, the detailed properties of the SNS potential and the consequent oscillatory dynamical wake potential for the robust dust crystal formation in a streaming homogeneous dusty magnetoplasma have been shown in exact numerical calculations [9]. From the exact solution, one notes that the SNS potential is elliptical in shape elongated across the external magnetic field [9].

However, in most of the laboratory situations of plasma experiments, plasmas involved will be nonuniform in nature. For an example, dust crystal experiments using rf-discharge plasmas, the plasma may be inhomogeneous along the direction of ion flow which maintains a sheath in the plasma. The plasma density inhomogeneity will produce an electric field $\mathbf{E}$ in the plasma causing continuous ion flows ($\hat{x}$-axis). The external magnetic field $\mathbf{B}_0$ applied along the ion flow direction cannot produce any $\mathbf{E}\times\mathbf{B}_0$ drift of plasma particles. However, the externally applied transverse magnetic field $\mathbf{B}_0 \parallel \hat{z}$ can cause $\mathbf{E}\times\mathbf{B}_0$ drift of plasma particles which might drive an electrostatic drift wave in the perpendicular direction (y-direction) due to the $\mathbf{E}\times\mathbf{B}_0$ drift of ions.

In this Letter, we make a theoretical investigation on the SNS potential in the inhomogeneous electron–ion plasma hav-

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ing uniformly drifting ions in the direction transverse to the externally applied static homogeneous magnetic field. We first derive the dielectric response function of the magnetoplasma. The resulting potential will involve the drift wave mode, plasma inhomogeneity and flowing distribution of ions, and the Boltzmann distribution of electrons.

Let a uniform magnetic field $\mathbf{B}_0 \parallel \hat{z}$ be applied across the electric field $\mathbf{E} \parallel \hat{x}$ produced due to the inhomogeneous ion distribution in the plasma. Thus, the ions will have $\mathbf{E}\times\mathbf{B}_0$ drift in the y-direction with the drift speed $u_{i0} = cE/B_0$. The inhomogeneity of plasma ions in the x-direction will give rise to a diamagnetic drift frequency to the ions. The electrons will satisfy the Boltzmann distribution.

In the presence of low-frequency (in comparison with the electron gyrofrequency ω_{ce}) electrostatic waves with parallel (to $\hat{\mathbf{z}}$) phase speed much smaller than the electron thermal speed v_{te}, hot electrons rapidly thermalize along $\hat{\mathbf{z}}$-direction and establish a Boltzmann distribution. The corresponding dielectric susceptibility is $1/k^2\lambda_{De}^2$, where k is the wavenumber and λ_{De} is the electron Debye radius.

Using the standard Vlasov–Poisson system of plasma equations for the Doppler-shifted ions, the ion susceptibility is given by [2,10,11]

$$\chi_i(\omega,\mathbf{k}) = \frac{1}{k^2\lambda_{Di}^2}\left[1 + \frac{\omega' - \omega_i^*}{\sqrt{2}k_\parallel v_{ti}}\sum_n Z\left(\frac{\omega' - n\omega_{ci}}{\sqrt{2}k_\parallel v_{ti}}\right) I_n(b_i)e^{-b_i}\right], \quad (1)$$

where $k_\perp$ ($k_\parallel$) is the component of $\mathbf{k}$ across (parallel) to $\hat{\mathbf{z}} \parallel \mathbf{B}_0$, $\omega_{pi} = (4\pi e^2 n_{i0}/m_i)^{1/2}$ is the ion plasma frequency, $\omega_{ci} = eB_0/m_i c$ is the ion gyrofrequency, $\omega_i^* = -k_y v_{ti}^2/L_{ni}\omega_{ci}$ is the diamagnetic drift frequency of ions, k_y is the component of the wave vector $\mathbf{k}$ along the y-axis which is transverse to $\hat{\mathbf{z}}$, $v_{ti} = (T_i/m_i)^{1/2}$ is the thermal velocity of ions, $L_{ni}(= -n_{i0}(x)/n'_{i0}(x), n'_{i0}(x) = dn_{i0}(x)/dx)$ is the scalelength of the density inhomogeneity of the ions, $\omega' = (\omega - \mathbf{k}\cdot\mathbf{u}_{i0})$ is the Doppler shifted frequency, and $\lambda_{Di} = (T_i/4\pi e^2 n_{i0})^{1/2} = v_{ti}/\omega_{pi}$ is the ion Debye length. In general, L_{ni} will be a function of x. However, for a particular choice of the ion density inhomogeneity in the x-direction, viz., $n_{i0}(x) = n_{i0}^0(1 \mp x/L_{ni})$, the scalelength of inhomogeneity is independent of x. Here, $-e, n_{i0}^0, m_i, c$ and T_i are the electronic charge, equilibrium number density of ions, mass of an ion, light speed and ion temperature, respectively.

For $kv_{ti} \ll \omega \ll \omega_{ci}$, the ions suffer the $\mathbf{E}\times\mathbf{B}_0$ drift, in addition to streaming along $\hat{\mathbf{x}}$, where ω is the wave frequency. The susceptibility for strongly magnetized ions with inhomogeneous distribution is given by [2]

$$\chi_i(\omega,\mathbf{k}) = \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega' k^2\lambda_{Di}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega'^2}. \quad (2)$$

Thus, the appropriate dielectric constant of such electrostatic waves in the magnetoplasma is given by

$$\epsilon(\omega,\mathbf{k}) = 1 + \frac{1}{k^2\lambda_{De}^2} + \frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega' k^2\lambda_{Di}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega'^2}$$

$$= \frac{1}{k^2}\left[k^2 + k_e^2 + k_\perp^2 f - \frac{k_\parallel^2\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{u}_{i0})^2} + \frac{k_y\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{u}_{i0})L_{ni}\omega_{ci}}\right], \quad (3)$$

where $k_e = 1/\lambda_{De}$ and $f = \omega_{pi}^2/\omega_{ci}^2$.

The electrostatic potential around a test dust particulate [12] in the presence of electrostatic mode $(\omega,\mathbf{k})$ in the uniform dusty magnetoplasma, whose dielectric response function is given by Eq. (1), is given by

$$\Phi(\mathbf{x},t) = \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega,\mathbf{k})}\exp(i\mathbf{k}\cdot\mathbf{r})\,d\mathbf{k}\,d\omega, \quad (4)$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of a test dust particulate, and q_t is its charge. In the following we study the SNS potential employing the dielectric response given by Eq. (3). Using the cylindrical coordinates (ρ,θ,z) and evaluating the θ-integration, we obtain the electrostatic potential as

$$\phi(\mathbf{x},t) = \frac{q_t}{\pi}\int \frac{J_0(k_\perp\rho)\exp(ik_\parallel z)\delta(\omega)}{k^2\epsilon(\omega,\mathbf{k})}k_\perp\,dk_\perp\,dk_\parallel\,d\omega, \quad (5)$$

where $v_t \approx 0$ for the static test dust particulate.

For a drift wave $\mathbf{k} \simeq \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_\parallel$ with $k_y \gg k_\parallel$ and performing the ω-integration, we obtain

$$\phi(\mathbf{x},t) = \frac{q_t}{\pi}\int \frac{J_0(k_\perp\rho)\exp(ik_\parallel z)k_\perp\,dk_\perp\,dk_\parallel}{k_\parallel^2 + k_\perp^2(1+f) + k_e^2(1 - u_{i0}M^{-2}/L_{ni}\omega_{ci})}, \quad (6)$$

where $M = u_{i0}/C_s$ is the Mach number, $C_s = \omega_{pi}\lambda_{De}$ being the ion sound velocity and u_{i0y} is the component of $\mathbf{u}_{i0}$ along $\hat{\mathbf{y}}$-direction. Here we have assumed the condition $k_\perp^2 \gg k_\parallel^2$. Following Ref. [1], we finally obtain performing $k_\parallel$- and $k_\perp$-integrations in Eq. (6)

$$\phi(\rho,z) = \frac{q_t}{\sqrt{1+f}}\frac{\exp[-\sqrt{\rho^2 + z^2(1+f)}/L_s]}{\sqrt{\rho^2 + z^2(1+f)}}, \quad (7)$$

where

$$L_s = \frac{\lambda_{De}\sqrt{1+f}}{\sqrt{1 - C_s^2/u_{i0}L_{ni}\omega_{ci}}}. \quad (8)$$

This is the static asymmetric SNS potential in an inhomogeneous magnetoplasma. It is clear that if we neglect the ion inhomogeneity effect ($L_{ni} \approx \infty$) in the dielectric function, we can retrieve the usual SNS potential [1]. In the direction perpendicular to the magnetic field ($z = 0$), the amplitude of the potential decreases by a factor $\omega_{pi}/\omega_{ci} \gg 1$ and the effective shielding length of the potential increases by the same factor. However, in the direction parallel to the magnetic field ($\rho = 0$), the amplitude of the SNS potential decreases faster by the factor $\omega_{pi}^2/\omega_{ci}^2$ but the effective length ($L_s \sim \lambda_{De}$) is not affected by the magnetic field significantly [6].

Since $L_{ni} < 0$ for positive density gradient ($\partial n_{i0}/\partial x > 0$) and if $1 < C_s^2/u_{i0}|L_{ni}|\omega_{ci}$ for stronger inhomogeneity (smaller $|L_{ni}|$), the effective shielding length of the SNS potential reduces to

$$L_s \simeq \lambda_{De}\sqrt{1+f}\cdot\sqrt{\frac{|L_{ni}|\omega_{ci}M}{C_s}}. \quad (9)$$

Here, we notice that L_s increases with M, ω_{ci}, and $|L_{ni}|$. Hence, the effective shielding length L_s is a sensitive function of the scale length L_{ni}, the uniform drift velocity of streaming ions u_{i0}, Mach number M and magnetic field $\mathbf{B}_0$. The scale length of inhomogeneity L_{ni} can be positive as well as negative depending upon the decreasing or increasing density gradient $\partial n_{i0}(x)/\partial x$.

For $L_{ni} > 0$ for the decreasing ion density gradient, the ion dynamics may prohibit the shielding mechanism, and the shielding becomes oscillatory [cf. Eq. (7)] for $1 < |C_s^2/u_{i0}L_{ni}\omega_{ci}|$. Consequently, the strong inhomogeneity with smaller $|L_{ni}|$ can even destroy the shielding effect. Thus, the plasma approximation is broken for this situation.[1] However, for $1 > |C_s^2/u_{i0}L_{ni}\omega_{ci}|$, the SNS potential is retrieved with modified asymmetric screening length in different direction.

In the summary, we have investigated analytically the detailed properties of the electrostatic SNS potential in an inhomogeneous and transversely magnetized plasma as in a laboratory discharge plasma. The potential becomes a sensitive function of the external static magnetic field, the scalelength of inhomogeneity, and the diamagnetic drift velocity of ions. For a negative ion density gradient ($L_{ni} > 0$), there is a limit of existence of the modified static SNS potential [cf. Eq. (7)].

It may be mentioned here that our results are valid for a relatively stronger external magnetic field in the plasma where the approximations $\omega_{ci} \gg \omega \gg kv_{ti}$ are valid. For the opposite limit of vanishingly smaller magnetic field, there would not be a sufficiently $\mathbf{E} \times \mathbf{B}_0$ drift velocity of ions, and consequently, the drift mode would cease to exist. Then, the ion susceptibility could be given by $\chi_i = -\omega_{pi}^2/\omega'^2$ where ω' would be Doppler-shifted along the ion flow direction. This situation is similar to the case of Vladimirov and Nambu [13] for unmagnetized plasmas.

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[1] It should be noted that one cannot simply take an analytic continuation of Eq. (7) for obtaining the oscillatory long-range potential. This is because the poles for $k_\parallel$ integration in Eq. (6) should be shifted by taking into account the boundary condition [8].