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Electromagnetic dust-lower-hybrid and dust-magnetosonic waves and their instabilities in a dusty magnetoplasma

M. Salimullah
Salam Chair, Government College University, Lahore, Pakistan, Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh, and Department of Physics, Government College University, Lahore, Pakistan

M. M. Rahman
Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh

I. Zeba and H. A. Shah
Department of Physics, Government College University, Lahore, Pakistan

G. Murtaza
Salam Chair, Government College University, Lahore, Pakistan

P. K. Shukla
Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780, Bochum, Germany

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The electromagnetic waves below the ion-cyclotron frequency have been examined in a collisionless and homogeneous dusty plasma in the presence of a dust beam parallel to the direction of the external magnetic field. The low-frequency mixed electromagnetic dust-lower-hybrid and purely transverse magnetosonic waves become unstable for the sheared flow of dust grains and grow in amplitude when the drift velocity of the dust grains exceeds the parallel phase velocity of the waves. The growth rate depends dominantly upon the thermal velocity and density of the electrons. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2400846]

I. INTRODUCTION

Micron or submicron sized dust grains of conducting and nonconducting materials are often present in the laboratory and space environments where high frequency radiation or plasma currents may charge them electrically to high values.[1,2] These relatively massive and highly charged impurities/grains are quite common in natural systems, such as the plasmas in the planetary atmospheres and rings, rocket exhausts, satellite burning, volcanic eruptions, asteroid zones, cometary tails and deposits, circumstellar regions, interstellar clouds etc. and laboratory plasmas, such as industrial processing plasmas, plasma etching, plasma furnaces, edge plasmas in some fusion devices, etc. Obviously, because of the dynamics of this massive and charged third component, the dusty plasma can introduce new time and space scales giving rise to modifications or additional waves and instabilities.[3]

In unmagnetized dusty plasmas, most of the plasma properties appear in the form of the low-frequency electrostatic dust-acoustic and dust-ion-acoustic waves. External uniform magnetic field is usually applied with a view to controlling the properties and confining dusty plasmas in the laboratory systems, whereas most dusty plasmas in astrophysical environments are immersed in ambient magnetic field.[4] Although dusty plasmas are rich in waves and instabilities, low-frequency electrostatic waves with short wavelengths have drawn much attention. To the best of our knowledge, extensive works on excitation of long wavelength low-frequency electromagnetic waves, viz., the dust-lower-hybrid or dust-magnetosonic waves have not been reported in the literature. It is also well known that the sheared velocity flows of charged particles exist widely in space plasmas, such as those in the magnetopause, polar cusp, comet tails, and solar wind streams. The space shuttle exhaust plume is an example[5,6] where a dust particle beam drifts through the background of electrons and ions. Sheared flow drifts of dust particles in nonuniform dusty plasmas have been studied extensively.[7] These flows are the important free energy sources of macro- and microinstabilities in space plasmas.[8,9] The parametric cascading of the kinetic Alfvén wave can lead to the emission of ULF waves observed in the Earth's magnetosphere.

However, in the electromagnetic regime, only a limited number of attempts have been made in dusty magnetoplasmas.[10,11] Extensive analyses are required for the study of waves and instabilities in dusty magnetoplasmas. In this paper, we examine the role of dynamics of the dust grains and gyrating motion of ions and electrons on the low-frequency electromagnetic waves.

In this paper, we have studied the excitation and damping of the low-frequency electromagnetic dust-lower-hybrid and dust-magnetosonic waves by a continuously drifting beam of dust particles along the direction of the magnetic field in a finite temperature magnetized dusty plasma. We restrict our attention to the frequency regime below the ion-cyclotron frequency, where the dust dynamics may play a vital role. We examine the general dispersion relation of any wave, either electrostatic or electromagnetic, using the Vlasov-Maxwell set of equations. In Sec. II, we solve the Vlasov equation in terms of guiding center coordinates to obtain the distribution function for any species in a magnetized dusty plasma. To obtain the dielectric tensor components, the conductivity tensor components in the presence of an electromagnetic wave perturbation have been derived first in Sec. III. The general wave equation is written in terms of dielectric tensor components in the same section. The wave dispersion relations of the dust-lower-hybrid and dust-magnetosonic waves and their instability conditions are derived in Sec. IV. Finally, a brief discussion of the results is given in Sec. V.

II. SOLUTION OF VLASOV EQUATION IN TERMS OF GUIDING CENTER COORDINATES

We consider the propagation of a small amplitude low-frequency electromagnetic wave perturbation in a collisionless dusty plasma in the presence of a uniform magnetic field ($\mathbf{B}_0 \| \hat{z}$). For convenience of solution of the Vlasov equation governing the dynamics of the various plasma species, we write the distribution function, F_α for a plasma species α in terms of the canonical gyrokinetic variables as[12]

$$F_\alpha(\mathbf{x},\mathbf{v},t) = F_\alpha(\mathbf{x}_g,\mu,\theta,p_z,t), \tag{1}$$

so that the Vlasov equation is written as

$$\frac{\partial F_\alpha}{\partial t} + \dot{\mathbf{x}}_g \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}_g} + \dot{\mu}\frac{\partial F_\alpha}{\partial \mu} + \dot{\theta}\frac{\partial F_\alpha}{\partial \theta} + \dot{p}_z\frac{\partial F_\alpha}{\partial p_z} + v_z\frac{\partial F_\alpha}{\partial z} = 0. \tag{2}$$

Here,

$$\mathbf{x}_g = \mathbf{x} - \frac{\mathbf{v} \times \underline{\omega}_{c\alpha}}{\omega_{c\alpha}^2}, \tag{3}$$

where the underline on ω denotes a vector, and

$$\begin{aligned} &\omega_{c\alpha} = q_\alpha B_0/m_\alpha c,\\ &\mu = m_\alpha v_{\alpha\perp}^2/2\omega_{c\alpha},\\ &\theta = \tan^{-1}(v_y/v_x),\\ &p_z = m_\alpha v_z,\\ &\alpha = e,i,d, \end{aligned} \tag{4}$$

and q_α, m_α, c are the charge, mass, and velocity of light in a vacuum, respectively.

The overdot in Eq. (2) denotes a derivative with respect to time. The linear equation of motion for any species is

$$\frac{d\mathbf{v}}{dt} = \frac{q_\alpha \mathbf{E}}{m_\alpha} + \mathbf{v} \times \underline{\omega}_{c\alpha} + \frac{q_\alpha}{m_\alpha}\frac{\mathbf{v}\times\mathbf{B}}{c}, \tag{5}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields of the electromagnetic wave perturbation.

Using the identity

$$e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})} = e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)}\sum_n J_n(k_\perp \rho)e^{in(\theta-\delta)}, \tag{6}$$

and Eqs. (3)–(5), we obtain

$$\dot{\mathbf{x}}_{g\perp} = -\frac{q_\alpha \mathbf{E}_\perp \times \underline{\omega}_{c\alpha}}{m_\alpha \omega_{c\alpha}^2} + \frac{q_\alpha(v_\perp \mathbf{B}\cdot\underline{\omega}_{c\alpha} - B_\perp \mathbf{v}\cdot\underline{\omega}_{c\alpha})}{m_\alpha c\omega_{c\alpha}^2}, \tag{7}$$

$$\dot{\mu} = \frac{q_\alpha}{\omega_{c\alpha}}\mathbf{E}_\perp\cdot\mathbf{v}_\perp + \frac{q_\alpha}{c\omega_{c\alpha}}(\mathbf{v}\times\mathbf{B})_\perp\cdot\mathbf{v}_\perp, \tag{8}$$

$$\dot{\theta} = \omega_{c\alpha} - \frac{q_\alpha}{m_\alpha v_\perp}\left[E_x \sin\theta - E_y\cos\theta + \frac{1}{c}(\mathbf{v}\times\mathbf{B})_x \sin\theta - \left(\frac{\mathbf{v}\times\mathbf{B}}{c}\right)_y \cos\theta\right], \tag{9}$$

$$\dot{p}_z = q_\alpha E_z + \frac{q_\alpha(\mathbf{v}\times\mathbf{B})_z}{c}, \tag{10}$$

where $\rho = v_\perp/\omega_{c\alpha}$, θ is the angle made by $\mathbf{v}_\perp$ with the x axis, δ is the angle made by $\mathbf{k}_\perp$ with the x axis, and the symbol $\perp(\|)$ denotes a quantity perpendicular (parallel) to the z axis.

Using the identity, Eq. (6), the electric and magnetic fields of the electromagnetic wave are expressed as

$$\mathbf{E} = \mathbf{E}' e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})} = \mathbf{E}' e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)}\sum_n J_n(k_\perp\rho)e^{in(\theta-\delta)}, \tag{11}$$

$$\mathbf{B} = c\mathbf{k}\times\mathbf{E}/\omega. \tag{12}$$

Substituting Eqs. (11) and (12) in Eqs. (7)–(10) and following Ref. 12, we obtain for the species α

$$\begin{aligned} \dot{\mu} = \frac{q_\alpha}{\omega_{c\alpha}} e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)}\sum_n e^{in(\theta-\delta)}\Bigg\{&\left(1 - \frac{k_\| v_\|}{\omega}\right)\frac{v_\perp}{2}\\ &\times[J_{n-1}(E_x - iE_y)e^{i\delta} + J_{n+1}(E_x+iE_y)e^{-i\delta}]\\ &+ \frac{n\omega_{c\alpha}}{\omega}J_n v_\| E_\|\Bigg\}, \end{aligned} \tag{13}$$

$$\dot{\theta} = \omega_{c\alpha}, \tag{14}$$

$$\begin{aligned} \dot{p}_z = q_\alpha e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)}\sum_n e^{in(\theta-\delta)}\Bigg\{&\left(1 - \frac{n\omega_{c\alpha}}{\omega}\right)J_n E_\|\\ &+ \frac{k_\| v_\perp}{2\omega}[J_{n-1}(E_x - iE_y)e^{i\delta} + J_{n+1}(E_x+iE_y)e^{-i\delta}]\Bigg\}, \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{x}_g = \frac{q_\alpha}{m_\alpha\omega_{c\alpha}} e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)}\sum_n e^{in(\theta-\delta)}\Bigg\{&\left(1 - \frac{k_\| v_\|}{\omega}\right)J_n E_y\\ &- \frac{n\omega_{c\alpha}}{\omega}J_n E_y + \frac{k_y v_\|}{\omega}J_n E_\| + \frac{k_y v_\perp}{2\omega}[J_{n-1}(E_x - iE_y)e^{i\delta}\\ &+ J_{n+1}(E_x+iE_y)e^{-i\delta}]\Bigg\}. \end{aligned} \tag{16}$$

From Eq. (2) we note that at the equilibrium, $\partial/\partial t = 0$, $\mathbf{E}=\mathbf{B}=0$, $\dot{\mu}=0$, $\dot{p}_z=0$, $\dot{\mathbf{x}}_g=0$, and $\dot{\theta}=\omega_{c\alpha}$. Then, the distribution function is a constant of motion and we may expand the total distribution function about the equilibrium

$$F_\alpha = f^0_{\alpha 0}(\mu, p_z, x_g, y_g) + f_\alpha(\mathbf{r}, \mathbf{v}, t). \tag{17}$$

We consider only one-dimensional density inhomogeneity,

$$n_{\alpha 0}(x) = n^0_{\alpha 0}(1 \pm x/L_{n\alpha}), \tag{18}$$

$L_{n\alpha}(=-n_{\alpha 0}(x)/n'_{\alpha 0}(x), n'_{\alpha 0}(x)=dn_{\alpha 0}(x)/dx)$ is the scale length of the density inhomogeneity of the species α. In general, $L_{n\alpha}$ is a function of x. However, for a particular choice of the density inhomogeneity in the x direction, viz., $n_{\alpha 0}(x) = n^0_{\alpha 0}(1 \mp x/L_{n\alpha})$, the scale length of inhomogeneity is independent of x. Hence, the equilibrium distribution function is taken as

$$f^0_{\alpha 0} = n^0_{\alpha 0}(1 \pm x_g/L_{n\alpha}) f_{M\alpha}(\mu, p_z), \tag{19}$$

where $f_{M\alpha}$ is the equilibrium distribution function which may be taken as a usual Maxwellian or a drifting Maxwellian.

Writing

$$f_\alpha = e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x}_g)} \sum_n e^{in(\theta-\delta)} f_{n\alpha}, \tag{20}$$

we solve the linearized Vlasov equation, Eq. (2) to obtain

$$f_{n\alpha} = \frac{n^0_{\alpha 0}}{i(\omega - n\omega_{c\alpha} - k_\parallel v_\parallel)} \Bigg\{ \frac{q_\alpha f_{M\alpha}}{m_\alpha \omega_{c\alpha} L_{n\alpha}} \Bigg[\left(1 - \frac{k_\parallel v_\parallel}{\omega} - \frac{n\omega_{c\alpha}}{\omega}\right) E_y J_n + \frac{k_y v_\parallel}{\omega} J_n E_\parallel + \frac{k_y v_\perp}{2\omega} \{ J_{n-1}(E_x - iE_y) e^{i\delta} + J_{n+1}(E_x + E_y) e^{-i\delta} \} \Bigg]$$
$$+ \frac{q_\alpha}{\omega_{c\alpha}} \left(1 + \frac{x_g}{L_{n\alpha}}\right) \frac{\partial f_{M\alpha}}{\partial \mu} \Bigg[\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) \frac{v_\perp}{2} \{ J_{n-1}(E_x - iE_y) e^{i\delta} + J_{n+1}(E_x + iE_y) e^{-i\delta} \} + \frac{n\omega_{c\alpha}}{\omega} J_n v_\parallel E_\parallel \Bigg]$$
$$+ q_\alpha \left(1 + \frac{x_g}{L_{n\alpha}}\right) \frac{\partial f_{M\alpha}}{\partial p_\parallel} \Bigg[\left(1 - \frac{n\omega_{c\alpha}}{\omega}\right) J_n E_\parallel + \frac{k_\parallel v_\perp}{2\omega} \{ J_{n-1}(E_x - iE_y) e^{i\delta} + J_{n+1}(E_x + iE_y) e^{-i\delta} \} \Bigg] \Bigg\}. \tag{21}$$

For a Maxwellian equilibrium distribution $f_{M\alpha}$, we simplify the above equation to obtain

$$f_{n\alpha} = -\frac{n^0_{\alpha 0} q_\alpha f_{M\alpha}}{iT_\alpha} \Bigg\{ -\frac{\omega^*_\alpha E_y J_n}{\omega k_y} + \frac{1 - \omega^*_\alpha/\omega}{\omega - n\omega_{c\alpha} - k_\parallel v_\parallel} \Bigg[J_n v_\parallel E_\parallel + \frac{v_\perp}{2} J_{n-1}(E_x - iE_y) e^{i\delta} + \frac{v_\perp}{2} J_{n+1}(E_x + iE_y) e^{-i\delta} \Bigg] \Bigg\}, \tag{22}$$

where $\omega^*_\alpha = k_y v^2_{t\alpha}/2\omega_{c\alpha} L_{n\alpha}$ is the diamagnetic drift frequency for the species α and $v_{t\alpha} = (2T_\alpha/m_\alpha)^{1/2}$. Thus,

$$f_\alpha = e^{-i(\omega t - \mathbf{k}\cdot\mathbf{x})} \sum_l \sum_n e^{i(n-l)(\theta-\delta)} J_n(k_\perp \rho) f_{n\alpha}. \tag{23}$$

III. DIELECTRIC TENSOR COMPONENTS

We consider a beam of dust grains flowing along an external magnetic field ($\underline{B}_0 \parallel \hat{z}$) in a collisionless, finite temperature electron-ion plasma with constant dust drift velocity, $\underline{u}_0 \parallel \underline{B}_0$. Following Bernhardt *et al.*,[5] we consider the frame of reference in which the dust is stationary and electrons and ions drift relative to the dust with constant drift velocity $-\underline{u}_0$. Thus, the free energy associated with the drift motions of electrons and ions can provide the source of an instability which excites the electromagnetic waves. In this frame of reference, the dust distribution can be described by a Maxwellian and electrons and ions distributions can be described by drifting Maxwellians.

We consider the equilibrium distribution function, f_{j0} for electrons and ions as the drifting Maxwellians

$$f_{j0} = (m_j/2\pi T_j)^{3/2} \exp(-v^2_\perp/v^2_{tj}) \exp[-(v_\parallel - u_{0j})^2/v^2_{tj}], \tag{24}$$

where $j = e, i$, $v_{tj} = (2T_j/m_j)^{1/2}$ is the thermal velocity, T_j being the temperature in energy units and m_j is the mass of the jth species. For simplicity we consider the equal drift velocity for electrons and ions ($u_{0e} = u_{0i} = u_0$) as they drift together with respect to the dust distribution along the external magnetic field.

The general wave equation for electromagnetic waves is given by

$$\nabla^2 \mathbf{E} - \nabla\nabla \cdot \mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E} = -\frac{4\pi i \omega}{c^2}\mathbf{J}, \tag{25}$$

or,

$$\underline{\underline{D}} \cdot \mathbf{E} = 0, \tag{26}$$

where

$$\underline{\underline{D}} = k^2 \underline{\underline{I}} - \mathbf{kk} - \frac{\omega^2}{c^2} \underline{\underline{\epsilon}}, \tag{27}$$

$$\underline{\underline{\epsilon}} = \underline{\underline{I}} + \frac{4\pi i}{\omega} \sum_\alpha \underline{\underline{\sigma}}_\alpha, \tag{28}$$

and $\underline{\underline{\sigma}}_\alpha$ is the conductivity tensor. The current density is given by

$$\mathbf{J}_\alpha \equiv \underline{\underline{\sigma}}_\alpha \cdot \mathbf{E} = q_\alpha \int \mathbf{v} f_\alpha d\mathbf{v}. \tag{29}$$

Using f_α from Eq. (23) with $f_{M\alpha}$ given by the parallel drifting Maxwellian, Eq. (24) in Eq. (29), one can easily obtain the components of the conductivity tensor as

$$\sigma_{\alpha xx} = \frac{-in_{0\alpha}^0 q_\alpha^2}{m_\alpha} \frac{1-\omega_\alpha^*/\omega'}{k_\| v_{t\alpha}} \sum_n Z\left(\frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}}\right) \cdot \left\{ \frac{n^2}{b_\alpha} I_n(b_\alpha) e^{-b_\alpha} \cos^2\delta + \left[b_\alpha \frac{d^2}{db_\alpha^2}(I_n e^{-b_\alpha}) + I_n' e^{-b_\alpha} \right] \sin^2\delta \right\}, \tag{30}$$

$$\sigma_{\alpha xy} = \frac{-n_{0\alpha}^0 q_\alpha^2}{m_\alpha} \frac{1-\omega_\alpha^*/\omega'}{k_\| v_{t\alpha}} \sum_n Z\left(\frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}}\right) \cdot \left\{ n \frac{d}{db_\alpha} I_n(b_\alpha) e^{-b_\alpha} \left(\frac{-q_\alpha}{e}\right) + i\cos\delta \sin\delta \times \left[\frac{n^2}{b_\alpha} I_n e^{-b_\alpha} - b_\alpha \frac{d^2}{db_\alpha^2}(I_n e^{-b_\alpha}) - I_n' e^{-b_\alpha} \right] \right\}, \tag{31}$$

$$\sigma_{\alpha xz} = \frac{-2n_{0\alpha}^0 q_\alpha^2}{m_\alpha} \frac{1-\omega_\alpha^*/\omega'}{k_\| v_{t\alpha}} \times \sum_n \left[1 + \frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}} Z\left(\frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}}\right) \right] \cdot \left[\frac{n\omega_{c\alpha}}{k_\perp v_{t\alpha}} i\cos\delta I_n e^{-b_\alpha} - \frac{k_\perp v_{t\alpha}}{2\omega_{c\alpha}} \sin\delta \frac{d}{db_\alpha} I_n e^{-b_\alpha} \right], \tag{32}$$

$$\sigma_{\alpha yx} = \frac{n_{0\alpha}^0 q_\alpha^2}{m_\alpha} \frac{1-\omega_\alpha^*/\omega'}{k_\| v_{t\alpha}} \sum_n Z\left(\frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}}\right) \cdot \left\{ n \frac{d}{db_\alpha} I_n(b_\alpha) e^{-b_\alpha} \left(\frac{-q_\alpha}{e}\right) - i\cos\delta \sin\delta \left[\frac{n^2}{b_\alpha} I_n e^{-b_\alpha} - b_\alpha \frac{d^2}{db_\alpha^2}(I_n e^{-b_\alpha}) - I_n' e^{-b_\alpha} \right] \right\}, \tag{33}$$

$$\sigma_{\alpha yy} = \sigma_{\alpha xx}|_{\delta\to\delta-\pi/2}, \tag{34}$$

$$\sigma_{\alpha yz} = \sigma_{\alpha xz}|_{\delta\to\delta-\pi/2}, \tag{35}$$

$$\sigma_{\alpha zx} = \sigma_{\alpha xz}|_{\delta\to-\delta}, \tag{36}$$

$$\sigma_{\alpha zy} = \sigma_{\alpha zx}|_{\delta\to\delta-\pi/2}, \tag{37}$$

$$\sigma_{\alpha zz} = \frac{-2in_{0\alpha}^0 q_\alpha^2}{m_\alpha} \frac{1-\omega_\alpha^*/\omega'}{k_\| v_{t\alpha}} \sum_n \frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}} \times \left[1 + \frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}} Z\left(\frac{\omega' - n\omega_{c\alpha}}{k_\| v_{t\alpha}}\right) \right] I_n e^{-b_\alpha}, \tag{38}$$

where $I_n'(b_\alpha) = dI_n(b_\alpha)/db_\alpha$, $b_\alpha = k_\perp^2 v_{t\alpha}^2 / 2\omega_{c\alpha}^2$, and $\omega' = \omega - k_\| u_0$.

IV. DUST-LOWER-HYBRID AND DUST-MAGNETOSONIC WAVES

In this section, we study the dispersive properties and damping or growth of the low frequency electromagnetic waves, viz. the mixed dust-lower-hybrid and purely transverse electromagnetic waves in a dusty plasma where sheared dust flow is present. Both the waves propagate nearly perpendicular to the external homogeneous magnetic field, having different direction of the electric field of the waves.

Since D_{yx} and D_{yz} are small compared to D_{yy} for the low-frequency electromagnetic wave $(\omega \ll \omega_{ci})$, we may assume $E_y = 0$, so that both **k** and **E** lie in the same plane (XZ plane). Here, the parallel electric component gives rise to the electromagnetic character of the propagating wave perturbation. Consequently, the dispersion relation of the low-frequency mixed electromagnetic dust-lower-hybrid wave is obtained from

$$\begin{bmatrix} D_{xx} & D_{xz} \\ D_{zx} & D_{zz} \end{bmatrix} = 0. \tag{39}$$

Since $D_{xz}, D_{zx} \ll D_{xx}, D_{zz}$, we have

$$D_{xx} \cdot D_{zz} \simeq 0. \tag{40}$$

Again, since D_{zz} is independent of the external magnetic field for nearly perpendicular propagation, the wave dispersion is obtained from $|D_{xx}| = 0$. Thus,

$$\frac{k_\|^2 c^2}{\omega^2} - \epsilon_{xx} = 0. \tag{41}$$

The sheared flow drift velocities of electrons and ions can be assumed to be equal and uniform $(u_{e0} = u_{i0} \equiv u_0)$ described by the equilibrium distribution, Eq. (24). Taking $\omega' = \omega - k_\| u_0$, we obtain ϵ_{xx} in the presence of shear flow drift of dust grains as

$$\epsilon_{xx}(\omega') = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega'^2} + iI, \tag{42}$$

where

$$I = \sqrt{\frac{\pi}{4}} \frac{2}{\omega'} \left[\frac{\omega_{pe}^2}{k_\| v_{te}} e^{-(\omega_{ce}/k_\| v_{te})^2} + \frac{\omega_{pi}^2}{k_\| v_{ti}} e^{-(\omega_{ci}/k_\| v_t)^2} \right]. \tag{43}$$

Neglecting damping or growth due to the free-energy of the drifting dust beam, we obtain the electromagnetic dust-lower-hybrid wave propagating nearly perpendicular to the external magnetic field from Eqs. (41) and (42) as

$$\omega_r^2 = \omega_{dlh}^2 \left(1 + \frac{k_\|^2 c^2}{\omega_{pd}^2} \right), \tag{44}$$

where the dust-lower-hybrid frequency is $\omega_{dlh}^2 = \omega_{pd}^2 / (1 + \omega_{pe}^2/\omega_{ce}^2 + \omega_{pi}^2/\omega_{ci}^2) \simeq \omega_{pd}^2 \omega_{ci}^2 / \omega_{pi}^2$.

Again, using Eqs. (41) and (42) and assuming $\omega = \omega_r + i\gamma$ with $\gamma \ll \omega_r$, we obtain the normalized damping rate of the electromagnetic dust-lower-hybrid wave as

$$\frac{\gamma}{\omega_r} = -\sqrt{\frac{\pi}{4}}\frac{\omega_r^2}{k_\parallel^2 c^2(\omega_r - k_\parallel u_0)}\frac{\omega_{pe}^2}{k_\parallel v_{te}} \times\left[e^{-\left(\frac{\omega_{ce}}{k_\parallel v_{te}}\right)^2} + \delta_{ie}\sqrt{\frac{T_e m_e}{T_i m_i}}e^{-\left(\frac{\omega_{ci}}{k_\parallel v_{ti}}\right)^2}\right], \quad (45)$$

where δ_{ie} is the non-neutrality parameter, $\delta_{ie}=n_{i0}/n_{e0}$ with n_{i0} and n_{e0} are the equilibrium density of ions and electrons, respectively, in the dusty plasma. Thus, the dust-lower-hybrid wave becomes unstable for the flow of electrons and ions and grows in amplitude when the uniform drift velocity exceeds the parallel phase velocity of the wave ($u_0 > \omega_r/k_\parallel$).

For the propagation of a purely transverse electromagnetic wave with frequency below the ion-cyclotron frequency, we assume $E_x=E_z=0, E_y\neq 0$ and $\mathbf{k}\simeq \hat{x}k_\perp + \hat{z}k_\parallel$, $k_\parallel^2 \ll k_\perp^2$. We consider the presence of similar sheared flow dust beam in the dusty magnetoplasma. Thus, using Eq. (26), the dispersion relation can be obtained from $|D_{yy}|=0$, that is

$$\frac{k^2c^2}{\omega^2} = \epsilon_{yy}. \quad (46)$$

Collecting real and imaginary parts, we obtain the dispersion relation of the purely electromagnetic dust-magnetosonic wave as

$$\omega^2 = \omega_{dlh}^2\left(1 + \frac{k^2c^2}{\omega_{pd}^2}\right), = \omega_{dlh}^2 + k^2 v_A^2, \quad (47)$$

where $v_A = c\omega_{ci}/\omega_{pi}$ is the Alfvén speed. It is noticed from Eq. (47) that in absence of the dust component, the dust magnetosonic wave reduces to the usual magnetosonic wave in the electron-ion plasma. The normalized damping rate of the pure electromagnetic transverse magnetosonic wave is obtained from Eq. (46) as

$$\frac{\gamma}{\omega_r} = -\sqrt{\frac{\pi}{4}}\frac{\omega_r^2}{k^2 c^2(\omega_r - k_\parallel u_0)}\frac{\omega_{pe}^2}{k_\parallel v_{te}} \times\left[e^{-\left(\frac{\omega_{ce}}{k_\parallel v_{te}}\right)^2} + \delta_{ie}\sqrt{\frac{T_e m_e}{T_i m_i}}e^{-\left(\frac{\omega_{ci}}{k_\parallel v_{ti}}\right)^2}\right]. \quad (48)$$

It may be mentioned here that the dust-magnetosonic wave at the frequency below the ion-cyclotron frequency having a finite parallel wave number will have the dispersion relation, Eq. (47) and the instability condition, Eq. (48). The other transverse electromagnetic wave, viz., the dust-kinetic Alfvén wave involving the dust dynamics propagating obliquely to the magnetic field has been studied earlier.[11,13,14]

It is also noted here that for the above phase velocity limit, the contribution of ions is dominant compared to that of electrons in contributing to the wave dispersions of the magnetized dusty plasma. The dust-lower-hybrid and dust-magnetosonic waves are unstable and grow in amplitude when the dust drift velocity exceeds the parallel phase velocity of the waves. The growth rate depends upon the thermal velocity and density of the lighter of the lightest particles, that is the electrons, Eqs. (45) and (48).

V. DISCUSSION

In this paper, we have made a rigorous analytical investigation on the low-frequency electromagnetic waves in a streaming homogeneous dusty magnetoplasma. The low-frequency electromagnetic dust-lower-hybrid and dust-magnetosonic waves become unstable for the sheared flow of dust grains and grow in amplitude when the drift velocity of the dust grains exceeds the parallel phase velocity of the waves. The growth rate depends dominantly upon the thermal velocity and density of the electrons.

It may be mentioned here that in the present paper, the dust grains are considered as an additional ion-like plasma component with a constant charge. The fluctuations of the dust charges are neglected. The "Tromso-damping," i.e., the damping due to fluctuations of grain charges are important for the very low-frequency electrostatic waves involving the dust dynamics. No charge density is associated with an electromagnetic wave in a dusty plasma, which may be affected due to the flow of fluctuating charge perturbation on the dust surfaces. Since the electromagnetic dust-lower-hybrid wave has a component of electric field in the direction of the magnetic field, it is a mixed electromagnetic and electrostatic mode. However, the dust magnetosonic wave is purely electromagnetic and transverse.

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