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PARALLEL PROTON HEATING IN SOLAR WIND USING GENERALIZED (r, q) DISTRIBUTION FUNCTION

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Abstract. Kinetic theory is used to calculate the power dissipated by obliquely propagating Alfvén waves to heat the solar wind protons, using the Generalized (r, q) distribution function. The evolution of power dissipation of protons with increasing heliocentric distance is subsequently determined. Comparison between theoretical and observational results with data shows good agreement, especially for the slow solar wind streams. Previous results where a Maxwellian distribution function was used to calculate the power dissipated did not match well with observations.

1. Introduction

During the last few decades the physical picture of the solar wind has been considerably enriched due to *in situ* observations of particles and wave processes. In particular, data obtained from the plasma analyzer of the *Helios* solar probe has provided a full three-dimensional resolution of the ion-velocity distributions. This data analysis was carried out for observations between 0.3 and 1 AU for helium protons and ions distribution by Marsch *et al.* (1982a,b). As the solar wind expands from the Sun toward the orbit of the Earth, one expects both the electrons and protons to cool. However, they do so less rapidly than would be expected on the basis of an adiabatic equation of state (Gary and Feldman, 1977; Marsch *et al.*, 1989). Observations (Parker, Kennel, and Lanzerotti, 1979; Marsch *et al.*, 1989) show that the solar wind proton temperature falls off as $T \sim r^{-\alpha}$, where α has values $0.3 \leq \alpha \leq 4/3$, being the value for the fastest and slowest streams at radial distance r of the solar wind. But the adiabatic equation of state gives the power law to be $T \sim r^{-\beta}$, where $\beta = 1.333$ for adiabatic cooling. So the slower streams tend to follow the adiabatic law while the faster streams show a significant departure from it. Evidently some supplementary heating mechanism is needed for the latter.

Marsch *et al.* (1982b) investigated the dependence of $T_{\parallel p}$ and $T_{\perp p}$ on heliocentric distance between 0.3 and one AU for various solar wind velocities.

Alfvén waves are known to dominate the high-velocity regions of the solar wind and, in particular, the stream trailing edges (Belcher and Davis, 1971). Because most of these waves propagate away from the Sun in the solar wind and their amplitude is sufficiently large at one AU, one expects that they play a major role in driving the high-speed solar wind streams. Collisionless heating by these waves through ion cyclotron resonance has indeed been used by many authors (Marsch, Goertz, and Richter, 1981; Schwartz, Feldman, and Gary, 1981; Isenberg and Hollweg, 1982) to obtain perpendicular heating. The general conclusion (Schwartz, Feldman, and Gary, 1981) is that this mechanism which involves only the high frequency part of the Alfvénic spectrum is insufficient to explain the heating. To examine this discrepancy, it was noted that the Alfvénic turbulence observed in the solar wind may indeed undergo oblique propagation, for which the dissipation mechanism is linear Landau damping. Indeed this mechanism was used earlier (Dobrowolny and Torricelli-Ciamponi, 1985) to account for the radial evolution of power law spectra of the magnetic field fluctuations as obtained from the *Helios* data (Bavassano *et al.*, 1982). The same linear Landau damping, which leads to wave dissipation, also causes particle heating since the field responsible for this effect is an electric field parallel to the interplanetary magnetic field, thus its impact is on the parallel temperature.

In this paper we propose that dissipation due to obliquely propagating Alfvén waves will adequately account for the discrepancy between the adiabatic law and the observed variation of the parallel temperature of the protons. Previously the power dissipated was calculated using a Maxwellian distribution function (Shah, Iess, and Dobrowolny, 1986) which did not give good agreement with the observations, since the magnitude of Alfvénic power that goes into parallel proton heating is of the order of 10^{-21} erg cm^{-3} s^{-1} at one AU, which is much smaller than the required proton heating power of the order of 10^{-16} erg cm^{-3} s^{-1} at one AU.

The Maxwellian distribution is the most popular plasma distribution and has become the default assumption when the detailed distribution function is unknown. However, with more and more empirical data becoming available, it is realized that in real plasma systems, the particle distributions deviate significantly from Maxwellian distributions implying that when we use theoretical models using these distribution functions to explain or to predict different waves and instabilities, the ensuing results do not give good quantitative fits with observations (Summers and Thorne, 1991; Xue, Thorne, and Summers, 1993; Qureshi *et al.*, 2004). It means that a Maxwellian is not a realistic distribution under all circumstances, and other distributions such as Kappa (Treumann, 1999), or Generalized (r, q) distribution (which is used in the present work) work better to reproduce the observed results. The use of the family of Kappa distributions to model the observed nonthermal features of electron and ion structures was frequently criticized due to lack of its formal derivation. A classical analysis addressing this problem was performed by Hasegawa, Mima, and Duong-van (1985) who demonstrated how the Kappa distributions emerge as a natural consequence of the presence of super-thermal radiation fields in plasmas.

In the present work, we use in place of Maxwellian distribution, Generalized (r, q) (or non-Maxwellian) distribution function (Qureshi *et al.*, 2004; Zaheer, Murtaza, and Shah, 2004), given by

$$f_{(r,q)} = \frac{3(q-1)^{\frac{-3}{2(1+r)}}\Gamma(q)}{4\pi\Psi_\perp^2\Psi_{\shortparallel}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)} \times\left(1+\frac{1}{q-1}\left(\left(\frac{v_{\shortparallel}}{\Psi_{\shortparallel}}\right)^2+\left(\frac{v_\perp}{\Psi_\perp}\right)^2\right)^{r+1}\right)^{-q} \tag{1}$$

where r and q are spectral indices, and thermal speed is related to parallel and perpendicular temperatures (with respect to the ambient magnetic field) through the following relationships

$$\Psi_{\shortparallel} = v_{T\parallel}\sqrt{\frac{3(q-1)^{\frac{-1}{(1+r)}}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}} \tag{2}$$

and

$$\Psi_\perp = v_{T\perp}\sqrt{\frac{3(q-1)^{\frac{-1}{(1+r)}}\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}} \tag{3}$$

The normalization condition for the (r, q) distribution function restricts r and q to $q > 1$ and $q(1 + r) > 5/2$. This distribution is a generalized form of the Kappa distribution function with index κ (Qureshi *et al.*, 2004), and reduces to a Kappa distribution function if $r = 0$ and $q \to \kappa + 1$, and to a Maxwellian when $q \to \infty$ and $r = 0$. We note here that the spectral index q contributes to the high-energy tails in the distribution functions and r gives rise to the flat or sharp top of the distribution function. We note that r is the same spectral index that appears in the Davydov–Druvestyen distribution functions (Manfredi, 1997) which have been used to model low-pressure electric discharge plasmas, and is responsible for the nonlinear (anomalous) damping which may occur in plasmas in quasi-thermodynamic equilibrium. These distributions are common both in natural and laboratory plasmas (Mushtaq and Shah, 2006) such as in galactic cosmic rays, solar flares, the magnetotail, shock waves, the earths plasma sheet, and the solar wind, *etc.*

Data analysis for solar wind plasmas suggests that (r, q) type distributions give better fits to data than the simpler Kappa distributions (Mushtaq and Shah, 2006).

The layout of this paper is as follows: in the second section we use the kinetic theory to calculate the expression for the power dissipated due to oblique Alfvén waves when a background plasma has an (r, q) velocity distribution, in the third section we derive an expression which gives an equation for power dissipated (which is needed in order to account for the amount of heating which is necessary for the solar wind protons), in section four we use data for the solar wind particle distribution function taken from the *Cluster* spacecraft, and have used the (r, q) distribution function. A brief summary of the results and discussion is given in the fifth section.

2. Power Dissipated Due to Landau Damping of Alfvén Waves with (r, q) Distribution Function

We follow the general formalism of the kinetic theory to calculate the power dissipated through obliquely propagating Alfvén waves. In a purely collisionless state the most fundamental equation describing the plasma dynamics is the Vlasov equation, taken separately for each species (Stix, 1962). The ambient magnetic field $\mathbf{B}_0$ is assumed to be constant and homogeneous and is taken along the z-axis, the obliqueness is taken with respect to $\mathbf{B}_0$. The wave vector $\mathbf{k}$ has components in the y and z directions and the fluctuating electric field $\mathbf{E}$ for an Alfvén wave has components in the x and z direction (Shah, Iess, and Dobrowolny, 1986). However, we consider small angle of propagation only, $E_z \ll E_x$. It is by virtue of the E_z component that Landau damping is operative.

The general expression for the power dissipated per unit volume (Stix, 1962) is

$$P_j = \frac{1}{16\pi} \sum_j \frac{\epsilon_j \omega_{Pj}^2}{\Omega_j} \mathbf{E}^* \cdot \mathbf{M}_j \cdot \mathbf{E} \tag{4}$$

where ϵ_j is sign of the charge, $\mathbf{M}_j$ the mobility tensor, $\omega_{\mathrm{P}j}$ the plasma frequency, and Ω_j the gyrofrequency of particles of the species j (Shah, Iess, and Dobrowolny, 1986). To determine the Landau damping, we need to calculate $\mathbf{M}_{zz}^{(r,q)}$ which, for the (r, q) distribution function, is

$$\begin{aligned}\mathbf{M}_{zz}^{(r,q)} = &-\sum_j \frac{i\Omega_j \epsilon_j 2q(1+r)A}{\Psi_{\parallel j}^2 (q-1)} \int v_{\parallel}^2 d^3 v \\ &\times \left(1 + \frac{1}{q-1}\left(\left(\frac{v_{\parallel}}{\Psi_{\parallel j}}\right)^2 + \left(\frac{v_{\perp}}{\Psi_{\perp j}}\right)^2\right)^{r+1}\right)^{-q-1} \\ &\times \left(\left(\frac{v_{\parallel}}{\Psi_{\parallel j}}\right)^2 + \left(\frac{v_{\perp}}{\Psi_{\perp j}}\right)^2\right)^{r} \left(1 - \frac{k_{\perp}^2 v_{\perp}^2}{2\Omega_j^2}\right) \frac{1}{(k_{\parallel} v_{\parallel} - \omega)}\end{aligned} \tag{5}$$

where

$$A = \frac{3(q-1)^{\frac{-3}{2(1+r)}} \Gamma(q)}{4\pi \Psi_{\perp j}^2 \Psi_{\parallel j} \Gamma\left(q - \frac{3}{2(1+r)}\right) \Gamma\left(1 + \frac{3}{2(1+r)}\right)} \tag{6}$$

Here we have used a series expansion of the Bessel function for small Larmor radius. We first perform the integration with respect to $v_{\perp}$ of Equation (5), by making the following change of variables

$$\frac{v_\perp^2}{\Psi_{\perp j}^2} + a = x, \quad \text{where} \quad a = \frac{v_\shortparallel^2}{\Psi_{\shortparallel j}^2}. \tag{7}$$

. Now the limits for the perpendicular integration with respect to x are between a and ∞, and we obtain

$$\mathbf{M}_{zz}^{(r,q)} = -\sum_j \frac{i\Omega_j \epsilon_j 2A_1}{\Psi_{\parallel j}^3}$$

$$\times \left[\begin{array}{c} \displaystyle\int_{-\infty}^{+\infty} \frac{v_\shortparallel^2 \mathrm{d}v_\shortparallel}{(k_\shortparallel v_\shortparallel - \omega)} \left(1 + \frac{1}{q-1}\left(\frac{v_\shortparallel^2}{\Psi_{\shortparallel j}^2}\right)^{r+1}\right)^{-q} \\ \displaystyle -\frac{q(1+r)(q-1)^q}{(-1+q+qr)} \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2} \int_{-\infty}^{+\infty} \frac{v_\shortparallel^2 \left(\frac{v_\shortparallel^2}{\Psi_{\shortparallel j}^2}\right)^{1-q-qr} \mathrm{d}v_\shortparallel}{(k_\shortparallel v_\shortparallel - \omega)} \times \\ \displaystyle {}_2F_1\left(q+1, q - \frac{1}{1+r}, q + \frac{r}{1+r}, -(q-1)\left(\frac{v_\shortparallel^2}{\Psi_{\shortparallel j}^2}\right)^{(-r-1)}\right) \\ \displaystyle + \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2 \Psi_{\parallel j}^2} \int_{-\infty}^{+\infty} \frac{v_\shortparallel^4 \mathrm{d}v_\shortparallel}{(k_\shortparallel v_\shortparallel - \omega)} \left(1 + \frac{1}{q-1}\left(\frac{v_\shortparallel^2}{\Psi_{\shortparallel j}^2}\right)^{r+1}\right)^{-q} \end{array} \right] \tag{8}$$

where

$$A_1 = \frac{3(q-1)^{\frac{-3}{2(1+r)}}\Gamma(q)}{4\Gamma\left(q - \frac{3}{2(1+r)}\right)\Gamma\left(1 + \frac{3}{2(1+r)}\right)} \tag{9}$$

We again make a change of variables for parallel integration by taking

$$s = \frac{v_\shortparallel}{\Psi_{\shortparallel j}} \quad \text{and} \quad \xi = \frac{\omega}{k_\parallel \Psi_{\parallel j}^2} \tag{10}$$

in Equation (8) and obtain for the zz-component of the mobility tensor

$$\mathbf{M}_{zz}^{(r,q)} = -\sum_j \frac{\Omega_j \epsilon_j 2i}{\Psi_{\shortparallel j} k_\shortparallel} \left(\begin{array}{c} \xi^2 \left(\mathbf{Z}_1^{(r,q)}(\xi) - \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2} \mathbf{Z}_2^{(r,q)}(\xi)\right) + \\ \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2} \xi^4 \mathbf{Z}_1^{(r,q)}(\xi) + \xi A_1 C_1 - \\ \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2} \xi A_1 B_1 C_3 + \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2} \xi A_1 C_2 + \\ \frac{k_\perp^2 \Psi_{\perp j}^2}{2\Omega_j^2} \xi^3 A_1 C_1 \end{array} \right) \tag{11}$$

where $\mathbf{Z}_1^{(r,q)}(\boldsymbol{\xi})$ and $\mathbf{Z}_2^{(r,q)}(\xi)$ are the plasma dispersion functions for the Generalized (r, q) distribution function (Qureshi *et al.*, 2004), which are given by

$$\mathbf{Z}_1^{(r,q)}(\xi) = \frac{3(q-1)^{\frac{-3}{2(1+r)}}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)}\int_{-\infty}^{\infty}\frac{\left[1+\frac{1}{(q-1)}(s^2)^{(1+r)}\right]^{-q}}{(s-\xi)}\,\mathrm{d}s \quad (12)$$

and

$$\mathbf{Z}_2^{(r,q)}(\xi) = \frac{3(q-1)^{\frac{-3}{2(1+r)}}\Gamma(q)}{4\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)} \times\left(\frac{q\,(1+r)\,(q-1)^q}{(q+qr-1)}\right)\int_{-\infty}^{\infty}\frac{(s^2)^{1-q-qr}}{(s-\xi)} \times {}_2F_1\left(q+1, q-\frac{1}{1+r}, q+\frac{r}{1+r}, -(q-1)\,(s^2)^{(-r-1)}\right)\mathrm{d}s \quad (13)$$

where ${}_2F_1(q+1, q-\frac{1}{1+r}, q+\frac{r}{1+r}, -(q-1)(s^2)^{(-r-1)})$ is the Hypergeometric function and B_1, C_1, C_2, and C_3 are defined as

$$B_1 = \frac{q\,(1+r)\,(q-1)^q}{(q+qr-1)} \quad (13a)$$

$$C_1 = \int_{-\infty}^{\infty}\left(1+\frac{1}{q-1}(s^2)^{(1+r)}\right)^{-q}\mathrm{d}s \quad (13b)$$

$$C_2 = \int_{-\infty}^{\infty} s^2\left(1+\frac{1}{q-1}(s^2)^{(1+r)}\right)^{-q}\mathrm{d}s \quad (13c)$$

$$C_3 = \int_{-\infty}^{\infty}(s^2)^{(1-q-qr)}{}_2F_1 \times \left(q+1, q-\frac{1}{1+r}, q+\frac{r}{1+r}, -(q-1)(s^2)^{(-r-1)}\right)\mathrm{d}s \quad (13d)$$

Now substituting the value of the $\mathbf{M}_{zz}^{(r,q)}$ from Equation (11) into Equation (4), the expression for the power dissipated becomes

$$\mathbf{P}_{(r,q)j} = -\frac{|E_z|^2}{8\pi}\sum_j \frac{i\omega_{Pj}^2}{\Psi_{\shortparallel j}k_{\shortparallel}} \begin{pmatrix} \xi^2\left(\mathbf{Z}_1^{(r,q)}(\xi) - \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\mathbf{Z}_2^{(r,q)}(\xi)\right) + \\ \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\xi^4\mathbf{Z}_1^{(r,q)}(\xi) \\ +\xi A_1C_1 - \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\xi A_1B_1C_3 + \\ \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\xi A_1C_2 + \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\xi^3 A_1C_1 \end{pmatrix} \tag{14}$$

For $E_z \ll E_x$, we use the following expression (Wu and Huba, 1975)

$$\frac{E_z}{E_x} \cong \frac{k_\perp}{k_{\shortparallel}} \cdot \frac{\omega^2}{\Omega_p^2} \cdot \frac{m_e}{m_p}. \tag{15}$$

Further by using Maxwell's equation.

$$\nabla \times \mathbf{E} = \frac{-1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{16}$$

we can express E_z in terms of B_y and obtain

$$E_z = \frac{k_\perp}{k_{\shortparallel}} \cdot \frac{\omega^2}{\Omega_p^2} \cdot \frac{m_e}{m_p}\frac{\omega}{ck_z}B_y \tag{17}$$

Substituting this expression in Equation (14), we get

$$\begin{aligned} \mathbf{P}_{(r,q)j} = {} & -\frac{1}{8\pi}\sum_j \frac{\omega_{Pj}^2 i}{\Psi_{\shortparallel j}k_{\shortparallel}} \\ & \times \begin{pmatrix} \frac{v_A^2}{\Psi_{\shortparallel j}^2}\left(\mathbf{Z}_1^{(r,q)}(\xi) - \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\mathbf{Z}_2^{(r,q)}(\xi)\right) + \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\frac{v_A^4}{\Psi_{\shortparallel j}^4}\mathbf{Z}_1^{(r,q)}(\xi) \\ +\frac{v_A}{\Psi_{\shortparallel j}}A_1C_1 - \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\frac{v_A}{\Psi_{\shortparallel j}}A_1B_1C_3 \\ +\frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\frac{v_A}{\Psi_{\shortparallel j}}A_1C_2 + \frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\frac{v_A^3}{\Psi_{\shortparallel j}^3}A_1C_1 \end{pmatrix} \\ & \times \left(\frac{k_\perp k_{\shortparallel}}{c\Omega_p^2}v_A^3\frac{m_e}{m_p}B_y\right)^2 \end{aligned} \tag{18}$$

We now use the asymptotic expansion of $\mathbf{Z}_1^{(r,q)}(\xi)$ and $\mathbf{Z}_2^{(r,q)}(\xi)$ for $\xi \gg 1$ (this is relevant for Landau damping of oblique Alfvén waves) and get

$$\mathbf{P}_{(r,q)j} = \frac{1}{8}\sum_j \frac{A_1\omega_{\mathrm{P}j}^2 i}{\Psi_{\shortparallel j}|k_\shortparallel|}\left(\frac{v_\mathrm{A}^2}{\Psi_{\shortparallel j}^2}\right) \times \left(\begin{array}{c} \left(1+\frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}\left(\frac{v_\mathrm{A}^2}{\Psi_{\shortparallel j}^2}\right)\right)\left(1+\frac{1}{q-1}\left(\frac{v_\mathrm{A}^2}{\Psi_{\shortparallel j}^2}\right)^{1+r}\right)^{-q} \\ -\frac{k_\perp^2\Psi_{\perp j}^2}{2\Omega_j^2}B_1\left(\frac{v_\mathrm{A}^2}{\Psi_{\shortparallel j}^2}\right)^{1-q-qr} \\ \times{}_2F_1\left(q+1, q-\frac{1}{1+r}, q+\frac{r}{1+r}, -(q-1)\left(\frac{v_\mathrm{A}^2}{\Psi_{\shortparallel j}^2}\right)^{-r-1}\right)\end{array}\right) \times\left(\frac{k_\perp k_\shortparallel}{c\Omega_p^2}\frac{m_\mathrm{e}}{m_\mathrm{p}}\left(v_\mathrm{A}^3\right)B_y\right)^2 \quad (19)$$

with the Alfvén velocity $v_\mathrm{A} = B_o/\sqrt{4\pi m_i n_o}$.

In order to calculate the total power dissipated in the solar wind, we must consider a full spectrum of magnetic field fluctuations and then integrate over wave-number space for Alfvén waves. Further, we assume that the spectrum is isotropic and is of the kind (Shah, Iess, and Dobrowolny, 1986)

$$B_y^2(\vec{k}) = a_\mathrm{p}L_\mathrm{o}^3\langle\delta B_y^2\rangle(kL_\mathrm{o})^{-P} \quad (20)$$

where $\langle\delta B_y^2\rangle$ is the average value of the square of the magnetic field in the y direction, L_o is the correlation length of the outer scale of turbulence, P is the spectral index of the power law, and a_p is a constant. Defining the total power dissipated per unit volume as

$$R_j^\mathrm{alf} = \int P_j\,\mathrm{d}\Omega\,k^2\,\mathrm{d}k \quad (21)$$

where

$$\mathrm{d}\Omega = \mathrm{d}\phi\sin\theta\mathrm{d}\theta$$

We obtain

$$\mathbf{R}_{(r,q)j}^\mathrm{alf} = \frac{\pi}{8}A_1\sum_j \frac{\omega_{Pj}^2}{\Psi_{\shortparallel j}^3}\frac{v_\mathrm{A}^2}{c^2}\frac{v_\mathrm{A}^3}{\Omega_\mathrm{p}^2}\frac{v_\mathrm{A}^3}{\Omega_\mathrm{p}^2}\left(\frac{m_\mathrm{e}}{m_\mathrm{p}}\right)^2\frac{a_\mathrm{p}}{(6-p)} \times\left(1+\frac{1}{q-1}\left(\frac{v_\mathrm{A}^2}{\Psi_{\shortparallel j}^2}\right)^{1+r}\right)^{-q}\left(\frac{L_\mathrm{o}}{\chi r_{(r,q)p}}\right)^{6-p} L_\mathrm{o}^{-3}\langle\delta B_y^2\rangle \quad (22)$$

TABLE I
Observed Plasma Parameters in Slow Solar Wind at One AU

Parameter	Observed variations with radial distance	Value at one AU
B_o	$B_o \frac{(x^{-2}+x^{-4})}{2}$	5×10^{-5} G
p	const.	7/2 or 11/3
a_p	const.	$(p-3)/4$
L_o	const.	3.2×10^{10} cm
n	$n_o x^{-2}$	$5\,\text{cm}^{-3}$
$\langle \delta B_y^2 \rangle$	$\langle \delta B_y^2 \rangle_o \left[\frac{(x^{-2}+x^{-4})}{2}\right]$	0.25
$T_\parallel$	$T_{\parallel o} x^{-\alpha}$	1.5×10^5 K

where $\chi >$ one is consistent with a small Larmor radius. Using the observed variations of the different physical quantities shown in Table I, where $\langle \delta B_y^2 \rangle_o$ is the normalized value of the square of the average magnetic field, we obtain total Alfvénic power dissipated as

$$\mathbf{R}^{\text{alf}}_{(r,q)j} = \frac{\pi}{8} A_1 \sum_j \frac{\omega^2_{Pjo}}{v_{\text{Ao}}} \left(\frac{v_{\text{Ao}}}{c}\right)^2 \left(\frac{v_{\text{Ao}}}{\Psi_{\parallel jo}}\right)^3 \left(\frac{v_{\text{Ao}}}{\Omega_{\text{po}}}\right)^4 \left(\frac{m_e}{m_p}\right)^2 \left(\frac{L_o}{\chi r_{(r,q)\text{po}}}\right)^{6-p}$$
$$\times L_o^{-3} \langle \delta B_y^2 \rangle_o B_o^2$$
$$\times \left(\frac{x^{-2}+1}{2}\right)^{6-\frac{p}{2}} x^{-6+P} \tilde{T}^{\left(\frac{p}{2}-\frac{9}{2}\right)} \left[1 + \frac{1}{q-1} \left(\frac{v_{\text{Ao}}\sqrt{2}}{A_2 v_{T\parallel jo}}\right)^{2+2r}\right]^{-q+q}$$
$$\times \left[1 + \left(\frac{1}{q-1}\right) \left(\left(\frac{v_{\text{Ao}}\sqrt{2}}{A_2 v_{\text{T}\parallel jo}}\right)^2 \left(\frac{x^{-2}+1}{2}\right) \tilde{T}^{-1}\right)^{1+r}\right]^{-q} \tag{23}$$

Here $x = r/r_o$ is the normalized radial distance from the Sun as the solar wind travels toward the Earth and $\tilde{T}$ is given by

$$\tilde{T} = \frac{T_{\parallel j}(x)}{T_{\parallel jo}} \tag{24}$$

where $T_{\parallel jo}$ is the observed value of temperature at one AU for a particular solar wind stream (Shah, Iess, and Dobrowolny, 1986), $T_{\parallel j}(x)$ is the temperature between 0.3 and 1 AU and

$$A_2 = \sqrt{\frac{3(q-1)^{\frac{-1}{1+r}} \Gamma\left(\frac{3}{2(1+r)}\right) \Gamma\left(q - \frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right) \Gamma\left(q - \frac{5}{2(1+r)}\right)}} \tag{25}$$

Using the numerical values of different physical parameters for the protons at one AU (Shah, Iess, and Dobrowolny, 1986), the power dissipated for the electrons turns out to be smaller than that of the protons. The reason is that the proton thermal velocity is almost the same as the Alfvén velocity, which means that the protons can resonate with the Alfvén waves and a large amount of energy can be exchanged, we note that the electron thermal velocity is 40 times the Alfvénic velocity and therefore no effective exchange of energy takes place. Using the numerical values given in Table I, we get the power dissipated at one AU

$$R^{\mathrm{alf}}_{(r,q)p} = (1.8 \times 10^{-19}) A_1 \left(\frac{1}{A_2}\right)^{\frac{11}{2}} \left(\frac{x^{-2}+1}{2}\right)^{\frac{17}{4}} x^{\frac{-5}{2}} \tilde{T}^{\frac{-11}{4}} \times \left[1 + \frac{1}{q-1}\left(\frac{2}{A_2^2}\right)^{1+r}\right]^{-q+q} \times \left[1 + \frac{1}{q-1}\left(\left(\frac{2}{A_2^2}\right)\left(\frac{x^{-2}+1}{2}\right)\tilde{T}^{-1}\right)^{1+r}\right]^{-q} \tag{26}$$

This is the power dissipated through obliquely propagating Alfvén waves using the (r, q) distribution function. Here we may substitute different values of r and q subject to the conditions $q > 1$ and $q(1 + r) > 5/2$. Next we calculate the power required for the solar wind proton heating and then compare it with the dissipated Alfvén power.

3. Power Requirements of the Solar Wind Due to Adiabatic Cooling

The solar wind appears to cool during expansion, "less rapidly" than would be expected, and therefore some source of extra heating is required. In this section we compute this extra power for heating the protons in the parallel direction. Firstly, we write an expression for the rate at which parallel thermal energy density varies on the basis of observations in the range 0.3 –1 AU. Secondly, we obtain an expression which gives the heating or cooling rate due to a known theoretical law or equation of state. The difference ($R^{\mathrm{req.}}$) between these two expressions gives the power required for the parallel proton heating (such a scenario has been considered by Schwartz, Feldman, and Gary (1981) for the case of perpendicular proton heating). It is this power that has to be compared with the power dissipated by obliquely propagating Alfvén waves as calculated in the previous section.

The observed variation of the parallel temperature is

$$T^{\mathrm{obs.}}_{\|} = T^{\mathrm{obs.}}_{\|o} x^{-\alpha} \tag{27}$$

where the α has values $0.3 \leqslant \alpha \leqslant 4/3$, from fast to slow solar wind streams. The observed rate of change of energy can be expressed as

$$R^{\text{obs.}} = v_{\text{sw}} \frac{\mathrm{d}}{\mathrm{d}r} \left(n k_{\text{B}} T_{\parallel}^{\text{obs.}} \right) = -v_{\text{sw}} n_{\mathbf{o}} k_{\text{B}} T_{\parallel \text{o}}^{\text{obs.}} \frac{(2+\alpha)}{r_{\text{o}}} x^{-3-\alpha} \tag{28}$$

where v_{sw} is the velocity of the solar wind.

Following Schwartz, Feldman, and Gary (1981), we use an adiabatic law based on the CGL theory to calculate the variation of $T_{\parallel}^{\text{adb.}}$, since CGL theory is regarded as a good approximation for the collisionless solar wind with no wave effect to be taken into account. This gives the adiabatic temperature behavior as

$$T_{\parallel}^{\text{adb.}} \left(\frac{B}{n} \right)^2 = \text{const.} \tag{29}$$

Using Table I, we obtain

$$T_{\parallel}^{\text{adb.}} = 2 T_{\parallel \mathbf{o}}^{\text{adb.}} (1 + x^2)^{-1} \tag{30}$$

which gives the heating rate (cooling rate if negative). We have assumed here that $T_{\parallel}^{\text{adb.}} = T_{\parallel}^{\text{obs.}}$ at 0.3 AU. Thus the heating (cooling) rate due to the CGL theory is given by

$$R^{\text{adb.}} = v_{\text{sw}} \frac{\mathrm{d}}{\mathrm{d}r} \left(n k_{\text{B}} T_{\parallel}^{\text{adb.}} \right) = \frac{v_{\text{sw}} n_{\text{o}} k_{\text{B}} T_{\parallel \text{o}}^{\text{obs.}}}{r_{\text{o}}} \left(\frac{-4}{x(1+x^2)^2} - \frac{4}{x^3(1+x^2)} \right) \tag{31}$$

Now we calculate the excess power which needs to be supplied to counteract the cooling which is given by the difference of these two powers

$$R^{\text{req.}} = R^{\text{adb.}} - R^{\text{obs.}} = \frac{v_{\text{sw}} n_{\text{o}} k_{\text{B}} T_{\parallel \text{o}}^{\text{obs.}}}{r_{\text{o}}} \times \left((2+\alpha) x^{-3-\alpha} - \frac{4}{x(1+x^2)^2} - \frac{4}{x^3(1+x^2)} \right) \tag{32}$$

Choosing the typical parameter values at one AU, *i.e.*, $T_{\parallel \text{o}}^{\text{obs.}} = 1.5 \times 10^5\,\text{K}$, $v_{\text{sw}} = 4.5 \times 10^7\,\text{cm}\,\text{s}^{-1}$, $\alpha = 4/3$, we obtain

$$R^{\text{req.}} = 1.0 \times 10^{-16} \text{erg}\,\text{cm}^{-3}\,\text{s}^{-1} \tag{33}$$

From Equation (26), we obtain the power dissipated due to the Landau damping of Alfvén waves for $r = -0.7$ and $q = 9.5$

$$R^{\text{alf}} = 4.4 \times 10^{-16}\,\text{erg}\,\text{cm}^{-3}\,\text{s}^{-1}. \tag{34}$$

Comparison of Equations (33) and (34) shows a good qualitative agreement between observations and our theoretical model. In Figure 1, we display the radial evolution of $R^{\text{req.}}$ (from Equation (32)) and of $R_{(r,q)p}^{\text{alf}}$ (from Equation (26)) for different values of spectral indices r and q. We notice a good agreement, both qualitative as well as quantitative, between the two powers. Also the variations of

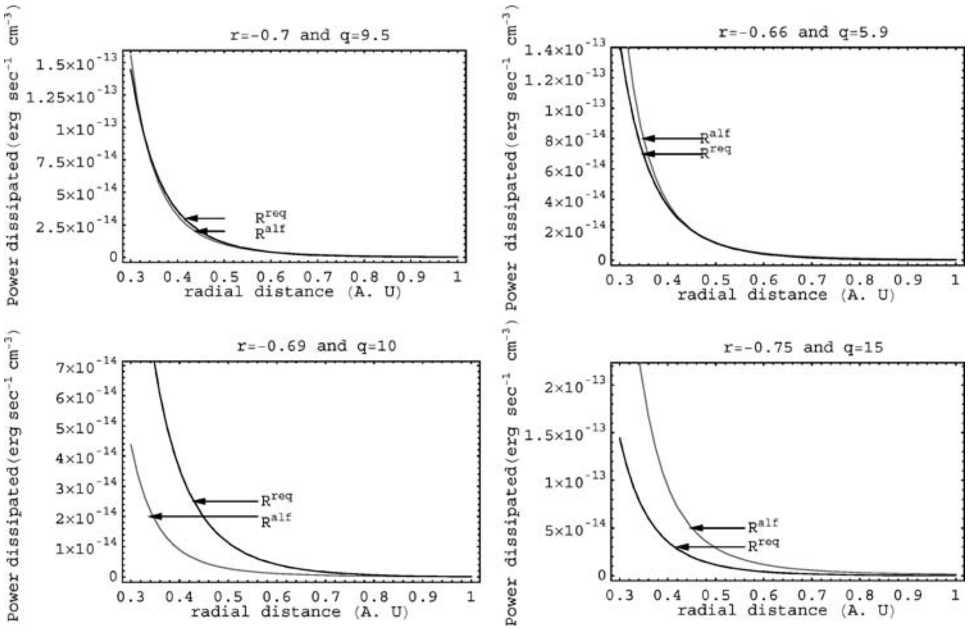

Figure 1. Radial evolution of power dissipated by Alfvén waves and the power which needs to be supplied to the solar wind are compared to one another, with different values of r and q.

r and q fit into a range of allowable values. This indicates that the use of (r, q) distribution function makes Landau damping a good candidate for explaining the power requirement of the solar wind.

4. Solar Wind Particle Distribution Function Fitted via the (r, q) Distribution Function

In this section we use the solar wind proton data from the *Cluster* spacecraft, which has already been used (Qureshi *et al.*, 2003) and get the best fit of the observed data. We note that in the above-mentioned reference, the distribution function was of the (r, q) type, but the proper normalization was not used. We note that the observed solar wind proton data is from the ion Composition and Distribution Function analyzer (CODIF), which is one of the sensors of the *Cluster* Ion Spectroscopy (CIS). Moreover, as the fluxes are high in the solar wind, the data is from the low sensitivity side of the sensor (CODIF). Each observed point of the distribution function is in the plasma reference frame and is characterized by a value of v (the velocity of the particles) and angle θ, where θ is measured with respect to an outward point vector taken along the direction of the ambient magnetic field. The details of transforming the data in the final form are given in Qureshi *et al.* (2003).

TABLE II

Observed Plasma Parameters in Slow Solar Wind at One AU

Day	a	b	c	q	r
Feb. 21, 2001	−17.4536	0.9	10^{-13}	11.219	−0.7000
Feb. 22, 2001	−15.3374	0.7	10^{-12}	05.651	−0.6600
Feb. 11, 2002	−15.0607	0.8	10^{-12}	05.967	−0.6665

In order to fit the data, we transform the (r, q) distribution function (Equation (1)) into a form which we find more convenient from the point of view of numerical modeling and which is given as

$$\log f_{(r,q)} = a - q \log \left[1 + \frac{1}{q-1} [cv^2 (b \cos^2 \theta + \sin^2 \theta)]^{(r+1)} \right] \tag{35}$$

where Equation (35) is in the form which is used to model the distribution function from the given data. And

$$a = \log \left(\frac{A_1}{\pi A_2^{3/2}} \right), \qquad b = \frac{v_{T\perp}^2}{v_{T\parallel}^2}, \quad \text{and} \quad c = \frac{1}{A_2^2 v_{T\perp}^2}$$

Here A_1 and A_2 are related to the gamma functions given in Equations (9) and (25), where b is an anisotropy ratio which is the same in all three intervals, and has a value $\approx$ one and a is the log of thermal speed. The parameters r and q obtained by fitting Equation (35) with the data, are listed in Table II.

Finally we see that as the solar wind approaches one AU, the value of $R_{(r,q)p}^{\text{alf}}$ and $R^{\text{req.}}$ come close to one another. In Figure 2 we have plotted the expression of these two powers given by Equations (26) and (32) respectively, and we have used the numerical values of the various parameters from Tables I and II.

5. Conclusions

In this work we have used kinetic theory to calculate the power dissipated through Landau damping of oblique propagating Alfvén waves by using the Generalized (r, q) distribution function. We have compared the observational results with the CGL (double adiabatic theory) and note that as the solar wind expands, the cooling is less than would be expected on the basis of the CGL theory, which means that some local heating mechanism is present to heat the solar wind plasma as it expands away from the Sun. From the dependence of the distribution function on the spectral indices r and q (Qureshi *et al.*, 2004), we see that for small values of q, high energy tails appear in the distribution function showing the presence of super thermal or energetic particles. As the value of r increases, the distribution function

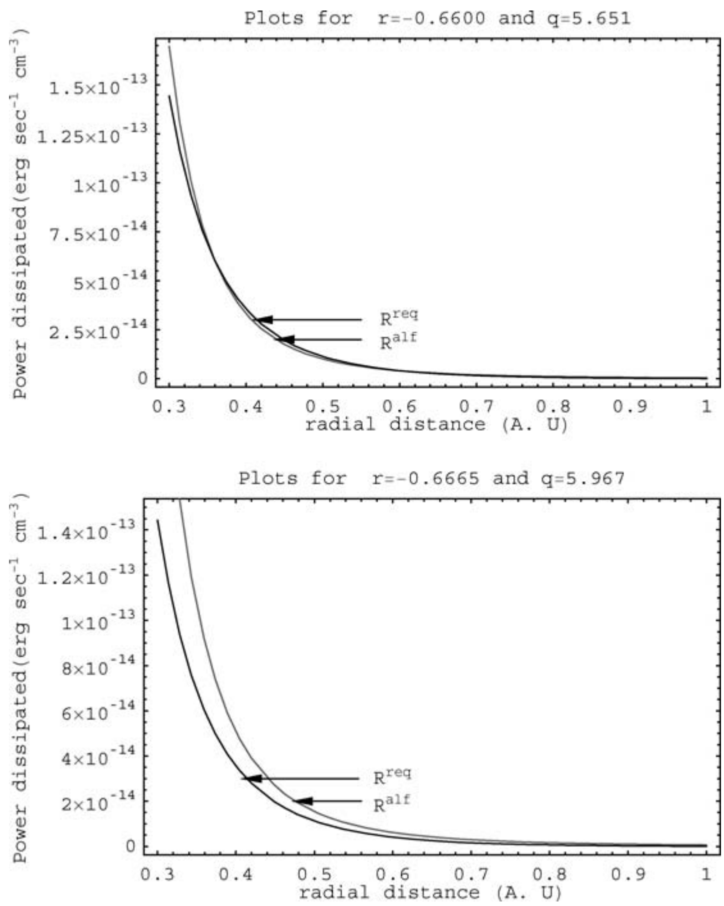

Figure 2. For the solar wind the data as given in Table I, for Feb. 22, 2001 and Feb. 11, 2002, which shows the good agreement with the Alfvénic power and the required solar wind power.

develops a flat top which is due to the anomalous damping of a non-equilibrium plasma. For negative value of r (as shown in Figure 3) the shape of the distribution function becomes peaked. As seen in the last section, we get a good quantitative fit with observed results for negative r. This is because higher values of damping are obtained for the obliquely propagating Alfvén waves.

The damping of the oblique propagating Alfvén waves in a non-Maxwellian plasma is proposed as a mechanism for heating the protons in the solar wind. We see that for different values of the spectral indices r and q occurring in the distribution function, the solar wind proton heating requirements in the parallel direction are met both qualitatively and quantitatively. These results are well represented in Figure 1 in which we have plotted the radial evolution of $R^{\text{req.}}$ and $R^{\text{alf}}_{(r,q)p}$ for different values of r and q. We notice a good agreement, both qualitative as well as quantitative, between the two powers. Also the variations of r and q fit into a range of allowable

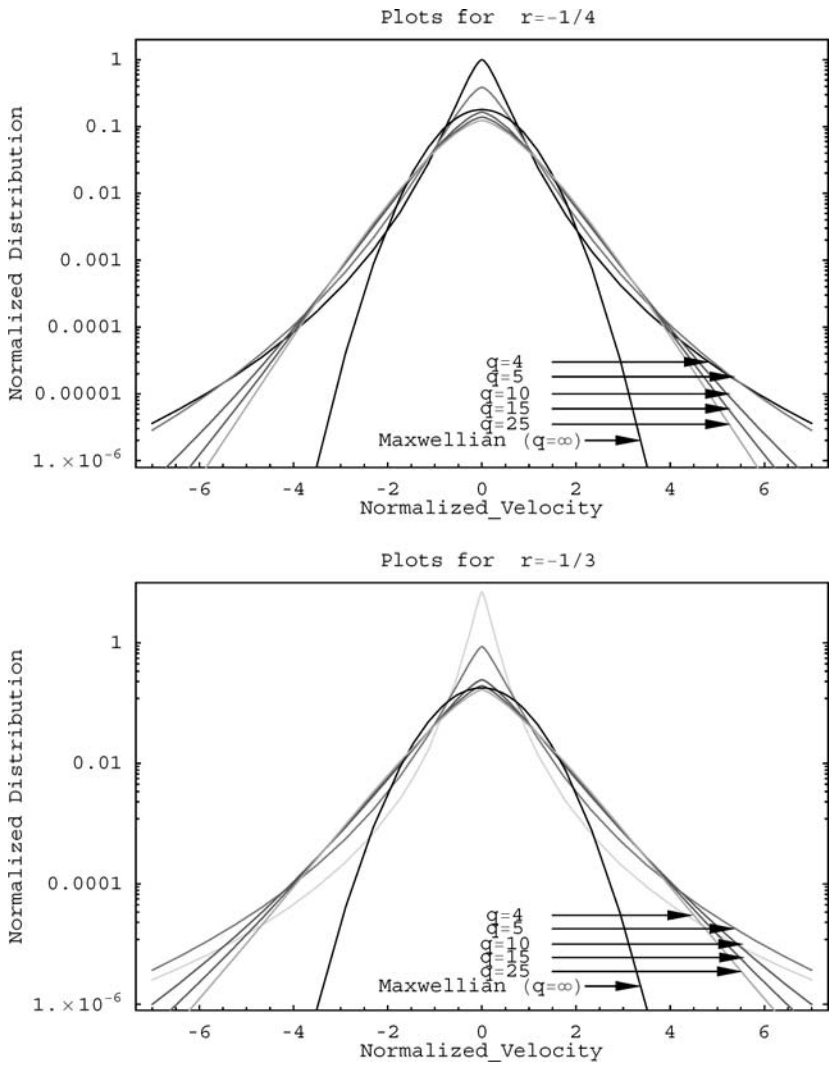

Figure 3. Generalized (r, q) distribution function for different negative values of the spectral index r.

values, *i.e.*, $(r = -0.66, q = 5.9)$, $(r = -0.69, q = 10)$ and $(r = -0.75, q = 15)$ in which the radial evolution of the power dissipated by Alfvén waves and the radial evolution of the power which needs to be supplied to the solar wind are compared to one another. These are in good agreement especially for the slow solar wind streams.

In the fourth section we use the solar wind proton data from *Cluster* spacecraft, which has already been used (Qureshi *et al.*, 2003) and get the best fit of the observed data.

More data analysis is required to get better fits with the proposed (r, q) distribution function. To make the approach more general, we need to consider that r and q are not constant, but are a functions of radial distance too. This may provide a more comprehensive picture of the solar wind.

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