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Perpendicularly propagating electromagnetic modes in a strongly magnetized hot plasma with non-Maxwellian distribution function

S. Zaheer
Salam Chair in Physics, Department of Physics, G. C. University Lahore, Lahore, Punjab 54000, Pakistan

G. Murtaza
Salam Chair in Physics, G. C. University Lahore, Lahore, Punjab 54000, Pakistan

H. A. Shah
Department of Physics, G. C. University, Lahore, Punjab 54000, Pakistan

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Electromagnetic modes (ordinary and extraordinary) for strongly magnetized plasma are studied and their damping factors γ_{or} and γ_{ex} are calculated using non-Maxwellian velocity distribution function. It is observed that for moderate values of the spectral indices r and q [used in (r, q) distribution functions], both the damping decrements show substantial change. As the value of the spectral index r increases for a fixed value of q, the damping increases for the O mode but decreases for the X mode. In the limiting case of $r=0$, $q\rightarrow\infty$, the damping factors reduce to the standard Maxwellian values. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2212830]

I. INTRODUCTION

Maxwellian distribution is the most popular plasma distribution and has become the standard assumption when the detailed distribution function is unknown. However, with more and more empirical data available, it is realized that in real plasma systems, the particle distributions may deviate significantly from Maxwellian distributions implying that when we use these distribution functions to explain or to predict different waves and instabilities, the ensuing results do not give good quantitative fits with observations, indicating that Maxwellian is not a realistic distribution under all circumstances.

Non-Maxwellian particle distribution functions are a common phenomenon in both laboratory and natural plasmas. In laboratory plasma, the highly energetic particles which are generated from alpha particles or fast ions from neutral beams can lead to non-Maxwellian particle distributions. But in natural (space) or fusion plasmas, such type of distributions are caused by wave-plasma interactions, either from natural turbulence or from externally driven sources. For sufficiently high powers, wave sources used for plasma heating and control can also lead to non-Maxwellian distributions.

Besides this empirical justification, Hasegawa demonstrated on the basis of classical analysis how the power distributions emerge as a natural consequence of the presence of superthermal radiation fields in plasmas.[1] Such a power-law functional dependence is an approximation to the more general distribution, like the Lorentzian or kappa distribution.[2] Measurements of electron energy spectra with spacecraft have been successfully modeled with kappa distributions.[3] Also plasma turbulence can interact with both thermal and superthermal particles and generate superthermal tails of the particle velocity distribution.[4] For plasmas in which both electrons and ions have kappa distribution as in Ref. 5, Summers and Thorne have calculated the modified plasma dispersion function theoretically and discussed general propagation properties of waves. More recently a justification for the power-law distributions in space plasma was provided by Ma and Summers as given in Ref. 6 in connection with study of electron acceleration by whistler mode and a kinetic theory was developed showing that kappa-like velocity space distributions present a particular thermodynamic equilibrium state, which is valid for a turbulent system.

In experiments, the electron and ion distribution may be isotropic, but not necessarily Maxwellian. It may be of power-law form, and well represented by a generalized Lorentzian (kappa) and generalized (r, q) distribution; the parameters kappa and (r, q) enable one to represent different power laws. The (r, q) distribution gives good fit for the magnetosheath and solar wind data available from cluster.[7] On the basis of observations and theory, it is expected that the space plasmas, e.g., the planetary magnetospheres and the solar wind, frequently contain particle components that exhibit high or superthermal energy tails with approximate power law distribution in velocity space. Such non-Maxwellian distribution with an over abundance of fast moving particles can be better fitted by non-Maxwellian distribution functions than by Maxwellian distribution function. So the non-Maxwellian distribution functions can lead to very different plasma wave characteristics. New modes are possible and wave absorption can increase or decrease with respect to Maxwellian. So the plasma stability can either be enhanced or degraded.

In this paper we adopt a more generalized non-Maxwellian distribution function as given Refs. 8 and 9 with two spectral indices r and q instead of one spectral index used in kappa distribution function, and has the form

$$f_{(r,q)} = \frac{3(q-1)^{-3/2(1+r)}\Gamma(q)}{4\pi\Psi_\perp^2\Psi_\parallel\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)} \times\left[1+\frac{1}{(q-1)}\left\{\left(\frac{v_\parallel}{\Psi_\parallel}\right)^2+\left(\frac{v_\perp}{\Psi_\perp}\right)^2\right\}^{r+1}\right]^{-q}, \quad (1)$$

where m is the mass of particles and $\Psi_\parallel$ and $\Psi_\perp$ are the respective thermal speeds with respect to the ambient magnetic field, and are calculated in the following manner:

We first note that

$$E_{\rm av} = \frac{\int_{-\infty}^{\infty}\frac{1}{2}mv_\parallel^2 f(\mathbf{v})d\mathbf{v}}{\int_{-\infty}^{\infty} f(\mathbf{v})d\mathbf{v}} = \frac{T_\parallel}{2}. \quad (2)$$

Now using the (r, q) distribution function and writing in cylindrical coordinates, we obtain

$$\int_{-\infty}^{\infty}\frac{1}{2}mv_\parallel^2 f(\mathbf{v})d\mathbf{v} = \frac{3m(q-1)^{-3/2(1+r)}\Gamma(q)}{4\Psi_\perp^2\Psi_\parallel\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)} \times\int_{-\infty}^{\infty} v_\parallel^2 dv_\parallel\int_0^\infty v_\perp \times\left[1+\frac{1}{(q-1)}\left\{\left(\frac{v_\parallel}{\Psi_\parallel}\right)^2 +\left(\frac{v_\perp}{\Psi_\perp}\right)^2\right\}^{r+1}\right]^{-q} dv_\perp.$$

After performing the integration, and putting the corresponding value in Eq. (2), we get

$$E_{\rm av} = \frac{m(q-1)^{1/r+1}\Psi_\parallel^2\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}{6\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)} = \frac{T_\parallel}{2}.$$

Solving the above relation for $\Psi_\parallel$, we get

$$\Psi_\parallel = \sqrt{\frac{T_\parallel}{m}} \times\sqrt{\frac{3(q-1)^{-1/1+r}\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}}.$$

Similarly for the perpendicular $\Psi_\perp$, we obtain

$$\Psi_\perp = \sqrt{\frac{T_\perp}{m}} \times\sqrt{\frac{3(q-1)^{-1/1+r}\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}}.$$

Here we note that $q>1$ and $q(1+r)>\frac{5}{2}$, are conditions which emerge from the normalization and definition of temperature for the distribution function given above; Γ is the gamma function and $f_{(r,\,q)}$ has been normalized so that $\int f_{(r,\,q)}d^3v=1$.

It may be noted here that we are using an empirical functional form Eq. (1) according to the one proposed on phenomenological grounds by Summers *et al.*[26,27] where the effective thermal speeds appear within the distribution already. This leads to a somewhat different functional form derived from nonextensive statistics,[10,11] which has the same form as the one derived in a different context by Hasegawa.[1] In Refs. 1, 10, and 11, it is seen that consistent with the evolution of suprathermal tails, the modified (effective) thermal spread Ψ increases for decreasing positive kappa values as compared to the Maxwellian thermal speed.

The (r, q) distribution is a generalized form of the kappa distribution function, and this evidently reduces to a kappa distribution function for $r=0$ and $q=\kappa+1$, and to a Maxwellian for $q\to\infty$ and $r=0$. We can see in the first panel in Fig. 1 of Ref. 8 how this distribution function reduces to a Lorentzian (kappa) distribution function when $r=0$ and approaches to Maxwellian when $r=0$ and $q\to\infty$. Different panels of Fig. 1 of Ref. 8 also show the dependence of the distribution function on r and q. If we fix the value of q and increase the value of r, then the contribution of high energy particles is reduced but the shoulders in the distribution function are broadened. On the other hand, if we fix the value of r and increase the value of q, then the contribution of high energy particles increase but the shoulders in the distribution function shrinks. Thus on varying the spectral index q, the high energy tails in the distribution function increases or decreases. Further it may be noted that the spectral index r is related to Davydov-Druvestyen distribution functions[12] and is due to a contribution which results from an anomalous dissipation wave-wave and wave-particle interactions. The Davydov-Druvestyen distribution functions have been used to model gas discharge plasmas and in general have the form $f\sim e^{-(v^2/v_{\rm th}^2)^r}$ or a combination of such exponential functions.[13–15] Thus we note here that the (r, q) distribution function that we propose to use in the present work is a generalization of both the kappa distribution function and the Davydov-Druvestyen distribution function. In fact we note that r can have negative values but $r>-1$ according to the conditions emerging from the integrations above. Thus small negative r values are permissible and these lead to a pointed top distribution function whereas for positive r values a flat top appears. These distributions are common both in natural and laboratory plasmas,[16] such as in galactic cosmic rays, solar flares, the magnetotail, shock waves, the earth plasma sheet, and the solar wind, etc.

Data analysis for solar wind plasmas[7,17] suggests that (r, q) distributions give better fits to data than the simpler kappa distributions. More recently such a distribution function has been used to account for the heating of solar wind protons due to the dissipation of Alfvén waves.[18] Good qualitative as well as quantitative fits were observed in this paper. We would like to emphasize here that the (r, q) distribution function is being used here on empirical grounds only. However we would like to point out that the kappa distribution functions were introduced phenomenologically, but later investigations by Hasegawa *et al.*[1] showed that these distribution functions emerged as a natural consequence from the diffusion equation when enhanced diffusion produces a power law distribution for the energy. Subsequent investigations (see Refs. 19–21) have attempted to establish these (kappa) distribution functions on thermodynamical grounds via the so-called nonextensive entropy representation.

In this paper, we adopt this (r, q) distribution function hoping it would give better data fits especially when there are shoulders in the profile of the distribution function along with high energy tail. Using this distribution function, we examine the damping factors for ordinary and extraordinary waves in a hot magnetized plasma. In the course of this derivation we obtain some new results and investigate some of their properties. The plots for ordinary and extraordinary modes are also presented for the imaginary part of the frequency.

II. DAMPING DECREMENTS FOR ELECTROMAGNETIC WAVES (*O* MODE AND *X* MODE) PROPAGATING ACROSS THE MAGNETIC FIELD

We follow the general formalism of kinetic theory to evaluate the damping decrement of the electromagnetic modes propagating in a hot magnetized plasma following standard texts end as in Refs. 22, 23 and 24. The special case of perpendicularly propagating waves in hot magnetized plasmas is considered and damping factors for the *O* mode and *X* mode for the (r, q) distribution function is calculated.

We consider a homogenous one-component electron plasma and assume propagation perpendicular to $B_{0,}$ that is, for which $k_\perp \gg k_\parallel$. The resulting matrix takes the form

$$\begin{bmatrix} R_{xx} & R_{xy} & 0 \\ R_{yx} & R_{yy} & 0 \\ 0 & 0 & R_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}. \tag{3}$$

The disturbance can thus be split into two mixed longitudinal and transverse "modes" and one pure transverse "mode."

Suppose we consider only the wave numbers such that

$$c^2k^2 \gg \omega_p^2. \tag{4}$$

The matrix elements R_{xy} and R_{yx} are of the order ω_p^2/c^2k^2 times either R_{xx}, R_{yy}, or R_{zz}. Thus the off diagonal elements can be neglected, the transverse and longitudinal waves completely decouple and we are left with two pure transverse waves with dispersion relations in the notation of Montgomery and Tidman,[22]

$$R_{zz} = 0 = -\omega^2 + c^2k^2 + 4\pi s\sigma_{zz} \quad (\text{ordinary wave, } E^{(1)} \parallel \text{ to } B_0),$$

$$R_{yy} = 0 = -\omega^2 + c^2k^2 + 4\pi s\sigma_{yy} \quad (\text{extraordinary wave, } E^{(1)} \perp \text{ to } B_0).$$

The quantities σ_{yy} and σ_{zz} are components of the conductivity tensor.

We shall approximately solve the above equations by noting $\omega = \omega_r + i\gamma$ where the real frequencies and the damping factors for the ordinary and extraordinary waves, respectively, are

$$\omega_r^2 - 4\pi\,\mathrm{Im}(\sigma_{zz})\omega_r - c^2k^2 = 0, \quad \gamma_{\mathrm{or}} = 2\pi\,\mathrm{Re}(\sigma_{zz}) \tag{5}$$

and

$$\omega_r^2 - 4\pi\,\mathrm{Im}(\sigma_{yy})\omega_r - c^2k^2 = 0, \quad \gamma_{\mathrm{ex}} = 2\pi\,\mathrm{Re}(\sigma_{yy}). \tag{6}$$

The relevant components of the conductivity tensor are[22]

$$(\sigma_{zz}, \sigma_{yy}) = -\frac{\omega_p^2}{2\Omega_e}\sum_n \int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp p_\perp \times \frac{\Omega e}{(-i\omega_r + ik_\parallel v_\parallel + in\Omega_e)}\left(p_\parallel J_n^2 \frac{\partial f_0}{\partial p_\parallel}, p_\perp (J_n')^2 \frac{\partial f_0}{\partial p_\perp}\right). \tag{7}$$

Here n represents the harmonic of the wave.

The parallel integration of Eq. (7) can be performed by following the standard procedure and separating the integrand into the principal part and a small imaginary part, i.e.,

$$-i\left[\frac{P}{(-\omega_r + k_\parallel v_\parallel + n\Omega_e)} + i\pi\delta(-\omega_r + k_\parallel v_\parallel + n\Omega_e)\right], \tag{8}$$

where P is the principal part.

Thus the real parts for ordinary and extraordinary waves are, respectively,

$$\omega_r^2 - 2\pi\omega_p^2 \sum_n \int_{-\infty}^{\infty} dp_\parallel \int_0^\infty dp_\perp p_\perp \times \frac{1}{(-\omega_r + k_\parallel v_\parallel + n\Omega_e)} \times \left(p_\parallel J_n^2 \frac{\partial f_0}{\partial p_\parallel},\ p_\perp (J_n')^2 \frac{\partial f_0}{\partial p_\perp}\right)\omega_r - c^2k^2 = 0 \tag{9}$$

and the corresponding damping decrements γ_{ex} and γ_{or} are

$$(\gamma_{\rm or},\gamma_{\rm ex}) = -\frac{\pi^2\omega_p^2}{\Omega_e}\sum_n \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} dp_\perp p_\perp \Omega_e \delta(\times \omega_r - k_\parallel v_\parallel - n\Omega_e)\left(p_\parallel J_n^2 \frac{\partial f_0}{\partial p_\parallel},\ p_\perp (J_n')^2 \frac{\partial f_0}{\partial p_\perp}\right). \quad (10)$$

Here J_n is the Bessel function and J_n' is its derivative with respect to its argument, and ω_p is the electron plasma frequency.

The argument of the Bessel function is $k_\perp p_\perp / m\Omega_e$, where

$$\Omega_e = \frac{eB_0}{m} \quad (11)$$

is the electron cyclotron frequency.

Assuming f_0 to be Maxwellian, the damping factors become

$$\gamma_{\rm or} = \frac{2\pi^{1/2}\omega_p^2}{m^5 v_{T_\parallel}^3 v_{T_\perp}^2}\sum_n \int_{-\infty}^{\infty} p_\parallel^2 \delta(\omega_r - k_\parallel v_\parallel - n\Omega_e) dp_\parallel \times \int_0^{\infty} p_\perp J_n^2 \exp\left[-\left(\frac{p_\parallel^2}{m^2 v_{T_\parallel}^2} + \frac{p_\perp^2}{m^2 v_{T_\perp}^2}\right)\right] dp_\perp = \pi^{1/2}\sum_n \wedge_n\left(\frac{k_\perp^2 v_{T_\perp}^2}{2\Omega_e^2}\right)\left(\frac{\omega_p^2(\omega_r - n\Omega_e)^2}{k_\parallel^3 v_{T_\parallel}^3}\right) \times \exp\left(-\frac{(\omega_r - n\Omega_e)^2}{k_\parallel^2 v_{T_\parallel}^2}\right) \quad (12)$$

where $\wedge_n(k_\perp^2 v_{T_\perp}^2/2\Omega_e^2) = I_n(k_\perp^2 v_{T_\perp}^2/2\Omega_e^2)\exp(-k_\perp^2 v_{T_\perp}^2/2\Omega_e^2)$, $I_n(k_\perp^2 v_{T_\perp}^2/2\Omega_e^2)$ being the modified Bessel function.

Similarly for the extraordinary mode,

$$\gamma_{\rm ex} = \pi^{1/2}\left(\frac{\omega_p^2}{k_\parallel v_{T_\parallel}}\right)\sum_n \frac{1}{2^{2n+5}}\exp\left(-\frac{(\omega_r - n\Omega_e)^2}{k_\parallel^2 v_{T_\parallel}^2}\right)\left(\begin{array}{c} 2^{3+2n}\exp\left(-\dfrac{k_\perp^2 v_{T_\perp}^2}{2\Omega_e^2}\right)\{nI_{n-1} - 2nI_n + (n+2)I_{n-1}\} \\ -\left(\dfrac{k_\perp^2}{m^2\Omega_e^2}\right)^n\left\{\dfrac{8(2n\Omega_e^2 F_1 - k_\perp^2 v_{T_\perp}^2 F_2)}{\Omega_e^2 n\Gamma(n)} + \dfrac{(k_\perp^4 v_{T_\perp}^4)F_3}{n(n+1)\Gamma(n)\Omega_e^4}\right\}\end{array}\right), \quad (13)$$

where F_1, F_2, and F_3 are hypergeometric functions

$$F_1 = {}_1F_1\left[n + \frac{1}{2}, 2n, -\frac{k_\perp^2 v_{T_\perp}^2}{\Omega_e^2}\right],$$

$$F_2 = {}_1F_1\left[n + \frac{3}{2}, 2 + 2n, -\frac{k_\perp^2 v_{T_\perp}^2}{\Omega_e^2}\right],$$

$$F_3 = {}_1F_1\left[n + \frac{5}{2}, 4 + 2n, -\frac{k_\perp^2 v_{T_\perp}^2}{\Omega_e^2}\right].$$

Equations (12) and (13) give the general expressions for the ordinary and extraordinary modes based on Maxwellian distribution function.

In the following we shall calculate ω_r and γ for the Maxwellian and generalized (r, q) distribution functions assuming the strong magnetic field, i.e., the argument of the Bessel function to be small i.e., $(k_\perp p_\perp / m\Omega_e) \ll 1$,[25] so that

$$J_n^2\left(\frac{k_\perp p_\perp}{m\Omega_e}\right) \rightarrow \frac{1}{\Gamma(n+1)}\left(\frac{k_\perp p_\perp}{2m\Omega_e}\right)^n. \quad (14)$$

We restrict our problem for two harmonics, i.e., $n=0, 1$.

Finally we shall compare the results with the Maxwellian in the same approximation of the strong magnetic field.

A. Ordinary mode

1. Maxwellian distribution function

Using the strong magnetic field limit Eq. (14), we obtain the real frequency and the damping rate of the ordinary mode:

a. $n=0$:

$$\omega_r^2 - 2\left(\frac{\omega_p^2\omega_r^2}{k_\parallel^2 v_{T\parallel}^2}\right) - c^2k^2 = 0 \quad (15)$$

and

$$\gamma_{\rm or} = (\pi^{1/2})\left(\frac{\omega_p^2 c^2 k^2}{k_\parallel^3 v_{T\parallel}^3}\right)\exp\left(-\frac{c^2k^2}{k_\parallel^2 v_{T\parallel}^2}\right). \quad (16)$$

b. $n=1$:

$$\omega_r^2 + 2\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^2\left(\frac{\omega_p^2(\omega_r - \Omega_e)}{k_\parallel^2 v_{T\parallel}^2}\right)\omega_r - c^2k^2 = 0 \quad (17)$$

and

$$\gamma_{\rm or} = (\pi^{1/2})\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^2\left(\frac{\omega_p^2(ck-\Omega_e)^2}{k_\parallel^3 v_{T\parallel}^3}\right)\times\exp\left(-\frac{(ck-\Omega_e)^2}{k_\parallel^2 v_{T\parallel}^2}\right). \quad (18)$$

2. (r, q) distribution function

In order to calculate the real part for (r, q) distribution, we differentiate Eq. (1) with respect to $p_\parallel$ and then use it in Eq. (9) to obtain

a. $n=0$:

$$\omega_r^2 - \left(\frac{3\omega_p^2}{m^5\Psi_\parallel^3\Psi_\perp^2}\right)\times\left(\frac{q(r+1)\Gamma(q)}{(q-1)^{3/2+2r+1}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\sum_n\int_{-\infty}^{\infty}\frac{p_\parallel^2}{(\omega_r+k_\parallel v_\parallel)}dp_\parallel\times\int_0^\infty p_\perp\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}+\frac{p_\perp^2}{m^2\Psi_\perp^2}\right)^r\times\left[1+\frac{1}{q-1}\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}+\frac{p_\perp^2}{m^2\Psi_\perp^2}\right)^{r+1}\right]^{-q-1}dp_\perp - c^2k^2 = 0. \quad (19)$$

Considering first the perpendicular integration only, we write

$$I = \int_0^\infty p_\perp\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}+\frac{p_\perp^2}{m^2\Psi_\perp^2}\right)^r\times\left[1+\frac{1}{q-1}\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}+\frac{p_\perp^2}{m^2\Psi_\perp^2}\right)^{r+1}\right]^{-q-1}dp_\perp. \quad (20)$$

Further we make a change of variables by taking $x=(p_\parallel^2/m^2\Psi_\parallel^2+p_\perp^2/m^2\Psi_\perp^2)$ so that Eq. (20) can be written as

$$I = \frac{(m^2\Psi_\perp^2)}{2}\int_{p_\parallel^2/m^2\Psi_\parallel^2}^{\infty} x^r\left(1+\frac{x^{r+1}}{q-1}\right)^{-q-1}dx. \quad (21)$$

We finally obtain for I

$$I = \frac{(m^2\Psi_\perp^2)}{2}\frac{q-1}{q(r+1)}\left(1+\frac{1}{q-1}\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}\right)^{r+1}\right)^{-q}dx. \quad (22)$$

Putting the value of I from Eq. (22) into the corresponding Eq. (19)

$$\omega_r^2 - \left(\frac{3\omega_p^2\omega_r}{m^5\Psi_\parallel^3\Psi_\perp^2}\right)\times\left(\frac{q(r+1)\Gamma(q)}{(q-1)^{3/2+2r+1}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\sum_n\int_{-\infty}^{\infty}\frac{p_\parallel^2}{(\omega_r+k_\parallel v_\parallel)}dp_\parallel\times\frac{(m^2\Psi_\perp^2)}{2}\frac{q-1}{q(r+1)}\times\left(1+\frac{1}{q-1}\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}\right)^{r+1}\right)^{-q}dx - c^2k^2 = 0.$$

After simplification and performing parallel integration, the real dispersion relation becomes

$$\omega_r^2 - \left(\frac{3\omega_p^2}{k_\parallel^2\Psi_\parallel^2}\right)\times\left(\frac{\Gamma\left(q-\frac{1}{2+2r}\right)\Gamma\left(1+\frac{1}{2+2r}\right)}{(q-1)^{1/r+1}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\omega_r^2 - c^2k^2 = 0. \quad (23)$$

Similarly the damping factor is

$$\gamma_{\rm or} = \left(\frac{3\pi\omega_p^2}{2m^5\Psi_\parallel^3\Psi_\perp^2}\right)\times\left(\frac{q(r+1)\Gamma(q)}{(q-1)^{3/2+2r+1}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\int_{-\infty}^{\infty}p_\parallel^2\delta(\omega_r-k_\parallel v_\parallel-n\Omega_e)dp_\parallel\times\int_0^\infty p_\perp\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}+\frac{p_\perp^2}{m^2\Psi_\perp^2}\right)^r\times\left[1+\frac{1}{q-1}\left(\frac{p_\parallel^2}{m^2\Psi_\parallel^2}+\frac{p_\perp^2}{m^2\Psi_\perp^2}\right)^{r+1}\right]^{-q-1}dp_\perp.$$

Performing the integrations by applying the same procedure as in the above case and after simplification, we get

$$\gamma_{\mathrm{or}} = \left(\frac{3\pi}{4}\right)\left(\frac{\omega_p^2 c^2 k^2}{k_\parallel^3 \Psi_\parallel^3}\right) \times \left(\frac{\Gamma(q)}{(q-1)^{3/2+2r}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right) \times \left[1+\frac{1}{q-1}\left(\frac{c^2k^2}{k_\parallel^2\Psi_\parallel^2}\right)^{r+1}\right]^{-q}. \tag{24}$$

b. $n=1$:

$$\omega_r^2 - 2\left(\frac{k_\perp \Psi_\perp}{\Omega_e}\right)^2\left(1+\frac{3\Gamma\left(q+\frac{r}{r+1}\right)}{q\Gamma\left(q-\frac{1}{r+1}\right)}\right) \times \left(\frac{\omega_p^2(\omega_r-\Omega_e)}{k_\parallel^2\Psi_\parallel^2}\right)\omega_r - c^2k^2 = 0 \tag{25}$$

and

$$\gamma_{\mathrm{or}} = \left(\frac{3\pi}{4}\right)\left(\frac{k_\perp\Psi_\perp}{\Omega_e}\right)^2\left(\frac{\omega_p^2}{k_\parallel\Psi_\parallel}\right)\left(\frac{q(r+1)\Gamma(q)}{(q-1)^{3/2+2r}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{2-q(r+1)} \times\left[-\frac{\left(1+q-1\left(\frac{c^2k^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right)^{-q}}{q(r+1)} + \frac{{}_2F_1\left(q+1,\frac{-1+q(r+1)}{r+1},\frac{q(r+1)+r}{r+1},(q-1)\left(-\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right)}{-1+q(r+1)}\right]. \tag{26}$$

For $r\to 0$ and $q\to\infty$, in Eqs. (23)–(26), we retrieve the results of the case when the Maxwellian distribution function is used in Eqs. (15)–(18), respectively. We note that when $r\to 0$ and $q\to\kappa+1$, we obtain results for the kappa distribution function used by Thorne *et al.*[5,26,27]

B. Extraordinary mode

In this section we derive expressions for the real frequency and damping rates of the X mode, first when the distribution function is a Maxwellian, and then with an $(r,\ q)$ distribution function.

1. Maxwellian distribution function

For a Maxwellian distribution function the derivation is straightforward and by proceeding in a manner similar to the previous case of a strong magnetic field, we obtain the following expression for the real and damping factors:

a. $n=0$:

$$\omega_r^2 - \frac{1}{2}\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^2\left(\frac{\omega_p^2\omega_r^2}{k_\parallel^2 v_{T\parallel}^2}\right) - c^2k^2 = 0 \tag{27}$$

and

$$\gamma_{\mathrm{ex}} = \left(\frac{\pi^{1/2}}{8}\right)\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^2\left(\frac{\omega_p^2}{k_\parallel v_{T\parallel}}\right)\exp\left(-\frac{c^2k^2}{k_\parallel^2 v_{T\parallel}^2}\right). \tag{28}$$

b. $n=1$:

$$\omega_r^2 - 4\left[1-\frac{1}{2}\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^2+\frac{3}{32}\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^4\right] \times\left(\frac{\omega_p^2(\omega_r-\Omega_e)}{k_\parallel^2 v_{T\parallel}^2}\right)\omega_r - c^2k^2 = 0 \tag{29}$$

and

$$\gamma_{\mathrm{ex}} = \left(\frac{\pi^{1/2}}{4}\right)\left[1-\frac{1}{2}\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^2+\frac{3}{32}\left(\frac{k_\perp v_{T\perp}}{\Omega_e}\right)^4\right] \times\left(\frac{\omega_p^2}{k_\parallel v_{T\parallel}}\right)\exp\left(-\frac{(ck-\Omega_e)^2}{k_\parallel^2 v_{T\parallel}^2}\right). \tag{30}$$

2. $(r,\ q)$ distribution function

Employing the distribution function given in Eq. (1) and proceeding as above we obtain the real and damping factors of the X mode as

a. $n=0$:

$$\omega_r^2 - \frac{1}{2}\left(\frac{k_\perp \Psi_\perp}{\Omega_e}\right)^2\left(\frac{\omega_p^2\omega_r^2}{k_\parallel^2 v_{T\parallel}^2}\right) - c^2k^2 = 0 \tag{31}$$

and

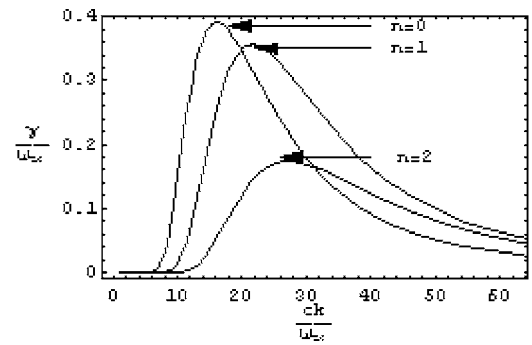

FIG. 1. Comparison of harmonics for $n=0$, 1, 2 for the ordinary mode.

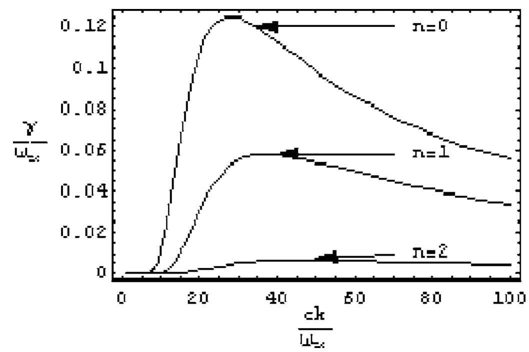

FIG. 2. Comparison of harmonics for $n=0$, 1, 2 for the extraordinary mode.

$$\gamma_{\mathrm{ex}}=\left(\frac{3\pi}{8}\right)\left(\frac{k_\perp\Psi_\perp}{\Omega_e}\right)^2\left(\frac{\omega_p^2}{k_\parallel\Psi_\parallel}\right)\times\left(\frac{q(r+1)\Gamma(q)}{(q-1)^{3/2+2r-q}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\left(\frac{c^2k^2}{k_\parallel^2\Psi_\parallel^2}\right)^{n-q(r+1)}\left\{-\frac{[1+(q-1)(c^2k^2/k_\parallel^2\Psi_\parallel^2)^{-r-1}]^{-q}}{q(r+1)}-\frac{F_4}{-1+q(r+1)}+\frac{F_5}{-2+q(r+1)}\right\} \tag{32}$$

b. $n=1$:

$$\omega_r^2-4\left\{1-\frac{1}{2}\left(\frac{k_\perp\Psi_\perp}{\Omega_e}\right)^2+\frac{3}{32}\left(\frac{k_\perp\Psi_\perp}{\Omega_e}\right)^4\times\left(\frac{\Gamma\left(\frac{5}{2+2r}\right)\Gamma\left(q-\frac{5}{2+2r}\right)}{(q-1)^{-1/r+1}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\right\}\times\left(\frac{\omega_p^2(\omega_r-\Omega_e)}{k_\parallel^2\Psi_\parallel^2}\right)\omega_r-c^2k^2=0 \tag{33}$$

and

$$\gamma_{\mathrm{or}}=\left(\frac{3\pi}{16}\right)\left(\frac{\omega_p^2}{k_\parallel\Psi_\parallel}\right)\left(\frac{q(r+1)\Gamma(q)}{(q-1)^{3/2+2r-q}\Gamma\left(q-\frac{3}{2+2r}\right)\Gamma\left(1+\frac{3}{2+2r}\right)}\right)\times\left[\left\{\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{1-q(r+1)}\left(-\frac{\left(1+q-1\left(\frac{c^2k^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right)^{-q}}{q(r+1)}+\frac{F_4}{-1+q(r+1)}\right)\right\}-\left(\frac{k_\perp\Psi_\perp}{2\Omega_e}\right)^2\left\{\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{2-q(r+1)}\left(\frac{\left(1+q-1\left(\frac{c^2k^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right)^{-q}}{q(r+1)}-\frac{2F_4}{-1+q(r+1)}+\frac{F_5}{-2+q(r+1)}\right)\right\}+\frac{1}{64}\left(\frac{k_\perp\Psi_\perp}{\Omega_e}\right)^4\left\{\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{3-q(r+1)}\times\left(-\frac{\left(1+q-1\left(\frac{c^2k^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right)^{-q}}{q(r+1)}+\frac{3F_4}{-1+q(r+1)}-\frac{3F_5}{-2+q(r+1)}+\frac{F_6}{-3+q(r+1)}\right)\right\}\right], \tag{34}$$

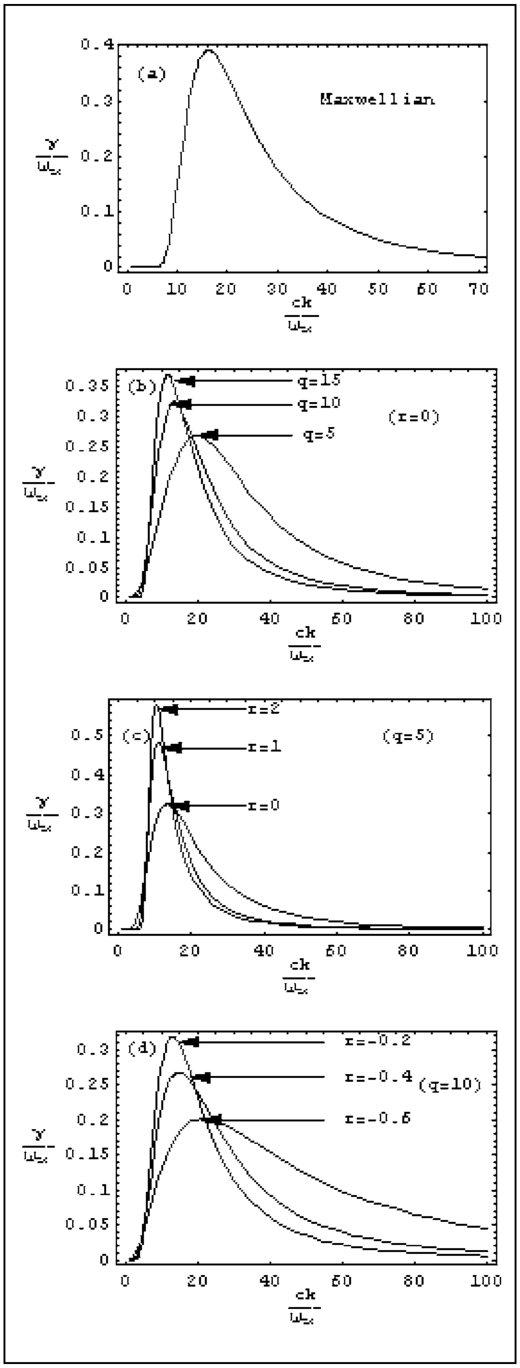

FIG. 3. Damping factors of the ordinary mode for $n=0$.

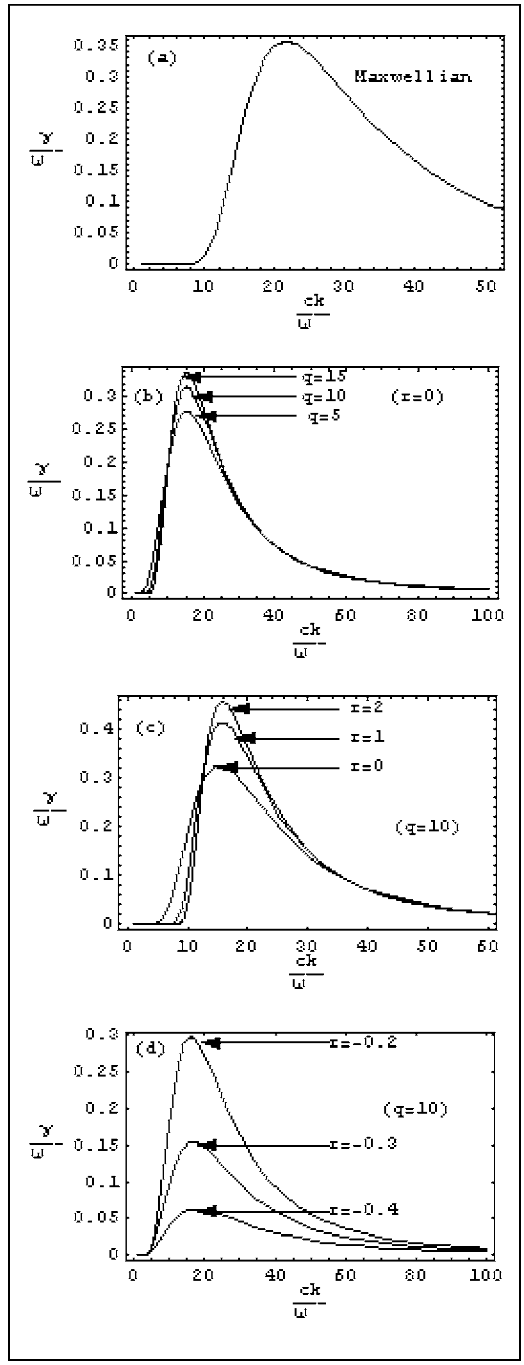

FIG. 4. Damping factors of the ordinary mode for $n=1$.

where F_4, F_5, and F_6 are the hypergeometric functions

$$F_4 = {}_2F_1\left(q+1, q-\frac{1}{r+1}, q+\frac{r}{r+1}, -(q-1)\right.$$
$$\left.\times\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right),$$

$$F_5 = {}_2F_1\left(q+1, q-\frac{2}{r+1}, 1+q-\frac{2}{r+1}, -(q-1)\right.$$
$$\left.\times\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right),$$

$$F_6 = {}_2F_1\left(q+1, q-\frac{3}{2+2r}, 1+q-\frac{3}{2+2r}, -(q-1)\right.$$
$$\left.\times\left(\frac{(ck-\Omega_e)^2}{k_\parallel^2\Psi_\parallel^2}\right)^{-r-1}\right).$$

Once again we note that when $r\to 0$ and $q\to\infty$, Eqs. (31)–(34) reduce to Eq. (27)–(30) which are the real and damping factors of the X mode with a Maxwellian distribution function for the background electrons.

We also note that when $r\to 0$ and $q\to\kappa+1$, we obtain results for the kappa distribution function used by Thorne *et al.*[5,26,27]

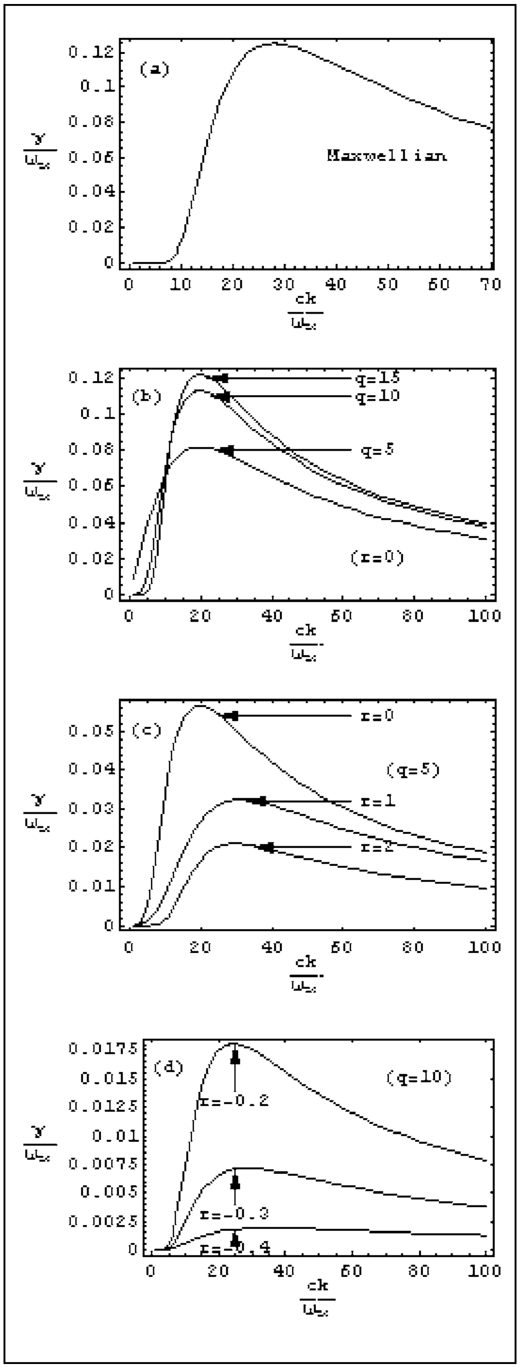

FIG. 5. Damping factors of the extraordinary mode for $n=0$.

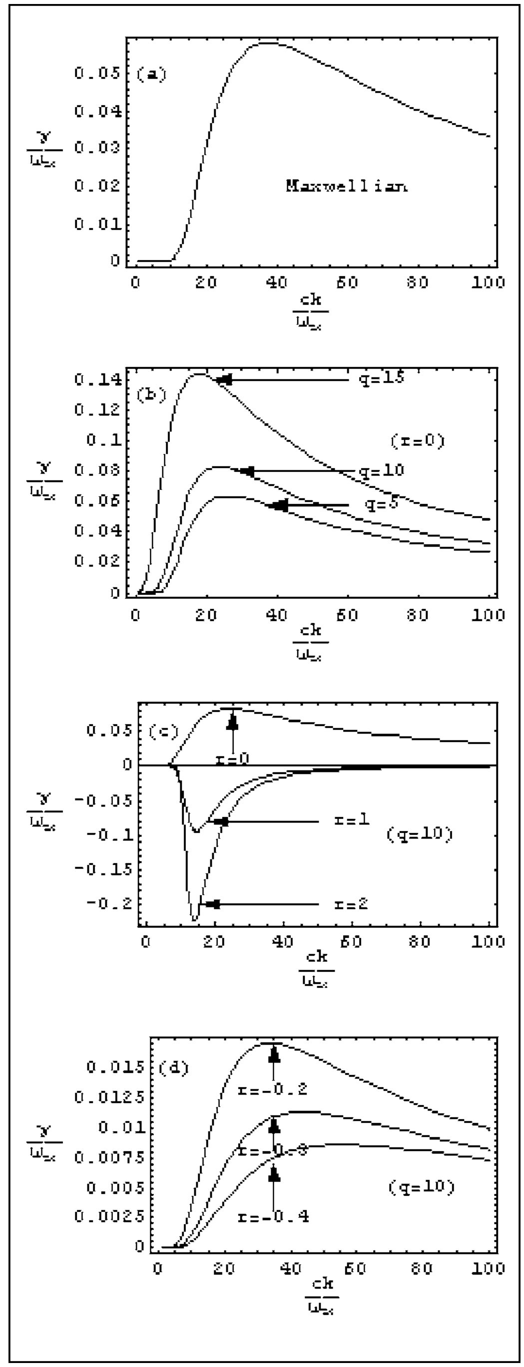

FIG. 6. Damping factors of the extraordinary mode for $n=1$.

III. RESULTS AND DISCUSSION

Propagating plasma waves contribute to the redistribution of energy and therefore to the transport of information. The various instabilities generate waves which are trapped in plasma. But there is radiation which can escape from the plasma. Such radiation is observed from natural plasmas, like the solar corona, magnetized stars, the large planets like Jupiter and also from magnetosphere. Radiation is emitted by accelerated electrons moving in the curved magnetic field. This synchrotron radiation requires highly energetic electrons, which exist only in radiation belts.[2]

However, there is also nonthermal radiation like the "Auroral Kilometric Radiation" observed in the magnetosphere during substorms. "Auroral Kilometric Radiation" is very impulsive and intense and cannot be generated as synchrotron-emission from trapped particles. Thus to get the instability in either the O mode or the X mode, a linear excitation mechanism is adopted as in Ref. 2. Mechanisms of this kind can exist only under extreme plasma conditions, because the escaping branches of both modes propagate at very high speeds and therefore need very long amplification lengths to reach reasonable amplitudes and require relativistic electrons to interact with.

As we know, the Maxwellian distribution describes the final state of thermodynamic equilibrium of a closed system which is perfectly relaxed. This situation may be difficult to realize in nature. Normally equilibria will result from competition between energy or momentum inflow and dissipation in a system. The plasma is very sensitive to both of these because of the high mobility of its free or quasifree particles. Hence, changes in the velocity distribution may occur, and the equilibrium may be disturbed and eventually evolve to another equilibrium which may not resemble the ideal Maxwellian distribution, but nevertheless represents an equilibrium. So, we need to employ distribution functions other than the Maxwellian.

We have calculated the damping decrements $\gamma_{\rm or}, \gamma_{\rm ex}$ for the so-called synchrotron modes for the two distribution functions. Expressions for the kappa distribution function can be simply obtained by taking $r=0$ and replacing q by $\kappa+1$. These modes have been calculated for high frequencies $\omega^2 \cong c^2k^2 \gg \omega_p^2$, under which circumstance, the plasma plays a very subordinate role in the propagation properties of the radiation; it is assumed to produce no refraction, and the quantities $\gamma_{\rm or}, \gamma_{\rm ex}$ are small.[22]

In this work, we have investigated the frequency spectrum of the radiations which consists of a series of the fundamental frequency Ω_e, when viewed in a direction perpendicular to B_0. These harmonics become Doppler shifted if the particle has a component of velocity $v_{T_\parallel}$ along B_0 and the radiation can be observed at some arbitrary angle to B_0.

We have derived, in computable form, expressions for electromagnetic waves propagating in a magnetized hot plasma modeled by the Maxwellian and (r, q) velocity distribution functions. To illustrate the behavior of these distributions, we present in Figs. 1–4, graphs of damping decrements of ordinary and extraordinary electromagnetic waves obtained for $n=0$ and $n=1$ by using Maxwellian and (r, q) distribution functions. For our numerical analysis we have used the following plasma parameters (see Ref. 28):

$\omega_{pe}=1.58\Omega_e$, $v_{T_\parallel}=0.094c$, $ck_\perp/\omega_{pe}=20$ eV, $T_\perp/T_\parallel=16$ to plot γ/ω_{pe} versus $ck_\parallel/\omega_{pe}$.

In Figs. 1 and 2, we show the comparison of harmonics for different integral values by using Maxwellian distribution functions for the above mentioned modes. From these figures, we note that as we increase the value of the integer n, the height of the harmonics is reduced and consequently the values of $\gamma_{\rm or}, \gamma_{\rm ex}$ for ordinary and extraordinary modes, respectively, decrease. The damping decrements in the case of the Maxwellian distribution function are depicted in graphs Figs. 3(a), 4(a), 5(a), and 6(a). The first two graphs are for the O mode and the latter two for the X mode for $n=0$ and $n=1$.

For the case of the generalized (r, q) distribution function we have carried out a similar analysis as above and have compared the results to those obtained for a Maxwellian. We note that the parameters r and q in general represent the flat part and the high energy tail of the distribution function, respectively. It is evident from the graphs shown in Figs. 3(b), 4(b), 5(b), and 6(b) that for $r=0$ and large values of q, the (r, q) distribution exhibits the same behavior as that of Maxwellian distribution. But, if we fix the value of q and increase the value of r, then the contribution of high energy particles is reduced and the shoulders become more prominent in the distribution function. When the damping rates of the various wave modes are compared we see from Figs. 3(c), 4(c), 5(c), and 6(c) that the damping increases for the O mode but decreases for the X mode, as the value of the spectral index r increases for a fixed value of q. For the negative values of r (with q fixed), this behavior can be seen from Figs. 3(d), 4(d), 5(d), and 6(d). This is as expected because as r increases the tail in the distribution function begins to vanish and the particles participating in the wave particle interaction become such that those giving energy to the wave become fewer than those receiving energy from the wave. (For different values of the spectral indices r and q the plots of the distribution function have been presented elsewhere, as in Ref. 8.)

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