AIP Physics of Plasmas

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A. Mushtaq and H. A. Shah

Citation: Phys. Plasmas **13**, 012303 (2006); doi: 10.1063/1.2154639
View online: http://dx.doi.org/10.1063/1.2154639
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v13/i1
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Study of non-Maxwellian trapped electrons by using generalized (r,q) distribution function and their effects on the dynamics of ion acoustic solitary wave

A. Mushtaq and H. A. Shah
Department of Physics, Government College University Lahore, P.O. Box 54000, Pakistan

(Received 11 July 2005; accepted 14 November 2005; published online 12 January 2006)

By using the generalized (r,q) distribution function, the effect of particle trapping on the linear and nonlinear evolution of an ion-acoustic wave in an electron-ion plasma has been discussed. The spectral indices q and r contribute to the high-energy tails and flatness on top of the distribution function respectively. The generalized Korteweg–de Vries equations with associated solitary wave solutions for different ranges of parameter r are derived by employing a reductive perturbation technique. It is shown that spectral indices r and q affect the trapping of electrons and subsequently the dynamics of the ion acoustic solitary wave significantly. © *2006 American Institute of Physics*.
[DOI: 10.1063/1.2154639]

I. INTRODUCTION

The ion-acoustic wave (IAW) in a two- or three-component plasma has long been studied, and both the linear[1–4] and nonlinear[5–8] dynamics associated with this wave have been investigated. It can propagate in both unmagnetized and magnetized plasma.

In the present paper we want to discuss the effect of particle trapping on the nonlinear evolution of an ion-acoustic wave, when the plasma background is represented by a non-Maxwellian distribution. It has been known from the early days of plasma physics that trapped particles exert a considerable influence on the nonlinear dynamics of plasma waves,[9] although in this pioneering work the trapping was considered directly by the longitudinal wave itself. However, trapping as a microscopic process was considered in 1967 by Gurevich[10] and involved the solution of the Vlasov equation together with the Maxwell equations. This microscopic formalism subsequently became the topic of investigation in Ref. 11; here, different aspects of wave-particle interactions were considered to show the effect of trapping while using Maxwellian distribution functions. The simulation of one-dimensional Vlasov-Poisson equations has also confirmed the existence of trapped particles[12] as have experiments carried out in the last two decades.[13]

Most studies of the waves in plasmas are based on the assumption of a Maxwellian distribution function. The Maxwellian distribution is the most popular plasma distribution and has become the default assumption when the detailed distribution function is unknown. However, with more and more empirical data becoming available, it is realized that in real plasma systems, the particle distributions deviate significantly from Maxwellian distributions,[14–16] implying that when we use theoretical models using these distribution functions to explain or to predict different waves and instabilities, the ensuing results do not give good quantitative fits with observations.[17–19] This means that a Maxwellian is not a realistic distribution under all circumstances and other distributions such as kappa,[20] or generalized (r,q) distribution (which is used in the present work) fit better for observed results.

Non-Maxwellian distributions are common both in natural and laboratory plasmas. Examples are found in galactic cosmic ray distributions, solar flares, the magneto tail, near-plasma shock waves,[21] the Earth's plasma sheet,[22] the solar wind,[23] etc. These distributions may have a significant high-energy tail arising from some external acceleration mechanism; i.e., a hard spectrum. These observed distribution functions contain a plentiful supply of superthermal particles that exhibit high- or superthermal-energy tails with approximate power-law distributions in velocity space, e.g., $4\pi v^2 f(v)dv \propto v^{-\alpha}dv$ for $|v| > v_{\text{th}}$ and can often be modeled by other distributions such as generalized Lorentzian (kappa) distributions or generalized (r,q) distribution and have been found to be more useful as compared to the Maxwellian distribution functions.[24] The use of the family of kappa distributions to model the observed nonthermal features of electron and ion structures was frequently criticized due to lack of its formal derivation. A classical analysis addressing this problem was performed by Hasegawa *et al.*,[25] who demonstrated how the kappa distributions emerge as a natural consequence of the presence of superthermal radiation fields in plasmas.

Recently, a modified version of the above mentioned generalized distribution functions has been used to model and theoretically investigate some electrostatic and electromagnetic modes (Ref. 18) by a non-Maxwellian distribution, which was referred to as the (r,q) distribution function, and its one-dimensional version for electrons has the following form:

$$f_{(r)}^{q} = \frac{n_0}{v_T} a \left[1 + \frac{1}{q-1} \left(\frac{v^2}{b^2 v_T^2} \right)^{r+1} \right]^{-q}, \qquad (1)$$

where $v_T^2 = T_e/m_e$ is the thermal speed of electron and a and b are dimensionless functions of r and q as follows:

$$a = \frac{1}{4\pi b^3}\frac{3(q-1)^{-3/2(1+r)}\Gamma(q)}{\left[\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(1+\frac{3}{2(1+r)}\right)\right]^{3/2}}, \quad (2)$$

$$b = \sqrt{3}(q-1)^{-1/2(1+r)}\left(\frac{\Gamma\left(q-\frac{3}{2(1+r)}\right)\Gamma\left(\frac{3}{2(1+r)}\right)}{\Gamma\left(\frac{5}{2(1+r)}\right)\Gamma\left(q-\frac{5}{2(1+r)}\right)}\right)^{1/2}. \quad (3)$$

It is noted that the restrictions on the indices r and q result from the normalization of the distribution function and are $r > -1$ and $q(r+1) > 3/2$ for real values of r and q. This distribution is a generalized form of the kappa distribution function with index k of Refs. 9–15 and 24, and reduces to a kappa distribution function if $r=0$ and $q \to \kappa+1$, and to a one-dimensional Maxwellian when $q \to \infty$ and $r=0$. We note here that the spectral index q contributes to the high-energy tails in the distribution functions and r gives rise to the flat or sharp top of the distribution function; it is the same spectral index that appears in the Davydov-Druvestyen distribution functions,[28] which have been used to model low-pressure electric discharge plasmas, and is responsible for the nonlinear (anomalous) damping that may occur in plasmas in quasithermodynamic equilibrium. Data analysis for solar wind plasmas[15] suggest that such (r,q) distributions give better fits to data than the simpler kappa distributions.

In this work we want to discuss the effect of particle trapping, when the plasma background is represented by a (r,q) distribution, on the nonlinear dynamics of ion-acoustic waves via Korteweg–de Vries (KdV)-like equation. We feel that using a more realistic distribution function will provide a better insight into the dynamics and effects of trapping on various wave particle interactions. The organization of the paper is as follows.

In Sec. II the basic set of nonlinear equations and dispersion relation are presented and the KdV equation is obtained by using the reductive perturbation technique along with its stationary solution. In Sec. III numerical results are presented along with a discussion of these results. Section IV gives a conclusion of our work.

II. BASIC EQUATIONS AND FORMULATION

We consider a one-dimensional, unmagnetized, and collisionless two-component plasma consisting of massive cold ions and adiabatically trapped electrons. The trapped electrons are assumed to follow the non-Maxwellian distribution function and propagation is considered in the x direction. Thus, at equilibrium we have $n_{eo}=n_{io}=n_o$. We neglect any transport properties such as viscosity, heat conduction, etc. Under these conditions the nonlinear dynamics of the low-frequency IAW in such a plasma are governed by the following set of equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \quad (4)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial}{\partial x} v_i = -\frac{\partial}{\partial x}\varphi, \quad (5)$$

$$\frac{\partial^2 \varphi}{\partial x^2} = n_e - n_i, \quad (6)$$

where n_e and n_i are the electron and ion number densities, respectively, normalized by equilibrium density n_o, v_i is the ion-fluid velocity normalized by the ion-acoustic speed $c_{si} = \sqrt{T_e/m_i}$, and φ is the electrostatic wave potential due to charge separation normalized by T_e/e, where T_e is the electron temperature, m_i is the mass of the ion, and is e is the magnitude of electron charge. The time and space variables are normalized by the ion-plasma period $\omega_{pi}^{-1} = \sqrt{m_i/4\pi n_o e^2}$ and Debye length $\lambda_d = \sqrt{T_e/4\pi n_o e^2}$, respectively.

To model the trapped electrons, we consider that the electrons in general follow the (r,q) distribution given by Eq. (1). The expression for the number density of adiabatically trapped electrons with Maxwellian distribution are given in Ref. 14, which for the case of $\varphi \ll 1$ is

$$n_e = 1 + \varphi - \frac{4}{3\sqrt{\pi}}\varphi^{3/2}. \quad (7)$$

Note that, according to Eq. (7), the trapping of electrons can change the ordering, giving rise to a half-integer rather than an integer power expansion (this is a particular characteristic of trapping in a Maxwellian plasma) of the number density in terms of weak potential energy φ. Using this type of expression [Eq. (7)], the KdV-type of nonlinear equation has been derived for electrostatic waves for electron-ion (e-i) plasma.[29]

We consider the case when free and trapped electrons follow the distribution function $f_{(r)}^{q}$. Thus, by following the procedure developed in Ref. 14 and in Refs. 26 and 27, the expression for the electron number density ($n_e = n_{(r)}^{q}$) in terms of potential energy φ for the (r,q) distribution function is

$$n_{(r)}^{q} = \frac{2a}{b^3}\frac{1}{\Gamma(q-1)}(q-1)^{-(2r+5)/(2r+2)} - \frac{1}{2}\varphi^2\Gamma\left(1-\frac{3}{2(1+r)}\right)\Gamma\left(q+\frac{3}{2(1+r)}\right) + b^4(q-1)^{2/(1+r)}$$
$$\times\Gamma\left(q-\frac{1}{2(1+r)}\right)\Gamma\left(1+\frac{1}{2(1+r)}\right) + \varphi b^2(q-1)^{1/(1+r)}\Gamma\left(q+\frac{1}{2(1+r)}\right)\Gamma\left(1-\frac{1}{2(1+r)}\right) - 4\frac{a}{b}\frac{q}{(1-q)^{1/2(1+r)}}\varphi$$
$$- B\left[\frac{1}{1-q}\left(\frac{2}{b^2}\varphi\right)^{1+r}, 1-\frac{1}{2(1+r)}, -q\right] + \left(\frac{1}{1-q}\right)^{1/(1+r)}\frac{\varphi}{b^2} rB\left[\frac{1}{1-q}\left(\frac{2}{b^2}\varphi\right)^{1+r}, 1-\frac{3}{2(1+r)}, -q\right]$$

TABLE I. The coefficients α_r^q, β_r^q, and γ_r^q for different values of r (i.e., 0, 1/4, 1/3, …, 2.

r	α_r^q	β_r^q	γ_r^q
0	$\frac{1-2q}{3-2q}$	$\frac{4}{3\sqrt{\pi}}\frac{\Gamma(q+1)\Gamma^{3/2}(q-3/2)}{\Gamma^{5/2}(q-1/2)}$	—
1/4	$\frac{1}{2}\frac{\Gamma(1/5)\Gamma(3/5)\Gamma(q-6/5)\Gamma(q+2/5)}{\Gamma^2(2/5)\Gamma^2(q-2/5)}$	$\frac{289}{168\times 2^{1/4}\times 5^{3/4}}\frac{\Gamma(q+1)\Gamma^{7/4}(1/5)\Gamma^{7/4}(q-6/5)}{\Gamma^{11/4}(2/5)\Gamma^{11/4}(q-2/5)}$	—
1/3	$\frac{1}{3}\frac{\Gamma(1/8)\Gamma(5/8)\Gamma(q-9/8)\Gamma(q+3/8)}{\Gamma^2(3/8)\Gamma^2(q-3/8)}$	$\frac{197}{495\times 4^{1/3}}\frac{\Gamma(q+1)\Gamma^{11/6}(1/8)\Gamma^{11/6}(q-9/8)}{\Gamma^{17/6}(2/5)\Gamma^{17/6}(q-2/5)}$	—
−1/3	$\frac{1}{3}\frac{\Gamma(1/4)\Gamma(13/4)\Gamma(q-9/4)\Gamma(q+3/4)}{\Gamma^2(7/4)\Gamma^2(q-3/4)}$	$\frac{(316)\times(2/3)^{1/6}}{315}\frac{\Gamma(q+1)\Gamma^{7/6}(13/4)\Gamma^{7/6}(q-9/4)}{\Gamma^{13/6}(7/4)\Gamma^{13/6}(q-3/4}$	—
−1/5	$\frac{1}{3}\frac{\Gamma(3/8)\Gamma(23/8)\Gamma(q-15/8)\Gamma(q+5/8)}{\Gamma^2(13/8)\Gamma^2(q-5/8)}$	$\frac{2434\times 2^{3/10}}{4095\times 3^{3/10}}\frac{\Gamma(q+1)\Gamma^{13/10}(23/8)\Gamma^{13/10}(q-15/8)}{\Gamma^{23/10}(13/8)\Gamma^{23/10}(q-5/8)}$	—
−1/6	$\frac{1}{3}\frac{\Gamma(3/7)\Gamma(19/7)\Gamma(q-12/7)\Gamma(q+4/7)}{\Gamma^2(11/7)\Gamma^2(q-4/7)}$	$\frac{4009}{3960\times 2^{5/8}\times 3^{3/8}}\frac{\Gamma(q+1)\Gamma^{11/8}(19/7)\Gamma^{11/8}(q-12/7)}{\Gamma^{19/8}(11/7)\Gamma^{19/8}(q-4/7)}$	—
−1/8	$\frac{1}{3}\frac{\Gamma(1/5)\Gamma(17/5)\Gamma(q-12/5)\Gamma(q+4/5)}{\Gamma^2(9/5)\Gamma^2(q-4/5)}$	$\frac{4009}{1512\times 2^{7/8}\times 3^{1/8}}\frac{\Gamma(q+1)\Gamma^{9/8}(17/5)\Gamma^{9/8}(q-12/5)}{\Gamma^{17/8}(9/5)\Gamma^{17/8}(q-4/5)}$	—
1	$\frac{1}{3}\frac{\Gamma^2(3/4)\Gamma(q-3/4)\Gamma(q+1/4)}{\Gamma^2(1/4)\Gamma^2(q-1/4)}$	—	$\frac{3\times(4q-1)\Gamma^2(q-3/4)}{5\times\Gamma^2(q-1/4)}$
2	$6\sqrt{\pi}\frac{\Gamma^2(5/6)\Gamma(q-1/2)\Gamma(q+1/6)}{\Gamma^2(1/6)\Gamma^2(q-1/6)}$	—	$\frac{\Gamma(q+1/2)\Gamma^2(q-1/2)}{10\times\Gamma^3(q-1/6)}$

$$+(1+q)(1+r)B\left[\frac{1}{1-q}\left(\frac{2}{b^2}\varphi\right)^{1+r},2-\frac{3}{2(1+r)},-(1+q)\right]-4\sqrt{2}a\varphi^{1/2}{}_2F_1 \times\left[q,\frac{1}{2(1+r)},1+\frac{1}{2(1+r)};\frac{1}{1-q}\left(\frac{2}{b^2}\varphi\right)^{1+r}\right]+2\sqrt{2}a\varphi^{1/2}, \tag{8}$$

where Γ, B, and ${}_2F_1$ are the, gamma, beta, and hypergeometric functions, respectively.

Expanding Eq. (8) for small φ and for $-1/2<r<1/2$, the following general form is obtained:

$$n_{(r)}^q = 1+\alpha_r^q\varphi-\beta_r^q\varphi^{r+3/2}, \tag{9}$$

where the coefficients α_r^q and β_r^q are positive functions of q and r and are listed for different values of r in Table I.

We now use the reductive perturbation technique and construct a weakly nonlinear theory for one-dimensional small but finite amplitude ion-acoustic waves. The stretched variables for this model in terms of spectral index r are

$$\xi=\epsilon^{(2r+1)/4}(x-v_o t),\qquad \tau=\epsilon^{3[(2r+1)/4]}t, \tag{10}$$

where ϵ is a small $(0<\epsilon\leqslant 1)$ expansion parameter characterizing the strength of the nonlinearity and v_o is the is the phase velocity of the ion-acoustic wave normalized by c_s the ion acoustic speed. The ion number density, velocity, and potential are expanded in the following manner:

$$n_i = 1 + \epsilon n_1 + \epsilon^{r+3/2} n_2 + \cdots,$$
$$v_i = \epsilon u_1 + \epsilon^{r+3/2} u_2 + \cdots, \quad (11)$$
$$\varphi = \epsilon \varphi_1 + \epsilon^{r+3/2} \varphi_2 + \cdots.$$

Substituting Eqs. (10) and (11) into Eqs. (4)–(6) and Eq. (9), and collecting terms of lowest order in ϵ (i.e., $\sim\epsilon$ and $\epsilon^{r/2+5/4}$,) we obtain

$$-v_0 \frac{\partial u_1}{\partial \xi} + \frac{\partial \varphi_1}{\partial \xi} = 0,$$
$$-v_0 \frac{\partial n_1}{\partial \xi} + \frac{\partial u_1}{\partial \xi} = 0, \quad (12)$$
$$n_1 - \alpha_r^q \varphi_1 = 0.$$

Using the boundary conditions $n_1 \to 0$, $u_1 \to 0$, and $\varphi_1 \to 0$ as $|\xi| \to \pm\infty$, from Eq. (12), we have

$$n_1 = \alpha_r^q \varphi_1,$$
$$u_1 = v_0 \alpha_r^q \varphi_1, \quad (13)$$

which in turn can be solved to give us the dispersion relation

$$v_0 = \frac{1}{\sqrt{\alpha_r^q}}. \quad (14)$$

This is the dispersion relation for IAW when the (r,q) distribution function is used. If $r=0$ and $q \to \infty$, then using the Table I, we retrieve the Maxwellian result $v_0=1$; i.e., $\omega/k = c_{si}$ of Ref. 29.

In the next order ($\sim\epsilon^{r+3/2}$) combining Eq. (6) and Eq. (9), we obtain

$$\frac{\partial^2 \varphi_1}{\partial \xi^2} = \alpha_r^q \varphi_2 - n_2 - \beta_r^q \varphi_1^{r+3/2}, \quad (15)$$

and from terms of order $\epsilon^{3r/2+7/4}$ from Eqs. (4) and (5) we obtain the following set of equations:

$$-v_0 \frac{\partial u_2}{\partial \xi} + \frac{\partial \varphi_2}{\partial \xi} = -\frac{\partial u_1}{\partial \tau}, \quad (16)$$

$$\frac{\partial u_2}{\partial \xi} - v_0 \frac{\partial n_2}{\partial \xi} = -\frac{\partial n_1}{\partial \tau}. \quad (17)$$

By eliminating quantities with subscript "2" from Eqs. (15)–(17) by means of Eqs. (13) and (14), we obtain

$$\frac{\partial \varphi_1}{\partial \tau} + A \varphi_1^{r+1/2} \frac{\partial \varphi_1}{\partial \xi} + B \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0, \quad (18)$$

which is a generalized KdV equation. Here, the coefficients A and B are given by

$$A = \frac{(r+3/2)\beta_r^q}{2(\alpha_r^q)^{3/2}},$$
$$B = \frac{1}{2}(\alpha_r^q)^{-3/2}. \quad (19)$$

If in Eq. (18), $r=0$, we then get the KdV equation of Ref. 29 in a Maxwellian plasma. The steady state solution of the generalized KdV equation [Eq. (18)] is obtained by shifting to a co-moving frame of reference $\eta=(\xi-u_0\tau)/\Delta$, where u_0 is a normalized constant velocity, and finally the steady state solution of Eq. (18) can be written as

$$\varphi_1 = \varphi_m \operatorname{sech}^{4/(2r+1)}[(\xi - u_0\tau)/\Delta], \quad (20)$$

where the amplitude φ_m and the width Δ (normalized by λ_D) in generic form are given by

$$\varphi_m = \left(\frac{u_0\left(1 + \frac{4}{2r+1}\right)\left(2 + \frac{4}{2r+1}\right)(2r+1)^2}{16A} \right)^{2/(2r+1)},$$
$$\Delta = \sqrt{\frac{16B}{u_0(2r+1)^2}}. \quad (21)$$

In Eq. (20), if we take $r=0$ and the limit $q \to \infty$, we retrieve Eq. (16) of Ref. 30. We note here that the above results are valid in the range $-1/2 < r < 1/2$.

We now expand Eq. (8) for small φ again, but this time for range $r > 1/2$, then we have the following generic form for the electron number density ($n_e = n_{(r)}^q$) in terms of weak potential energy φ:

$$n_{(r)}^q = 1 + \alpha_r^q \varphi - \gamma_r^q \varphi^2, \quad (22)$$

where α_r^q and γ_r^q are positive functions of q and r, which are listed in Table I for $r=1$ and 2. For this case, we have the stretched variables $\xi=\epsilon^{1/2}(x-v_o t)$ and $\tau=\epsilon^{3/2}t$, and the quantities are expanded in terms of ϵ in the following manner:

$$n_i = 1 + \epsilon n_1 + \epsilon^2 n_2 + \cdots,$$
$$v_i = \epsilon u_1 + \epsilon^2 u_2 + \cdots, \quad (23)$$
$$\varphi = \epsilon \varphi_1 + \epsilon^2 \varphi_2 + \cdots.$$

Then, using the same procedure as was followed for the case $-1/2 < r < 1/2$, the nonlinear evolution equation for $r > 1/2$ is obtained as

$$\frac{\partial \varphi_1}{\partial \tau} + C \varphi_1 \frac{\partial \varphi_1}{\partial \xi} + D \frac{\partial^3 \varphi_1}{\partial \xi^3} = 0, \quad (24)$$

which is the standard KdV equation, and here the coefficients C and D are given by

$$C = \frac{\gamma_r^q + \frac{3}{2}(\alpha_r^q)^2}{(\alpha_r^q)^{3/2}},$$
$$D = \frac{1}{2}(\alpha_r^q)^{-3/2}. \quad (25)$$

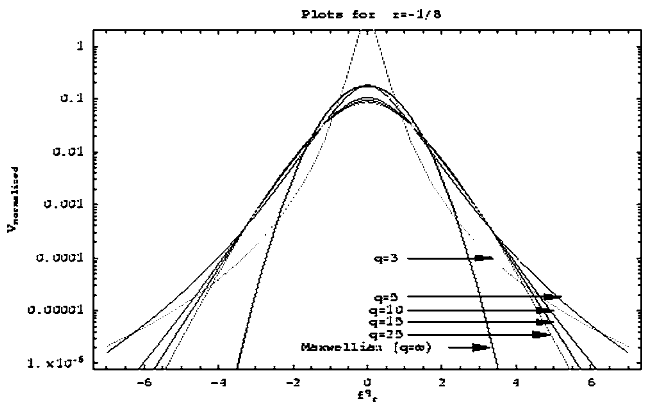

FIG. 1. Comparison of (r,q) generalized distribution function for different values of spectral index q and for negative value of r with Maxwellian distribution function.

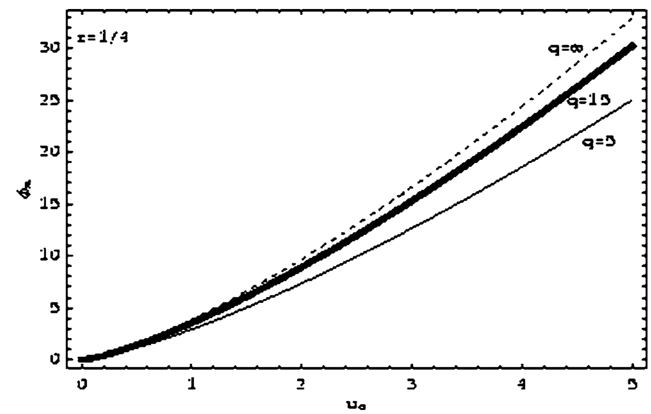

FIG. 3. The effect of spectral index q on the amplitude of solitary wave solution potential φ_m as a function of velocity u_0.

The steady state solution of Eq. (24) can be written as

$$\varphi_1 = \Psi_m \operatorname{sech}^2[(\xi - u_0\tau)/\delta], \tag{26}$$

where the normalized amplitude Ψ_m and the width δ are given by

$$\Psi_m = \left(\frac{3u_0(\alpha_r^q)^{3/2}}{\gamma_r^q + \frac{3}{2}(\alpha_r^q)^2} \right), \tag{27}$$

$$\delta = \sqrt{\frac{2(\alpha_r^q)^{-3/2}}{u_0}}. \tag{28}$$

We note here that although Eqs. (24)–(28) do not include trapped particle effects, but they do, however, contain the spectral indices of the (r,q) distribution function.

III. RESULTS AND DISCUSSION

We have derived in computable form the expressions for solitary ion-acoustic waves propagating in unmagnetized e-i plasmas with electron trapping with a generalized (r,q) particle distribution. To illustrate the behavior of distribution, we present in Fig. 1, the graphs of this distribution function against the normalized velocity [Eq. (1)]. In these graphs it is shown that r (the spectral index) affects the shoulder and q affects the tail of the distribution function. It is also shown that for negative values of r, the function becomes more spiky as compared to the positive values of r where shoulders appear as in Ref. 25. The dependence of solitary wave solution amplitude on different values of r against η for a fixed value of $q=5$ is shown in Fig. 2. It is observed that for decreasing values of r, the amplitude of solitary wave solution increases. Figure 3 shows the dependence of solitary wave solution amplitude on q as a function of velocity u_o. It shows that the amplitude increases with increased values of q. Figures 4 and 5 are the graphical results of the width of solitary wave solution Δ against the solitary wave solution velocity u_o for different values of positive q and negative r, respectively. These figures show that a decreasing trend for u_o when plotted against width for a fixed value of u_o increases with respect to q (for fixed $r=1/4$) and decreases for increased negative value of r (for fixed $q=0$), respectively. Figure 6 shows the dependence of width on positive r and q. It is evident that for increasing q and decreasing r, the width of solitary wave solution increases. Solitary wave solution velocity u_o is plotted against q, for negative r in Fig. 7, and shows that u_o increases with q, and for a fixed value of q it also increases with a decreasing negative value of r.

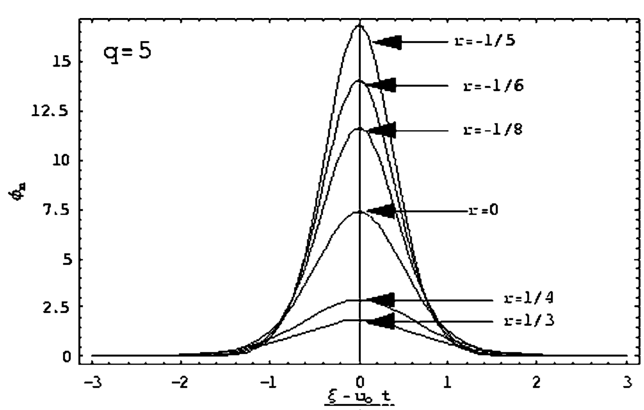

FIG. 2. The amplitude of solitary wave solution potential φ_m against the $\eta\,[=(\xi-u_0\tau)/\Delta]$ for different values of spectral index r.

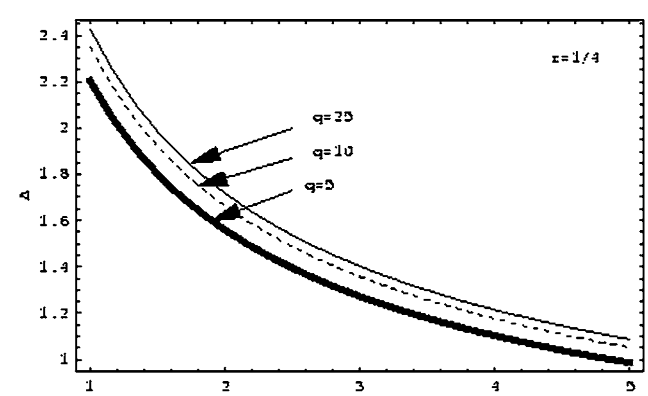

FIG. 4. Variation of the width of solitary wave solution Δ with its velocity u_0 for different values of spectral index q.

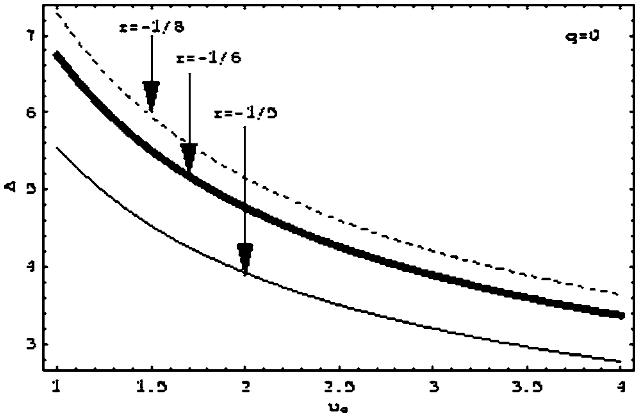

FIG. 5. Variation of the width of solitary wave solution Δ with its velocity u_0 for different negative values of spectral index r.

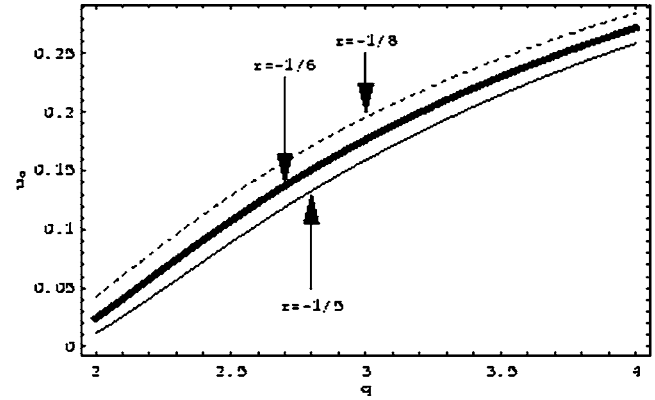

FIG. 7. The effect of negative values of r on the solitary wave solution velocity u_0 as a function of q.

IV. CONCLUSION

We have studied the ion-acoustic solitary wave solution with electron trapping with a generalized (r,q) distribution function. By using the well-known reductive perturbation technique, the generalized KdV equation with a solitary wave solution has been derived for different ranges of the parameter r. Both from the analytical and graphical results, it is concluded that for positive values of r, the ion-acoustic wave becomes supersonic, and for negative values of r it is subsonic. For $r=0$ and $q\to\infty$, the dispersion relation, the KdV equation, along with the solution and expressions for amplitude and width, reduces to those with a Maxwellian distribution function of Refs. 29 and 30. The amplitude of the solitary wave solution increases for decreasing r and increasing q with respect to its velocity u_o. The width of the solitary wave solution increases for decreasing positive values of r and increasing q with decreasing velocity u_o. However, we also see that for negative values of r, the width decreases, which in turn means that negative r makes the solitary wave solution more spiky. In a nutshell, we conclude that the spectral index r of the trapped electrons distribution effects the shoulder of the ion-acoustic solitary wave solution (makes the profile slightly spiky for positive r and more spiky for negative r in the range $-1/2<r<1/2$) and spectral index q effects the tail of it making the solitary wave faster than the thermal solitons, which are believed to be exist in near-plasma shock waves[21] and in the solar wind.[23]

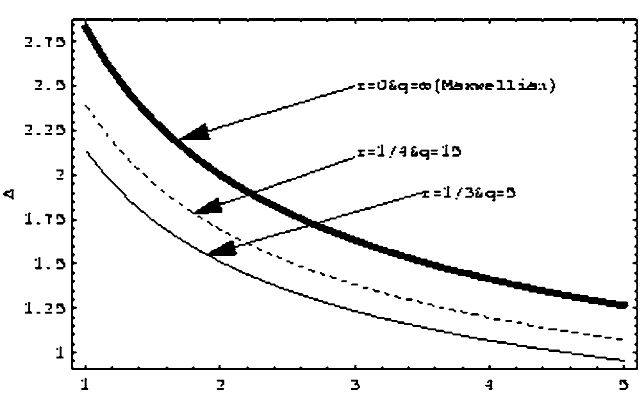

FIG. 6. The effects of spectral indices q and r on the width of solitary wave solution Δ as a function of velocity u_0.

ACKNOWLEDGMENTS

One of the authors (M. A.) thanks the Higher Education Commission (HEC) and the PAEC for financial support of this work.

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