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Debye shielding in quantum plasmas

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Abstract
Quantum effects through the Fermi temperature and the Bohm potential on the Debye–Hückel shielding potential have been examined in a supercold quantum plasma. The Bohm potential due to collective interaction of the quantum plasma modifies the Debye length significantly under an appropriate condition. The relevance of our results in supercold dense plasmas, viz. the microelectronic systems, laser-produced plasmas, compact astrophysical objects, etc has been pointed out.

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In recent years, there has been growing interest in the quantum mechanical effects in plasmas and microelectronic devices ([1] and references therein). The importance of quantum plasmas has been shown in microelectronic devices [2], in dense astrophysical systems [3] and in laser-produced plasmas [4]. New quantum mechanical results have been predicted in super-cooled dusty plasmas by a number of workers [5–8].

Basically, if a plasma is cooled to an extremely low temperature, the de Broglie wavelength of the charge carriers may be comparable to the dimension of the systems, viz. the Debye length of the plasma. In such situations, the ultracold plasmas must behave as a Fermi gas and the quantum mechanical effects are expected to play a vital role in the behavior of collective interactions of the charged particles. However, a plasma is a plasma if the Debye length is smaller than the size of the plasma systems. So, one could say that when the de Broglie wavelength of carriers is comparable to the Debye length, the quantum mechanical effect must be significant in Debye shielding.

Recently, Shukla *et al* [8] have calculated near and far field potentials of a slowly moving test charge in a quantum plasma. They have utilized the already derived dielectric function of Pines [9] considering the Fermi temperature of the quantum plasma gas. However, they have ignored the quantum effect arising out of the Bohm potential term due to the collective interaction in the equation of motion. In this paper, we show the modification of the usual Debye shielding in a supercooled Fermi gas plasma using a 'quantum hydrodynamic model' of the plasmas.

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We consider a zero-temperature Fermi gas, an electron–ion plasma with motion of plasma particles in one-dimension (x-direction). Fermi gas obeys the pressure law [1, 10]

$$p_j = m_j V_{Fj}^2 n_j^3 / 3n_{j0}^2, \tag{1}$$

where $j = e$ for electrons and $j = i$ for ions, m_j is the mass, $V_{Fj} = (2k_B T_{Fj}/m_j)^{1/2}$ is the Fermi speed, k_B is the Boltzmann constant, and T_{Fj} is the Fermi temperature. Here, n_j is the total number density with its equilibrium value n_{j0}.

The linearized equation of motion for the jth species with quantum Bohm potential term is

$$m_j n_{j0}\left(\frac{\partial}{\partial t} + \nu_j\right)\mathbf{v}_{j1} = -q_j n_{j0}\nabla\phi_1 - \nabla p_{j1} + \frac{\hbar^2}{4m_j}\nabla(\nabla^2 n_{j1}), \tag{2}$$

where q_j, m_j and ν_j are the charge, mass and collision frequency with the immobile neutrals for the jth species. Here, ϕ_1 is the potential of an electrostatic wave, $\hbar$ is the Planck's constant divided by 2π. The quantum correction in equation (2) appears through the Fermi temperatures T_{Fj} and the last term of the Bohm potential.

The continuity equation for the jth species is

$$\frac{\partial n_{j1}}{\partial t} + n_{j0}\frac{\partial v_{j1x}}{\partial x} = 0. \tag{3}$$

Assuming that the variation in $\mathbf{v}_{j1x}, n_{j1}$ and ϕ_1 is proportional to $\exp[-\mathrm{i}(\omega t - kx)]$ where ω and $\mathbf{k}$ are the wavefrequency and the wavenumber vector, equation (2) reduces to

$$(\omega + \mathrm{i}\nu_j)v_{j1x} = \frac{q_j k}{m_j}\phi_1 + \frac{k V_{Fj}^2}{n_{j0}}(1+\gamma_j)n_{j1}, \tag{4}$$

where

$$\gamma_j = \frac{\hbar^2 k^2}{8m_j k_B T_{Fj}}. \tag{5}$$

Combining equations (3) and (4) and writing the density perturbation as $n_{j1} = -\chi_j k^2 \phi_1 / 4\pi q_j$, the susceptibility function with quantum mechanical effects is given by

$$\chi_j = -\frac{\omega_{pj}^2}{\omega(\omega + \mathrm{i}\,\nu_j) - k^2 V_{Fj}^2 (1+\gamma_j)}, \tag{6}$$

where $\omega_{pj} = (4\pi q_j^2 n_{j0}/m_j)^{1/2}$ is the plasma frequency of the jth species.

The dielectric function of the plasma can be obtained from $\epsilon(\omega, \mathbf{k}) = 1 + \chi_e + \chi_i$. Taking $V_{Fj} > \omega/k$, we can immediately obtain

$$\epsilon = 1 + \frac{1}{k^2 \lambda_D^2}, \tag{7}$$

where

$$\frac{1}{\lambda_D^2} = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2} \tag{8}$$

and

$$\lambda_{Dj}^2 = \frac{V_{Fj}^2}{\omega_{pj}^2}(1+\gamma_j). \tag{9}$$

The electrostatic potential around a test charge in the presence of an electrostatic mode $(\omega, \mathbf{k})$ in a uniform plasma, whose dielectric response function is given by equation (7), is [11]

$$\Phi(\mathbf{x}, t) = \int \frac{q_t}{2\pi^2 k^2} \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega, \mathbf{k})} \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})\, \mathrm{d}\mathbf{k}\, \mathrm{d}\omega, \tag{10}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of a test charge particulate, and q_t is its charge.

Substituting equation (7) in (10), one can easily find the Debye–Hückel potential for the quantum plasma as

$$\Phi(r) = \frac{q_t}{r} \exp(-r/\lambda_D), \tag{11}$$

where the modified Debye length turns out to be

$$\lambda_D = \left(\frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2} \right)^{-1/2}, \tag{12}$$

where $\lambda_{Dj} = V_{Fj}\sqrt{1+\gamma_j}\,/\,\omega_{pj}$, V_{Fj}, is defined earlier, and γ_j is given by equation (5).

In summary, we have analyzed the Debye–Hückel problem in a zero-temperature supercooled Fermi gas plasma with quantum effects. Quantum hydrodynamic equations for the electrons and ions are employed to find the dielectric function of the quantum plasma. By employing this appropriate dielectric function for such a quantum plasma, we have derived an expression for the quantum mechanically modified shielding potential for a static or a slowly moving test charge. It is noticed from equations (9) and (11) that the Debye length does depend on the Fermi temperature (T_{Fj}) and the quantum correction factor γ_j. However, for arbitrary γ_j, the Debye length is given by equation (12) and for $\gamma_j \sim 1$, the Debye length includes a correction due to the Bohm potential. Obviously, our results yield the Debye length derived by Shukla *et al* [8] in absence of the Bohm potential correction.

We now discuss two extreme conditions. (i) First, we consider $\gamma_j \ll 1$, i.e. $\hbar^2 k^2 \ll 8m_j k_B T_{Fj}$. Thus, if we neglect the Bohm potential term compared to the Fermi temperature term in the equation of motion, we retrieve the results of Shukla *et al* [8]. Here, $\lambda_{Dj} = V_{Fj}/\omega_{pj}$. (ii) For $\gamma_j \gg 1$, one can have $\hbar^2 k^2 \gg 8m_j k_B T_{Fj}$. Under this condition, the quantum mechanical effect through the Bohm potential in the equation of motion becomes more dominant than the collective effect arising through the Fermi temperature of the quantum system. Then, the Debye shielding is due to quantum mechanical effect explicitly and $\lambda_{Dj} \simeq \hbar k / \sqrt{16\pi e^2 n_{j0} m_j}$. A similar quantum mechanical shielding of electrons in metals was earlier shown by Bohm and Pines [12]. Thus, the quantum mechanical effects become important for the supercooled plasma due to the collective behavior of the quantum plasma. Our results would be useful in understanding the modified Debye–Hückel potential around a test charge in ultracold quantum plasmas, e.g. in micro- and nano-systems, dense laser-produced plasmas, and dense astrophysical objects like white dwarfs and neutron stars.

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