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Longitudinal photons in a relativistic magneto-active plasma

N. L. Tsintsadze
Salam Chair in Physics, Government College University, Lahore 54000, Pakistan, Institute of Physics, Tbilisi, Georgia, and Department of Physics, Government College University, Lahore 54000, Pakistan

Ayesha Rehman
Salam Chair in Physics and Department of Physics, Government College University, Lahore 54000, Pakistan

G. Murtaza
Salam Chair in Physics, Government College University, Lahore 54000, Pakistan

H. A. Shah
Department of Physics, Government College University, Lahore 54000, Pakistan

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This paper presents some aspects of interaction of superstrong high-frequency electromagnetic waves with strongly magnetized plasmas. The case in which the photon-photon interaction dominates the photon-plasma particle interaction is considered. Strictly speaking, the photon and photon bunch interaction leads to the self-modulation of the photon gas. Assuming that the density of the plasma does not change, the dispersion relation, which includes relativistic self-modulation, is investigated. The existence of longitudinal photons in a strong magnetic field has the well-known Bogoliubov-type energy spectrum. The stability of the photon flow is investigated and an expression for Landau damping of the photons is obtained. Finally, it has been shown that the interaction of even a very strong electromagnetic radiation with a plasma does not always lead to instability, but causes only a change in plasma properties, whereby the plasma remains stable. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2768511]

I. INTRODUCTION

It was shown in Refs. 1–8 that the interaction of a relativistic intense electromagnetic (EM) radiation with plasmas leads the initially narrow spectrum to eventually broaden due to different kinds of instabilities. Hence, the natural state of strong radiation of the EM field has a broad spectrum, which allows us to treat such strong radiation as a dense photon gas. In a recent review by Marklud and Shukla,[9] nonlinear collective effects were considered in detail. This analysis involved the Heisenberg-Euler Lagrangian, which describes strong field vacuum effects.

Since photons in plasma acquire an effective rest mass, the authors of Refs. 10–14 considered blackbody radiation by taking relativistic effects into account. In the last couple of years, a remarkable new aspect of a dense photon gas was explored in Refs. 15 and 16 in the absence of the magnetic field where Bose-Einstein condensation (BEC) and an intermediate state of the photon gas were investigated. In these papers, the existence of "Compton"-type scattering in a nonlinear photon gas was established for the case in which the plasma density remains constant, and this may be considered a new physical phenomenon. Further, the generation of longitudinal photons (photoniko) was demonstrated, and a Bogoliubov-type energy spectrum was derived for them. In the above papers, the intensity of radiation that was considered was such that the photon-photon interactions were more likely than the photon-plasma particle interactions. In Ref. 16, a new version of the Pauli equation or the master equation for a photon gas was derived from a general kinetic equation for the electromagnetic spectral intensity (see, for example, Refs. 1–7). The kinetic equations for plasmons and photons in Vlasov's approximation were considered in Ref. 17 for the case of broad spectrum Langmuir plasmons. A more general kinetic equation was derived and the ion-sound wave excitation was investigated in Ref. 18.

It is well known that the role of relativistic effects increases when a strong magnetic field is present in plasma. This occurs when the cyclotron frequency $\Omega_c = eB_0/m_0\gamma c$ is of the order of the photon frequency ω, i.e., $\Omega_c = 1.76 \times 10^7 B_0/\gamma \sim \omega$, where m_0 is the particle rest mass and $\gamma = (1-v^2/c^2)^{-1/2}$ is the relativistic factor. The study of the problem of the influence of a strong or superstrong magnetic field on the dynamics of relativistic intense EM radiation is immensely important. Ginzburg[19] was the first to consider this and showed that the collapse of a star must be accompanied by the generation of a superstrong magnetic field. We would like to point out that there are a variety of well known effects of strong magnetic fields in plasmas and that the astrophysical data suggest that the surface magnetic field of a neutron star is $B \sim 10^{12} - 10^{13}$ G while the internal field can reach values of $B \sim 10^{15}$ G or even higher (as explained in Refs. 20 and 21).

Recently, in Ref. 22 a new mechanism for the generation of a magnetic field by superstrong radiation was proposed, and under this proposition the magnetic field could attain values of $B \sim 10^7 - 10^9$ G. In Ref. 23, a novel kinetic equation for the spectral function of high-frequency relativistic intense electromagnetic waves in the presence of a strong magnetic field was derived. In the same paper, a dispersion relation for relativistic self-modulation of the photon gas with arbitrary amplitude of the EM waves was obtained.

In the present paper, we consider the interaction of relativistic intense incoherent EM waves (photon gas) with plasmas in the presence of a strong magnetic field. We first obtain the dispersion relation and show that there exist new longitudinal photons in a strong magnetic field, which we refer to as longitudinal magnetic photons.

In the limit of a vanishing magnetic field, we obtain the expression of the Bogoliubov type of spectrum (see Ref. 16). These waves are fully relativistic and have no analog in the nonrelativistic limit.

In the present paper, we also show that the interaction of relativistic intense EM waves with plasmas does not always lead to instabilities. There exists a condition under which the radiation changes properties of the plasma and the plasma remains stable. This in turn leads to the spatial dispersion of the longitudinal waves, even when the temperature is neglected, (or $T \ll m_0 c^2$). Thus we have shown that the spectrum of these longitudinal plasma waves is quite different from the case in which there is no radiation.

II. KINETIC EQUATION FOR THE PHOTON GAS

In Ref. 23, circularly polarized relativistic intense EM waves propagating in the direction of a constant magnetic field B_0 were considered. There it was assumed that all quantities have both fast and slow temporal and spatial scales. By using the fully relativistic Maxwell equation for the vector potential

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = \frac{4\pi e n}{m c \gamma}\vec{p} \tag{1}$$

under the action of an intense high-frequency (HF) field, only the electrons contribute to the current density, i.e., $\vec{J} = -en\vec{p}/\gamma$. Here $\vec{p}$ is the momentum due to rapidly varying EM fields, and a simple calculation of $\vec{p}$ follows from the equation of motion to yield

$$\vec{p} = -\frac{e\gamma}{c}\int \frac{d\omega}{2\pi}\int \frac{d\vec{k}}{(2\pi)^3}\left[\frac{\omega}{(\gamma\omega - \omega_c)}\vec{A} - i\frac{\omega_c}{(\gamma\omega-\omega_c)^2}\frac{\partial \vec{A}}{\partial t}\right] \times \exp[i(\vec{k}\cdot\vec{x} - \omega\tau)]. \tag{2}$$

We shall consider Eq. (1) at two distinct points and instants of time. Following the procedure adopted in Ref. 23, by taking into account that when photon-photon interactions take place the phases of the wave are in general random functions of time, which can be averaged over, we can then derive an equation for the dimensionless correlation function, $e^2/(m_0c^2)^2\langle \vec{A}(\vec{r}_1,t_1)\vec{A}(\vec{r}_2,t_2)\rangle = \Pi(\vec{r}_1,t_1;\vec{r}_2,t_2)$. For this we use Eq. (1) for $\vec{A}(\vec{r}_1,t_1)$ and $\vec{A}(\vec{r}_2,t_2)$, multiply by the first by $\vec{A}(\vec{r}_2,t_2)$ and the latter by $\vec{A}(\vec{r}_1,t_1)$, respectively, and then subtract the resulting two equations from each other, and further by introducing the following new variables:

$$t = \tfrac{1}{2}(t_1+t_2), \quad \tau = t_1 - t_2, \quad \vec{r} = \tfrac{1}{2}(\vec{r}_1+\vec{r}_2), \quad \vec{\xi} = \vec{r}_1 - \vec{r}_2.$$

We obtain Eq. (7) of Ref. 23 for $\Pi(\vec{r}+\vec{\xi}/2, t+\tau/2; \vec{r}-\vec{\xi}/2, t-\tau/2)$, where $\Pi = e^2\langle \vec{A}(\vec{r},t)\vec{A}^*(\vec{r},t)\rangle/(m_0c^2)^2$.

Performing a Fourier transformation of $\Pi(\vec{r},\vec{\xi},t,\tau)$ on the variables $(\vec{\xi},\tau)$, we can introduce the dimensionless spectral function Q or Wigner representation

$$Q(\omega,\vec{k},\vec{r},t) = \frac{e^2|\vec{A}(\omega,\vec{k},\vec{r},t)|^2}{(m_0c^2)^2} = \int d\tau \int d\vec{\xi}\,\exp[i(\omega\tau - \vec{k}\cdot\vec{\xi})] \times \Pi\left(\vec{r}-\frac{\vec{\xi}}{2}, t-\frac{\tau}{2}; \vec{r}+\frac{\vec{\xi}}{2}, t+\frac{\tau}{2}\right). \tag{3}$$

Taking the double Fourier transformation of Eq. (7) from Ref. 23, and assuming that all the derivatives of the density $n_\pm = n(\vec{x}\pm\vec{\xi}/2, t\pm\tau/2)$ and the relativistic factor $\gamma_\pm = \gamma(\vec{x}\pm\vec{\xi}/2, t-\tau/2)$ exist, we then expand these quantities around $\vec{x}$ and t to obtain an evolution equation for the spectral function $Q(\omega,\vec{k},\vec{r},t)$ in the form

$$\left(\omega + \frac{\omega_{ce}\omega_{pe}^2}{2(\gamma\omega-\omega_{ce})^2}\frac{n_e}{n_0}\right)\frac{\partial Q}{\partial t} + c^2(\vec{k}\cdot\vec{\nabla})Q = \omega_p^2\sum_{l=0}^{\infty}\frac{(-1)^l}{(2l+1)!2^{2l+1}}\left(\vec{\nabla}\cdot\vec{\nabla}_{\vec{k}} - \frac{\partial}{\partial\omega}\frac{\partial}{\partial t}\right)^{2l+1} \times \left(\frac{\omega}{(\gamma\omega-\omega_{ce})}\frac{n_e}{n_0}\right)Q, \tag{4}$$

where n_0 is the equilibrium plasma density, $\omega_{ce} = eB_0/m_0c$, and $\omega_{pe}^2 = 4\pi n_0 e^2/m_{0e}$.

Note that in this equation, the time and space derivatives in the brackets do not act on Q. Further, the authors of Ref. 23 derived the ponderomotive force using Eq. (2), which is the equation for the momentum of electrons and is given by

$$\vec{F}_p = -m_0c^2\int\frac{d\omega}{2\pi}\int\frac{d\vec{k}}{(2\pi)^3}\left[\frac{\omega\vec{\nabla}Q}{(\gamma\omega-\omega_{ce})} - \frac{\omega_c\vec{k}}{(\gamma\omega-\omega_{ce})^2}\frac{\partial Q}{\partial t}\right]. \tag{5}$$

Here the relativistic factor γ can be expressed in terms of the spectral function Q and is given by

$$\gamma^2 = 1 + \gamma^2\int\frac{d\omega}{2\pi}\int\frac{d\vec{k}}{(2\pi)^3}\frac{\omega^2 Q}{(\gamma\omega-\omega_{ce})^2}. \tag{6}$$

It is important to emphasize that in Eq. (4) there are two forces, having a distinct nature, which can change the spectral function Q of photons. One force appears due to the redistribution of electrons in space, $\nabla n_e/n_o$, and time, $\partial/\partial t n_e/n_o$, and the other due to variations of the shape of the wave packet and is proportional to $\nabla 1/\gamma\omega-\omega_{ce}$ and $\partial/\partial t 1/\gamma\omega-\omega_{ce}$. We further note that the latter force takes into account the variation of the relativistic factor γ.

III. SELF-MODULATION OF WAVES IN A STRONG MAGNETIC FIELD

In Ref. 16, it was shown that if the modulation frequency of waves is much less than the Langmuir frequency of electrons ω_{pe}, then the photon flow can no longer excite Langmuir waves and the contribution of perturbation of the electron density is small in comparison with the perturbation of the photon density. In this section, therefore, we shall suppose that the density of the plasma particles remains constant.

In order to consider self-modulation of EM waves, we linearize Eqs. (4) and (5) with respect to perturbations and seek plane-wave solutions proportional to $\exp i(\vec{q}\cdot\vec{r}-\Omega t)$. We consider the range of frequencies for which the inequality $\Omega \ll \omega \sim \omega_{ce}/\gamma$ holds, and as stated earlier, the density perturbation δn_e is zero. In other words, only the perturbation of the wave packet is considered. With these assumptions, we obtain the dispersion relation for the self-modulation of the photon gas in magnetized plasma, which is given by

$$1-\left(\frac{\omega_{pe}^2\gamma_0}{g_0}\right)\left[\int \frac{d\vec{k}\omega^4 Q_0(k)}{\Gamma(\gamma_0\omega-\omega_{ce})^4}\left(\frac{P}{\Omega-\vec{q}\cdot\vec{v}_g+\frac{q^2c^2}{2\Gamma}}-\frac{P}{\Omega-\vec{q}\cdot\vec{v}_g-\frac{q^2c^2}{2\Gamma}}\right) - i\pi\left[\delta\left(\Omega-\vec{q}\cdot\vec{v}_g+\frac{q^2c^2}{2\Gamma}\right)-\delta\left(\Omega-\vec{q}\cdot\vec{v}_g-\frac{q^2c^2}{2\Gamma}\right)\right]\right]=0, \quad (7)$$

where

$$g_0 = 1+\omega_{ce}\int \frac{d\vec{k}\omega^2 Q_0(\vec{k})}{(\gamma_0\omega-\omega_{ce})^3},$$

$$\Gamma = \omega + \frac{1}{2}\frac{\omega_{ce}\omega_{pe}^2}{(\gamma_0\omega-\omega_{ce})^2} \quad \text{and } \vec{v}_g = \frac{\vec{k}c^2}{\Gamma}.$$

Here we have used the well-known relation

$$\lim_{\varepsilon\to 0}\frac{1}{x+i\varepsilon} = \frac{P}{x} - i\pi\delta(x),$$

where P denotes the prescription that at the singularity ($x=0$), the principal value is to be taken. In Eq. (7), $Q_0(\vec{k})$ is the spectral function in the equilibrium state and is represented by

$$Q_0(\vec{k}) = \frac{\alpha(\vec{k}_0)}{(2\pi\sigma_k^2)^{3/2}}\exp\left(-\frac{(\vec{k}-\vec{k}_0)^2}{2\sigma_k^2}\right). \quad (8)$$

This is a spectral Gaussian distribution, with the average wave vector $\vec{k}_0$ and spectral width σ_k. Further, assuming that $\vec{k}=\vec{k}_0+\vec{\chi}$ and $|\vec{q}|,|\vec{\chi}|\ll|\vec{k}_0|$, and then upon integration of Eq. (7), we obtain the dispersion relation in the form

$$1+\frac{\omega_{pe}^2}{\gamma_0 g_0}\frac{\omega^2(k_0)}{\Gamma^2(k_0)}\frac{(\gamma_0^2-1)}{(\gamma_0\omega-\omega_c)^2} \times\left[\frac{q^2c^2}{[\Omega-\vec{q}\cdot\vec{v}_g(k_0)]^2-q^2U_s^2-\delta^2q^4}\right] - 3i\sqrt{\frac{\pi}{2}}\frac{1}{q\sigma_k U_s^2}\frac{\omega_{pe}^2}{\gamma_0 g_0}\frac{\omega^2}{\Gamma(k_0)}\frac{(\gamma_0^2-1)}{(\gamma_0\omega-\omega_c)^2}[\Omega - \vec{q}\cdot\vec{v}_g(k_0)]\exp\left(-\frac{3[\Omega-\vec{q}\cdot\vec{v}_g(k_0)]^2}{2q^2U_s^2}\right)=0, \quad (9)$$

where

$$\delta = \frac{c^2}{2\Gamma}, \quad g_0 = 1+\frac{\omega_{ce}}{(\gamma_0\omega-\omega_{ce})}\frac{(\gamma_0^2-1)}{\gamma_0^2},$$
$$\frac{\gamma_0^2-1}{\gamma_0^2} = \alpha(k_0)\frac{\omega^2(k_0)}{(\gamma_0\omega-\omega_{ce})^2}. \quad (10)$$

The velocity $U_s=\sqrt{3}\sigma_k c/\Gamma c$ can be treated as the magnetic sound velocity (or magnetized sound velocity) of the longitudinal photons.

Further, by neglecting the imaginary term in Eq. (9), we investigate the dispersion relation in the hydrodynamic limit, which then has the form

$$(\Omega-\vec{q}\cdot\vec{v}_g)^2 = q^2[U_s^2-V_E^2+\delta^2q^2], \quad (11)$$

where

$$V_E^2 = \frac{\omega_L^2}{g_0}\frac{(\gamma_0^2-1)}{\gamma_0^2\Gamma^2}\frac{\omega^2c^2}{(\gamma_0\omega-\omega_{ce})^2}. \quad (12)$$

We note that g_0 can be positive when $\omega > \omega_{ce}/\gamma$ and negative in the opposite case. In the latter case, the longitudinal waves are always stable, i.e., for the case $V_E^2=-|V_E^2|$ we obtain the Bogoliubov type of spectrum for the longitudinal photons in the magnetized plasma (as noted earlier, the spectrum without the magnetic field or in an isotropic plasma was obtained in Ref. 16).

If V_E^2 is positive and $V_E^2 > U_s^2 + \alpha^2 q^2$ or $q < 2\Gamma / c^2 \sqrt{V_E^2 - U_s^2}$, then Eq. (9) has an unstable solution. Taking $\Omega = \Omega' + i\Omega''$, where

$$\Omega' = \vec{q} \cdot \vec{v}_g, \tag{13}$$

we obtain the growth rate for the unstable mode given by

$$\Omega'' = q\sqrt{V_E^2 - U_s^2 - \alpha^2 q^2}. \tag{14}$$

From Eqs. (13) and (14) it follows that the emission of magneto-longitudinal photons takes place due to a bunch of photons inside a resonance cone ($\cos\theta = \Omega' / q v_g$). If V_E^2 is negative or $V_E^2 < U_s^2 + \alpha^2 q^2$, we have a stable Bogoliubov type of spectrum with damping, which follows from Eqs. (9) and (11) for real Ω in a comoving frame of reference ($\Omega - \vec{q} \cdot \vec{v}_g \to \Omega$),

$$\Omega' = q\sqrt{U^2 + \alpha^2 q^2}, \tag{15}$$

where $U^2 = U_s^2 - V_E^2$.

The decrement Ω'' is derived from Eq. (9), and is given by

$$\Omega'' = -\sqrt{\frac{\pi}{8}}\left(\frac{\omega_{pe}}{\sigma_k c}\right)^3 \left(\frac{\omega_{pe}}{\Gamma}\right) \frac{qc}{g_0^2}\left(\frac{\omega^2}{(\gamma_0\omega - \omega_{ce})^2}\right)^2 (\gamma_0^2 - 1)^2 \times \exp\left(-\frac{3(\Omega - \vec{q}\cdot\vec{v}_g)^2}{2q^2 U_s^2}\right). \tag{16}$$

If $\omega(k_0) \sim \omega_{ce}/\gamma_0$, the imaginary frequency of the magneto-longitudinal photons is

$$\Omega'' = -\sqrt{\frac{\pi}{8}} qc \left(\frac{\omega_p}{\sigma_k c}\right)^3 \frac{\omega_{ce}}{\omega_{pe}}. \tag{17}$$

We note that the decrement in this case is maximum and $\exp[-3(\Omega - \vec{q}\cdot\vec{v}_g)^2 / 2q^2 U_s^2] \approx 1$. High power levels are required for the observation of these effects in the laboratory, and modern HF power supplies (strong laser) can produce fields in a plasma strong enough for the electrons to acquire relativistic velocities. In astrophysical situations, these strong fields may be produced by the EM radiation from various astrophysical objects (galactic nuclei, radio galaxies, and quasars, for example).

IV. SPATIAL DISPERSION

In the preceding section, the density of plasma was assumed constant. However, we now allow for the variation of the plasma density due to strong radiations, and we will show that the relativistic intense EM wave does not always lead to instabilities in a plasma. In order to show this, we linearize the ponderomotive force given by Eq. (5) and the gamma factor Eq. (6), and after *straightforward* calculations, we obtain the following expression for the ponderomotive force:

$$\delta\vec{F}_p = -im_o c^2 \frac{\omega_{pe}^2 G_o^2 \vec{q} D}{1 + \omega_{pe}^2 \frac{\gamma_0}{g_0} G_0^4 D}\left(\frac{\delta n_e}{n_0}\right), \tag{18}$$

where δn_e is the electron density variation,

$$D = \int \frac{d\vec{k}}{(2\pi)^3}\left[\frac{Q_0^+ - Q_0^-}{(\vec{q}\cdot\vec{k}c^2 - \omega\Omega)}\right], \quad Q_0^{\pm} = Q_0\left(\vec{k} \pm \frac{\vec{q}}{2}\right),$$

and

$$G_0^2 = \left(\frac{\omega}{\omega\gamma_0 - \omega_c}\right)^2. \tag{19}$$

In Eq. (18), if $\omega_{pe}^2 \gamma_0 / g_0 G_0^4 D \gg 1$, then the ponderomotive force reduces to

$$\delta\vec{F}_p = -im_0\gamma_0 V_0^2 \left(\frac{\delta n_e}{n_0}\right)\vec{q}, \tag{20}$$

where $V_0^2 = c^2 g_0 / \gamma_0^2 (\gamma_0\omega - \omega_c/\omega)^2$.

We note that near resonance, $\omega \sim \omega_{ce}/\gamma_0$, $\delta\vec{F}_p$ becomes

$$\delta\vec{F}_p = -i\frac{m_0 c^2}{\gamma_0}\frac{\omega_c}{(\gamma_0\omega - \omega_c)}\left(\frac{\delta n}{n_0}\right)\vec{q}. \tag{21}$$

When $\omega_c > \gamma_0\omega$, the ponderomotive force changes signature as expected.

In the limiting case of a vanishing magnetic field, i.e., when $\omega_{ce} = 0$ or $\omega \gg \omega_c$, Eq. (19) reduces to $\delta\vec{F}_p = -im_0\gamma_0 c^2 (\delta n_e / n_0)\vec{q}$, which was obtained in Ref. 1.

Further, we consider an underdense plasma, i.e., when $\omega \gg \omega_{pe}, \omega_{ce}$ and $V_0 = c$. From Eq. (20), it is evident that the ponderomotive force changes the character of propagation of the longitudinal plasma waves when the plasma temperature equals zero or is much less than the rest energy of electrons, i.e., $m_0 c^2 \gg T$. In other words, when we assume temperature $T = 0$, then in the initial state the plasma perturbations are without spatial dispersion. However, when we apply the force given by Eq. (20), then in the equation of motion of electrons there appears a pressure term with an effective temperature $T_0 = m_0 c^2$. This term leads to spatial dispersion of the plasma. The question that one may ask is that when the waves in a cold magneto-active plasma are longitudinal in the presence of the ponderomotive force given by Eq. (20), what then is their frequency spectrum? By using the linear hydrodynamic set of equations for magnetized plasma for both electrons and ions and taking into account that the ponderomotive force acts only on the electrons, we obtain after a standard procedure the following linear dispersion relation for obliquely propagating waves:

$$1 - \frac{\omega_L^2 + \omega_{pi}^2}{\Omega^2}\cos^2\theta - \left(\frac{\omega_L^2}{\Omega^2 - \Omega_{ce}^2} + \frac{\omega_{pi}^2}{\Omega^2 - \omega_{ci}^2}\right)\sin^2\theta + \frac{q^2 c^2 \omega_{pi}^2}{\Omega^4}\left(\cos^2\theta + \frac{\Omega^2 \sin^2\theta}{\Omega^2 - \Omega_{ce}^2}\right)\left(\cos^2\theta + \frac{\Omega^2\sin^2\theta}{\Omega^2 - \omega_{ci}^2}\right) = 0, \tag{22}$$

where

$$\omega_L^2 = \tilde{\omega}_{pe}^2 + q^2c^2, \quad \tilde{\omega}_{pe} = \left(\frac{4\pi e^2 n_0}{m_{oe}\gamma_0}\right)^{1/2}, \quad \omega_{ci} = \frac{eB_0}{m_i},$$

and $\omega_{pi} = \left(\frac{4\pi e^2 n_0}{m_i}\right)^{1/2}$,

and θ is the angle between the wave vector $\boldsymbol{q}$ and the ambient magnetic field B_0. It is important to emphasize here that in the absence of the ponderomotive force (which implies that $q \to 0$), the last term in Eq. (22) disappears and we are left with the standard equation for the longitudinal plasma oscillation in a magnetized plasma.

First we consider the case in which the magnetic field $B_0 = 0$. The dispersion relation given by Eq. (22) decouples, and in this case we obtain a two-frequency spectrum. The first is a novel Langmuir wave spectrum due to the strong relativistic effect (see, for example, Ref. 1) and is given by

$$\Omega = \sqrt{\tilde{\omega}_{pe}^2 + q^2c^2}, \tag{23}$$

and the second describes a new type of ion sound wave, where the dispersion spectrum is given by

$$\Omega = \left(\frac{m_{oe}\gamma_o}{m_i}\right)^{1/2} \frac{qc}{\left(1 + \frac{q^2c^2}{\tilde{\omega}_{pe}^2}\right)^{1/2}} = \frac{qU_s}{\sqrt{1 + \frac{q^2c^2}{\tilde{\omega}_{pe}^2}}}. \tag{24}$$

From Eq. (22), it is clear that when the intensity of the radiation increases, the ion sound velocity $U_s = (m_{0e}/m_i)^{1/2} qc\gamma_0^{1/2}$ also increases.

We now consider the case $B_0 \neq 0$, but we take the ions to be immobile. Then we obtain from Eq. (20) the dispersion relation for the high-frequency longitudinal wave spectrum as

$$\Omega^2 = \frac{\omega_L^2 + \Omega_{ce}^2}{2} \pm \frac{1}{2}[(\omega_L^2 + \Omega_{ce}^2)^2 - 4\omega_L^2\Omega_{ce}^2 \cos^2\theta]^{1/2}. \tag{25}$$

If the radiation (the ponderomotive force term) is absent, then Eq. (25) describes the oscillations of the upper hybrid mode. On the other hand, when q is large, Eq. (25) reduces to the following two uncoupled modes:

$$\Omega_+ \approx q^2c^2 + \tilde{\omega}_{pe}^2 + \Omega_{ce}^2 \sin^2\vartheta \approx q^2c^2,$$
$$\Omega_-^2 = \Omega_{ce}^2 \cos^2\vartheta. \tag{26}$$

Thus our Eqs. (25) and (26) are new and are quite different from the case in which radiation is not taken into account.

The θ dependence Eq. (25) is shown in Fig. 1 for $\tilde{\omega}_{pe} > \Omega_{ce}$.

We would like to state here that in view of the consideration above, we now naturally expect that new branches of waves appear in the low-frequency range when we include ion density variations. To realize this statement, we consider the frequency range $\Omega_{ce} > \Omega > \Omega_{ci}$, and from Eq. (22) for waves, which propagate strictly across the magnetic field ($\theta = \pi/2$), we obtain

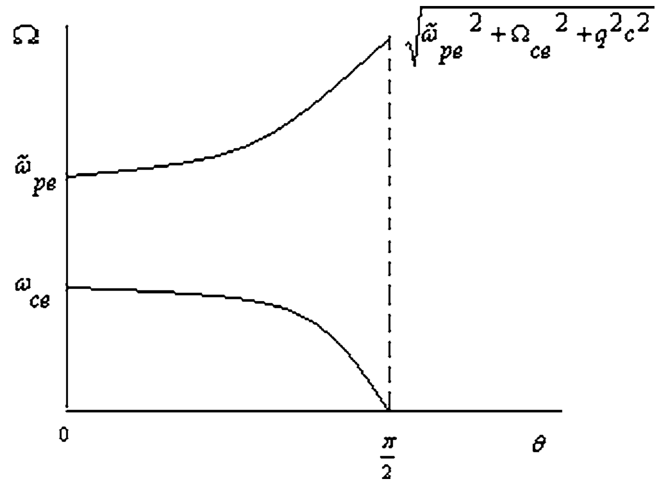

FIG. 1. Dependence of frequency Ω on angle of propagation θ.

$$\Omega^2 = \frac{\omega_{pi}^2}{\left(1 + \frac{\omega_L^2}{\Omega_{ce}^2}\right)}\left(1 + \frac{q^2c^2}{\Omega_{ce}^2}\right). \tag{27}$$

If $\omega_L^2 \gg \Omega_{ce}^2$, then Eq. (25) reduces to a new type of lower hybrid wave and is given by

$$\Omega^2 = \frac{\Omega_{ce}\omega_{ci}}{\left(1 + \frac{q^2c^2}{\tilde{\omega}_{pe}^2}\right)}\left(1 + \frac{q^2c^2}{\Omega_{ce}^2}\right). \tag{28}$$

From Eq. (28), we obtain the interesting results that when $q \to \infty$, $\Omega \to \omega_{pi}$, which is also evident from Fig. 2.

There exists another low-frequency spectrum at $\theta = \pi/2$ and $\tilde{\omega}_{pe}^2 \ll \Omega_{ce}^2$, which is given by

$$\Omega^2 = \omega_{ci}^2 + \frac{q^2c^2\omega_{pi}^2}{(\Omega_{ce}^2 + q^2c^2)}. \tag{29}$$

This reduces to $\Omega^2 = \omega_{ci}^2 + \omega_{pi}^2$ as $q \to \infty$.

As shown in Ref. 24, the resonant nature of the wave causes the electrons to become ultrarelativistic even when the EM radiation is weak, i.e., even if the amplitude E_0 of EM waves satisfies the inequality $E_0 < B_0$. Because we consider a homogeneous collisionless cold plasma, with right-hand circularly polarized EM waves, directed along the ex-

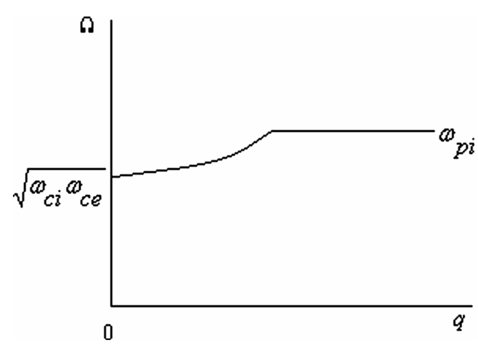

FIG. 2. Dependence of frequency Ω on wave number q.

ternal magnetic field B_0, from Maxwell equations the following standard expression for refraction coefficient can be obtained:

$$N^2 = \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega(\gamma_0\omega - \omega_{ce})}. \tag{30}$$

Introducing $\omega_E = eE_0/m_0c$, we may now rewrite the expression for γ_0 in Eq. (8), in the following form given by

$$\gamma_0^2 - 1 = \gamma_0^2\frac{\omega_E^2}{(\gamma_0\omega - \omega_{ce})^2}. \tag{31}$$

Generally speaking, this is a fourth-order equation for γ_0, but when $\gamma_0^2 \gg 1$, it takes a simple form given by

$$\gamma_0 = \frac{\omega_{ce} \pm \omega_E}{\omega}. \tag{32}$$

It is clear that the inequality $(\gamma_0^2 \gg 1)$ may be fulfilled even for a relatively small amplitude E_0 when $\omega_E < \omega_c$ and $\omega_{ce} \ll \omega$. Thus it is shown from Eqs. (30)–(32) that two types of waves can exist in an electron plasma in this case. It is important to emphasize that based on Eq. (32), pure cyclotron resonance is absent, but the difference $(\gamma_0\omega - \omega_{ce})$ can reach the cyclotron resonance point, and it can be shown that the factor $(\gamma_0\omega - \omega_{ce}) = \omega_E$. Here we note that the relativistic effects forbid a sharp resonance.

Using Eq. (32), we can express the following parameters: U_s, δ, g_0, v_g and V_E, through ω_E, in the following manner:

$$U_s = 2\sqrt{3}\frac{\sigma_k c^2}{\omega_{ce}} \cdot \frac{\omega_E^2}{\omega_{pe}^2}, \quad \delta = \frac{c^2\omega_E^2}{\omega_{ce}\omega_{pe}^2}, \quad g_0 = \pm\frac{\omega_{ce}}{\omega_E},$$

$$v_g = 2\frac{kc^2\omega_E^2}{\omega_{ce}\omega_{pe}^2} \quad \text{and } V_E^2 = \pm\frac{\omega^3\omega_E^3}{\omega_{ce}^4\omega_{pe}^2}c^2.$$

Here we have used $\omega^3\omega_E^3 \ll \omega_{ce}^4\omega_{pe}^2$, i.e., $V_E^2 \ll c^2$. We can see that these parameters depend strongly on the amplitude of the electromagnetic waves. We would like to emphasize here that these parameters have smaller values than in the case in which the magnetic field is absent, which in turn indicates that the frequency of the Bogoliubov spectrum in the presence of magnetic field is lower than without a magnetic field, if the wave vectors are equal in both cases.

V. CONCLUSIONS

In the present work, we investigated first the interaction of a spectrally broad and relativistic intense EM radiation in strongly magnetized plasma in the case in which photon-photon interactions dominate the photon particle interactions. By neglecting the variations of the plasma density, but taking into account the variation of the wave packet, we obtain a dispersion relation, from which follows a Bogoliubov frequency spectrum, and we have thus reestablished the relation between the frequency and the magnetic field. In the ultrarelativistic limit [$\gamma_0^2 \gg 1$, see Eq. (32)], the frequency of the Bogoliubov spectrum depends entirely on the amplitude of the wave.

In the second case, we have shown that even very strong radiation does not always lead to instabilities, and plasma with radiations remains stable. We further note that in the presence of strong radiations, the plasma waves have a spatial dispersion, which is absent when radiations are not taken into account.

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