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Wake potential in a nonuniform self-gravitating dusty magnetoplasma in the presence of ion streaming

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A detailed investigation of the electrostatic asymmetric shielding potential and consequent generation of the dynamical oscillatory wake potential has been examined analytically in an inhomogeneous self-gravitating dusty magnetoplasma in the presence of uniform ion streaming. It is found that the wake potential depends significantly on the test particle speed, ambient magnetic field, ion streaming velocity, and the plasma inhomogeneity. The periodic oscillatory potential might lead to an alternative approach to the Jeans instability for the formation of dust agglomeration leading to gravitational collapse of the self-gravitating systems. © 2007 American Institute of *Physics*. [DOI: 10.1063/1.2801413]

Wake potential formation^{1–3} seems to be the best contender for explaining the dust-Coulomb crystals in laboratory conditions.^{4–8} It can clearly explain the periodic forces causing attraction among the same polarity dust grains with effective lengths comparable to experimentally observed lattice spacings of dust crystals. It is anticipated that this wake potential theory might also explain the attractive forces among dust grains causing coagulations/agglomerations in a self-gravitating dusty plasma leading to gravitational collapse. The conventional theory of gravitational collapse is the Jeans instability.^{9–13} However, the Jeans instability is a considerably fast process, whereas structures such as stars and galaxies require a scale of a billion years for their formation.

The sudden local changes of energy on account of magnetic reconnections, shock waves, lightning, or transient heavenly events may cause sufficient hot electrons and dust grains to have drift motion.^{14,15} Ions in a self-gravitating dusty plasma may have a uniform motion for a number of reasons.^{16–18} The ion streaming is due to some electric field E_0 , which might cause the density inhomogeneities also in the self-gravitating plasma in the same direction. In addition, an ambient magnetic field will magnetize the appropriate plasma particles.

In this Brief Communication, we study the modification of the usual symmetric Debye-Hückel potential and the consequent generation of the dynamical wake potential in an inhomogeneous self-gravitating dusty plasma in the presence of continuous ion streaming and ambient magnetic field. Such an attractive wake potential might be the origin of the grain attraction giving rise to the dust agglomeration. We first derive the appropriate dielectric susceptibilities for different species with appropriate conditions of the selfgravitating dusty magnetoplasma. Next, the derivation of the modified shielding potential and the dynamical wake potentials are presented. It is noticed that the wake potential depends significantly on the test particle speed, ambient magnetic field, ion streaming velocity, and the plasma inhomogeneity in the self-gravitating dusty plasma. Finally, a brief discussion of the results is given.

We consider an inhomogeneous self-gravitating dusty plasma consisting of electrons, ions, and charged dust grains in the background of neutral atoms in the presence of a homogeneous magnetic field ($\mathbf{B}_0 \| \hat{z}$). At equilibrium, the gravitational force among the grains is balanced by the electromagnetic force acting on them. The dusty plasma is assumed to satisfy everywhere the quasineutrality condition $n_{i0}(x)$ $=n_{e0}(x)+Z_dn_{d0}(x)$, where Z_d is the number of electronic charge residing on the grains and $n_{\alpha 0}(x)$ with $\alpha = e, i, d$ is the nonuniform equilibrium number density of the α species. For mathematical simplicity, all grains are assumed to have the same radius a_0 with constant charge $(-Z_d e)$. The intergrain distance r_0 is assumed to be smaller than the plasma Debye length ($a_0 \ll r_0 \ll \lambda_D$), where λ_D is the effective Debye length of the self-gravitating dusty plasma. We first derive the dielectric response function of the self-gravitating dusty magnetoplasma. The resulting potential will involve the drift wave mode, plasma inhomogeneity and flowing distribution of magnetized ions, and the Boltzmann distribution of electrons.

Let the electric field $\mathbf{E} \| \hat{x}$ produced due to the inhomogeneous ion distribution be transverse to the ambient magnetic field \mathbf{B}_0 in the plasma. Thus, the inhomogeneity of plasma ions in the *x* direction will give rise to a diamagnetic drift frequency to the ions in the *y* direction. In addition, the presence of the uniform electric field \mathbf{E}_0 responsible for the uniform ion drift and the magnetic field \mathbf{B}_0 causes the uniform ion streaming in the *y* direction and will cause a Doppler shift of the electrostatic drift wave frequency in the selfgravitating dusty plasma. However, the electrons will satisfy the Boltzmann distribution.

In the presence of low-frequency (in comparison with the electron gyrofrequency ω_{ce}) electrostatic waves with parallel (to \hat{z}) phase speed much smaller than the electron thermal speed v_{te} , hot electrons rapidly thermalize along \hat{z} direction and establish a Boltzmann distribution. The corresponding dielectric susceptibility is $1/k^2\lambda_{De}^2$, where k is the wavenumber and λ_{De} is the electron Debye radius.

Using the standard Vlasov-Poisson system of plasma equations for the Doppler-shifted ions, the ion susceptibility is given by 19,20

$$\chi_{i}(\boldsymbol{\omega}, \mathbf{k}) = \frac{1}{k^{2} \lambda_{\mathrm{D}i}^{2}} \left[1 + \frac{\boldsymbol{\omega}' - \boldsymbol{\omega}_{i}^{*}}{\sqrt{2} k_{\parallel} \boldsymbol{v}_{ti}} \right] \times \sum_{n} Z \left(\frac{\boldsymbol{\omega}' - n \boldsymbol{\omega}_{ci}}{\sqrt{2} k_{\parallel} \boldsymbol{v}_{ti}} \right) I_{n}(b_{i}) e^{-b_{i}} , \qquad (1)$$

where k_{\perp} (k_{\parallel}) is the component of **k** across (parallel) to $\hat{\mathbf{z}} \| \mathbf{B}_0, \ \omega_{pi} = (4\pi e^2 n_{i0}/m_i)^{1/2}$ is the ion plasma frequency, ω_{ci} $=eB_0/m_ic$ is the ion gyrofrequency, $\omega_i^* = -k_y v_{ii}^2/L_{ni}\omega_{ci}$ is the diamagnetic drift frequency of ions, k_v is the component of the wave vector **k** along the y axis, which is transverse to \hat{z} , $v_{i} = (T_i/m_i)^{1/2}$ is the thermal velocity of ions, $L_{ni}[=-n_{i0}(x)/n'_{i0}(x), n'_{i0}(x)=dn_{i0}(x)/dx]$ is the scalelength of the density inhomogeneity of the ions, $\omega' = \omega - \mathbf{k} \cdot \mathbf{u}_{i0} = \omega$ $-k_y u_{i0}$ with $u_{i0} = cE_0/B_0$ is the Doppler shifted frequency, and $\lambda_{\text{D}i} = (T_i/4\pi e^2 n_{i0})^{1/2} = v_{ti}/\omega_{pi}$ is the ion Debye length. In general, L_{ni} will be a function of x. However, for a particular choice of the ion density inhomogeneity in the x direction, viz., $n_{i0}(x) = n_{i0}^0 (1 \pm x/L_{ni})$, the scalelength of inhomogeneity is independent of x. Here, -e, n_{i0}^0 , m_i , c, and T_i are the electronic charge, equilibrium number density of ions, mass of an ion, light speed, and ion temperature, respectively. For $kv_{ti} \ll \omega \ll \omega_{ci}$, the ions have the diamagnetic drift and the susceptibility for strongly magnetized ions with inhomogeneous distribution is given by²⁰

$$\chi_i(\omega, \mathbf{k}) = \frac{k_\perp^2}{k^2} \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega' k^2 \lambda_{\mathrm{D}i}^2} - \frac{k_\parallel^2}{k^2} \frac{\omega_{pi}^2}{\omega'^2}.$$
 (2)

The dynamics of the cold dust grains with negligible dustneutral collision frequency in the presence of any electrostatic wave (ω, \mathbf{k}) , the electrostatic force, and gravitational force is governed by the dust momentum balance equation

$$\frac{\partial \mathbf{v}_d}{\partial t} = -\frac{q_d}{m_d} \, \nabla \, \phi - \nabla \psi, \tag{3}$$

the dust continuity equation

$$\frac{\partial n_d}{\partial t} + \boldsymbol{\nabla} \cdot (n_d \mathbf{v}_d) = 0, \qquad (4)$$

and the Poisson equation in terms of the gravitational potential ψ and the dust mass density $m_d n_d$

$$\nabla^2 \psi = 4\pi G m_d n_d,\tag{5}$$

where n_d and \mathbf{v}_d are the number density and the velocity of the dust grains, respectively, m_d is the mass of the dust grains, and G is the gravitational constant. Solving Eqs.

(3)–(5), we obtain the dust density perturbation for the cold and unmagnetized dust grains as

$$n_{d1} = -\frac{\chi_d k^2}{4\pi q_d}\phi,\tag{6}$$

where

$$\chi_d = -\frac{\omega_{pd}^2}{\omega^2 + \omega_{\mathrm{J}d}^2},\tag{7}$$

with $\omega_{\rm Jd} = \sqrt{4\pi G m_d n_{d0}}$, the Jeans frequency.

The dielectric function of the magnetized self-gravitating dusty plasma in the presence of an electrostatic dust mode is given by

$$\boldsymbol{\epsilon}(\omega, \mathbf{k}) = 1 + \frac{1}{k^2 \lambda_{\mathrm{D}e}^2} + \frac{k_{\perp}^2}{k^2} \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega' k^2 \lambda_{\mathrm{D}i}^2} - \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pi}^2}{\omega'^2} - \frac{\omega_{pd}^2}{\omega^2 + \omega_{\mathrm{J}d}^2},$$
(8)

where $\lambda_{De}^2 = T_e/4\pi n_{e0}e^2$. Equation (8) represents the mixed Jeans-dust-drift wave in the magnetized self-gravitating dusty plasma.

The electrostatic potential around an isolated test charged particulate in the presence of the electrostatic mode in a plasma is given by 21

$$\phi(\mathbf{x},t) = \frac{q_t}{2\pi^2} \int \frac{\delta(\boldsymbol{\omega} - \mathbf{k} \cdot \mathbf{v}_t)}{k^2 \epsilon(\mathbf{k}, \omega)} \exp[i\mathbf{k} \cdot \mathbf{r}] d\mathbf{k} \, d\omega, \qquad (9)$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, \mathbf{v}_t is the velocity vector of a test charged particulate, and q_t is its charge.

We now assume (i) $k_{\perp}^2 \ge k_{\parallel}^2$ to take into account the maximum effect of the magnetic field; (ii) $\omega \ll k_y u_{i0}$, ω_{ci} since we consider the low-frequency electrostatic wave involving the dust dynamics also; and (iii) we consider the presence of an ion beam with uniform speed \mathbf{u}_{i0} . Because of the diamagnetic drift, the drift wave will have $\mathbf{k} = \hat{y}k_y + \hat{z}k_z$, with $k_z \ll k_y$. Therefore, $\omega' = \omega - k_y u_{i0} \approx -k_y u_{i0}$ since $\omega \ll k_y u_{i0}$. We may then rewrite Eq. (8) as

$$\boldsymbol{\epsilon}(\boldsymbol{\omega}, \mathbf{k}) = \frac{1 + (1 + f_i k_\perp^2 / k^2) k^2 \lambda_{\rm D}^2}{k^2 \lambda_{\rm D}^2} \left[1 - \frac{\omega_k^2}{\omega^2 + \omega_{\rm Jd}^2} \right], \qquad (10)$$

where

$$\omega_k^2 = \frac{k^2 C_d^2}{1 + (1 + k_\perp^2 f_i / k^2) k^2 \lambda_D^2}, \quad C_d^2 = \omega_{pd}^2 \lambda_D^2, \quad f_i = \omega_{pi}^2 / \omega_{ci}^2,$$
(11)

and

$$\lambda_{\rm D}^2 = \frac{\lambda_{\rm De}^2}{1 - C_s^2 / u_{io} L_{ni} \omega_{ci}}, \quad C_s = \omega_{pi} \lambda_{\rm De}.$$
(12)

The inverse of the real part of the dielectric constant, i.e., $\epsilon(\omega, \mathbf{k})$, can be written as

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$$\frac{1}{\epsilon(\omega,\mathbf{k})} = \frac{k^2 \lambda_{\rm D}^2}{1 + (1 + f_i k_{\perp}^2 / k^2) k^2 \lambda_{\rm D}^2} \left[1 + \frac{\omega_k^2}{\omega^2 - (\omega_k^2 - \omega_{\rm Jd}^2)} \right].$$
(13)

Substituting Eq. (13) into Eq. (9) and following the standard mathematical techniques,^{22–24} we obtain the total electrostatic potential as

$$\Phi = \Phi_{\rm I} + \Phi_{\rm II},\tag{14}$$

where for finite v_t ,

$$\phi_{\rm I}(\rho,\xi) = \frac{q_t}{\sqrt{1+f_i}} \frac{\exp[-\sqrt{\rho^2 + \xi^2 (1+f_i)}/L_{\rm D}]}{\sqrt{\rho^2 + \xi^2 (1+f_i)}},$$
(15)

which is the strongly anisotropic Yukawa-type static Debye-Hückel screening potential²⁴ with the effective screening length $L_{\rm D}$,

$$L_{\rm D} \equiv \frac{\lambda_{\rm De} \sqrt{1 + f_i}}{\sqrt{1 - C_s^2 / u_{i0} L_{ni} \omega_{ci}}},\tag{16}$$

where $\xi = z - v_t t$. In Eq. (15), we have used (ρ, θ, z) as the cylindrical coordinates of **r**, where $r = (\rho^2 + z^2)^{1/2}$. This asymmetric modified shielding potential is a sensitive function of the ambient magnetic field, the scalelength of inhomogeneity, and the ion streaming velocity. If we neglect the inhomogeneity $(L_{ni} = \infty)$ in the dielectric function, we can retrieve the usual modified shielding potential.²⁴ This shielding potential, Eq. (15) is responsible for the generation of the wakefield, which we derive below.

The additional part of the potential involving the collective effects between the electrostatic wave and a test dust ion, after evaluating θ - and ω -integrations, is

$$\begin{split} \phi_{\mathrm{II}}(\rho,z,t) &= \frac{q_t \lambda_{\mathrm{D}}^2 C_d^2}{2 \,\pi^2} \int \frac{k^2 \delta(\omega - k_{\parallel} v_t) e^{i\mathbf{k} \cdot \mathbf{r}} d\mathbf{k} d\omega}{\{1 + (1 + k_{\perp}^2 f_t / k^2) k^2 \lambda_{\mathrm{D}}^2\} [(\omega^2 + \omega_{\mathrm{J}d}^2) \{1 + (1 + k_{\perp}^2 f_t / k^2) k^2 \lambda_{\mathrm{D}}^2\} - k^2 C_d^2]} \\ &= \frac{q_t M^{-2}}{\pi (1 + f_t)} \int \frac{k^2 J_0(k_{\perp} \rho) e^{ik_{\parallel} \xi} k_{\perp} dk_{\perp} dk_{\parallel}}{(k^2 + k_{\mathrm{D}}^2) [(1 + f_t) \lambda_{\mathrm{D}}^2 (k_{\parallel}^2 + \omega_{\mathrm{J}d}^2 / v_t^2) (k^2 + k_{\mathrm{D}}^2) - k^2 M^{-2}]}, \end{split}$$
(17)

where $\mathbf{v}_t \| \hat{z}$ is assumed, $k_{\rm D} = 1/\sqrt{1+f_i}\lambda_{\rm D}$, and $M = v_t/C_d$. Taking $k_{\perp}^2 \gg k_{\parallel}^2$ and introducing a dimensionless notation **K** = $\mathbf{k}\sqrt{1+f_i}\lambda_{\rm D}$. The above equation then reduces to

$$\Phi_{\rm II}(\rho,\xi) = \frac{q_i M^{-2}}{\pi \lambda_{\rm D} (1+f_i)^{3/2}} \int \frac{J_0(K_{\perp}\rho/\sqrt{1+f_i}\lambda_{\rm D})e^{iK_{\parallel}\xi/\sqrt{1+f_i}\lambda_{\rm D}}K^2 K_{\perp} dK_{\perp} dK_{\perp}}{(1+K^2)[(1+K^2)(K_{\parallel}^2+\alpha^2)-K^2 M^{-2}]},\tag{18}$$

where

$$\alpha = \frac{\omega_{\mathrm{J}d}}{v_t} \sqrt{1 + f_i} \lambda_{\mathrm{D}}.$$
(19)

The above equation can be recast as

$$\Phi_{\rm II}(\rho,\xi) = \frac{q_t M^{-2}}{\pi \lambda_{\rm D} (1+f_i)^{3/2}} \int \frac{J_0(K_{\perp}\rho/\sqrt{1+f_i}\lambda_{\rm D})e^{iK_{\parallel}\xi/\sqrt{1+f_i}\lambda_{\rm D}}K^2 K_{\perp} dK_{\perp} dK_{\perp} dK_{\parallel}}{(1+K^2)(K_{\parallel}^2+K_0^2)(K_{\parallel}^2-K_1^2)},\tag{20}$$

where

$$K_{0,1}^{2} = \pm \frac{\{1 + K_{\perp}^{2} + \alpha^{2} - M^{-2}\}}{2} + \sqrt{\frac{\{1 + K_{\perp}^{2} + \alpha^{2} - M^{-2}\}^{2}}{4} + [K_{\perp}^{2}M^{-2} - (1 + K_{\perp}^{2})\alpha^{2}]}.$$
(21)

It is noted here that the wake potential [Eq. (20)] involves the effects of plasma inhomogeneity (L_{ni}) , magnetic field (B_0) , ion streaming velocity (u_{i0}) , and self-gravitational effect (ω_{Jd}) via the definitions of λ_D and α in Eqs. (12) and (19), respectively. All these effects modify the strongly anisotropic Debye-Hückel shielding potential [Eq. (15)], which in turn affects the wake potential. Taking $K_{\perp} < 1$ with $M \ge 1$ and carrying out the K_{\parallel} -integration, we obtain

$$\frac{\Phi_{\rm II}(\rho',\xi')}{q_t/\lambda_{\rm D}} = -\frac{2M^{-2}}{(1+f_i)^{3/2}} \\ \times \int_0^\infty \frac{K_\perp^2 + K_1^2}{K_1} \frac{J_0(K_\perp\rho')\sin(K_1\xi')}{(K_0^2 + K_1^2)(1+K_1^2)} K_\perp dK_\perp,$$
(22)

where

$$K_{0,1}^{2} = \pm \frac{\{1 + \alpha^{2}\}}{2} + \sqrt{\frac{\{1 + \alpha^{2}\}^{2}}{4} + [K_{\perp}^{2}M^{-2} - \alpha^{2}]}, \quad (23)$$

where
$$\alpha^2 = (1+f_i)M^{-2}F$$
, $F = \omega_{Jd}^2 / \omega_{pd}^2$, $f_i = \omega_{pi}^2 / \omega_{ci}^2$, and

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$$\rho' = \frac{\rho}{\sqrt{1+f_i}\lambda_{\rm D}}, \quad \xi' = \frac{\xi}{\sqrt{1+f_i}\lambda_{\rm D}}, \tag{24}$$

where λ_D is given by Eq. (12).

The exact value of the wake potential could be calculated numerically from Eq. (22) with detailed dependence on the plasma inhomogeneity, magnetic field, ion streaming velocity, and the self-gravitational effects. However, we here study the wake potential explicitly at some important parameter regimes of the self-gravitating dusty plasma systems in three cases for ω_{Jd} approximately equal to, greater than, or less than ω_{pd} .

Case (i). For $K_{\perp} < 1$, $\omega_{Jd}^2 \simeq \omega_{pd}^2$, and $M \ge \omega_{pi}/\omega_{ci}$, we have $\alpha \ll 1$. At these conditions, the self-gravitational effect becomes insignificant. Then,

$$K_0^2 \simeq 1, \quad K_1^2 = K_\perp^2 M^{-2}.$$
 (25)

Under these conditions and for $\xi > M\lambda_D$, the oscillatory wake potential turns out to be

$$\Phi_{\rm II}(\rho=0,\xi) \simeq \frac{2q_t}{1+f_i} \frac{\cos(\xi/L_s)}{\xi},\tag{26}$$

where the effective attraction length is given by

$$L_s = \frac{M\sqrt{1 + f_i\lambda_{\mathrm{D}e}}}{\sqrt{1 - C_s^2/u_{io}L_{ni}\omega_{ci}}}.$$
(27)

Here, we note that the effective attraction length L_s increases with M and f_i . Moreover, the plasma inhomogeneity and ion streaming also play a vital role in L_s . The wake potential with real effective attraction length is possible for $u_{i0}L_{ni}\omega_{ci}$ $>C_s^2$. It is noticed from Eqs. (26) and (27) that the wake potential depends strongly on M, f_i , u_{i0} , L_{ni} , ω_{ci} , and the electron temperature.

Case (ii). For $\omega_{Id}^2 \ge \omega_{pd}^2$ and $f_i F > M^2$ with $f_i \ge 1$, we note $\alpha^2 \ge 1$. In this case, K_1 becomes imaginary and hence we do not get the wakefield generation, but only the Jeans instability.

Case (iii). For $\omega_{Jd}^2 \ll \omega_{pd}^2$, $K_{\perp} < 1$, and $f_i F \ll M^2$, we find $\alpha^2 \ll 1$. In this case also, we obtain the wake potential given by Eq. (26), which is again independent of the self-gravitational effect. One could find numerically the dependence of all parameters on the wake potential from Eq. (22).

We have investigated analytically the detailed properties of the strongly asymmetric electrostatic shielding potential and the consequent generation of the wake potential in a nonuniform self-gravitating dusty magnetoplasma in the presence of ion streaming. The potentials become sensitive functions of the static ambient magnetic field, the scalelength of inhomogeneity, and the transverse component of the ion streaming velocity. The scalelength of inhomogeneity can be positive as well as negative depending upon the decreasing or increasing ion density gradient. For a negative ion density gradient $(L_{ni} > 0)$, there is a limit of existence of the modified shielding potential. For the strong inhomogeneous selfgravitating dusty plasma, the effective attraction length of the dynamical oscillatory wake potential increases with increasing $M = v_t/C_d$ and $f_i = \omega_{pi}^2/\omega_{ci}^2$. The plasma inhomogeneity and ion streaming also play an important role on the effective attraction length. However, for $\omega_{Jd}^2 \leq \omega_{pd}^2$ and $M > \omega_{pi}/\omega_{ci}$, we note that the wake potential is given by Eq. (26) and is nearly independent of the self-gravitational effects. For $\omega_{Jd}^2 \geq \omega_{pd}^2$, the generation of the wakefield is not possible and the gravitational collapse of a self-gravitating plasma can be due solely to the Jeans instability.

Our wake potential model presented here provides a qualitative possibility of how the same polarity charged grains might form agglomerates as seeds for gravitational collapse of a self-gravitating dusty plasma. This dynamical wake potential model provides an alternative approach to the pure Jeans instability forming structures, such as stars and galaxies, etc., in self-gravitating dusty plasma systems.

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