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Acoustic-gravity waves in the Earth's ionosphere

T.D. Kaladze [a,b,*], O.A. Pokhotelov [c], H.A. Shah [a], M.I. Khan [a], L. Stenflo [d]

[a] Physics Department, GC University, Lahore 54000, Pakistan
[b] I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2 University Str., 0186 Tbilisi, Georgia
[c] Institute of Physics of the Earth, 10 B. Gruzinskaya Str., 123995 Moscow, Russia
[d] Department of Physics, Umeå University, SE-90 187 Umeå, and Department of Physics, Linkoping University, SE-58183 Linkoping, Sweden

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ABSTRACT

Taking into account the existence of charged particles in the Earth's ionosphere the propagation of acoustic-gravity waves is investigated. The influence of the Coriolis force is also taken into account. The weakly ionized ionospheric *D*, *E*, and *F*-layers are considered. The existence of a cut-off frequency at $2\Omega_0$ (Ω_0 is the value of the angular velocity of the Earth's rotation) is noted. It is shown that the linear waves are damped because of the Pedersen conductivity. When the acoustic-gravity waves are excited by external events (volcanic eruptions, earthquakes, lightning strikes, etc.) their amplitudes grow until self-organization of these waves into nonlinear vortex solitary structures is admitted. Taking into account the interaction of the induced ionospheric current with the geomagnetic field the governing nonlinear equations are deduced. The formation of dipole vortex solitary structures of low-frequency internal gravity waves is shown for the stable stratified ionosphere. The dynamic energy equation for such nonlinear structures is obtained. It is shown that nonlinear solitary vortical structures damp due to Joule losses.

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1. Introduction

Acoustic-gravity waves (AGWs) play an important role for the interpretation of a large number of physical phenomena in the troposphere, and for the dynamics of the ionospheric plasma (Hines, 1960; Hooke, 1968). Traditional methods for observing the ionospheric state have been improved, the models used for theoretical calculations of wave characteristics have become more complicated, better algorithms and programs for numerical calculations have been worked out, and new nonlinear effects for AGW radiation and propagation have been found. Another reason for the importance of AGW studies, related to their practical applications, is that energy and momentum fluxes transported by AGWs from the lower to the upper ionosphere are comparable or even larger than those coming from the solar wind or other sources (Francis, 1975; Ebel, 1984; Fritts et al., 1990; Kim and Mahrt, 1992; Alexander and Pfister, 1995). Theoretical and experimental studies have showed that AGW sources in the ionosphere can be earthquakes, volcanic eruptions, tornadoes, thunderstorms, solar eclipses, terminators, jet flows, polar and equatorial electrojets, meteors, strong explosions, and powerful rocket launches (Cole and Greifinger, 1969; Tolstoy and Lau, 1971; Gossard and Hooke, 1975; Richmond, 1978; Kato, 1980; Röttger, 1981; Fovell et al., 1992; Igarashi et al., 1994; Grigor'ev, 1999; Kanamori, 2004).

AGWs consist of relatively high-frequency acoustic and low-frequency internal gravity (IG) branches. AGWs have typical periods of $10^2\,\text{s} \leqslant \tau < 1$ day and are strongly affected by the Earth's gravitational field. Such waves

* Corresponding author at: I. Vekua Institute of Applied Mathematics, Tbilisi State University, 2 University Str., 0186 Tbilisi, Georgia.
Tel.: +995 32 375664.
E-mail address: tamaz_kaladze@yahoo.com (T.D. Kaladze).

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have typical wavelengths $\lambda \approx 10\,\mathrm{km}$ and propagation velocities $v_p \approx 30\,\mathrm{m/s}$. Various methods of measuring the parameters of the atmosphere indicate the presence of AGWs in a large range of heights extending from the troposphere to $z \leqslant 500\,\mathrm{km}$. A most complete survey of the observations of these waves in the troposphere has been provided by Gossard and Hooke (1975). The modern theory of small-amplitude AGWs in the Earth's atmosphere has been presented recently by Grigor'ev (1999).

The Earth's ionosphere consists of electrons, ions and neutral particles and is weakly ionized. Its behavior is determined on the whole by its massive neutral particles due to strong collisional coupling between the ionized and neutral components. The basis for such a statement is the condition $n/N \ll 1$, where n and N are the equilibrium number densities of the electrons and neutrals, respectively. The presence of charged particles causes however the medium to be electrically conducting. In addition the ionospheric plasma is immersed in the geomagnetic field $\mathbf{B}_0$ and under influence of the Coriolis force due to the rotation of the Earth with the angular velocity $\mathbf{\Omega}_0$. Therefore, the interaction of the inductive current with the geomagnetic field has also to be taken into account. For typical ionization fractions, the Ampere force will be comparable to the Coriolis force.

The propagation of AGWs under such conditions for the conductive ionosphere has not yet been studied properly. Hence in our investigation we must take into account the effects of the interaction of the induced ionospheric current with the geomagnetic field and the Earth's rotation, which are inherent to the ionosphere. A systematic investigation of the influence of the charged particles in the ionosphere was first made by Kaladze and Tsamalashvili (1997), Kaladze (1998, 1999) and Kaladze et al. (2004) to study nonlinear solitary vortical motions caused by planetary Rossby waves.

In the present paper we will focus our attention on the influence of the charged particles on the linear and nonlinear propagation peculiarities of AGWs in the weakly ionized conductive ionospheric D-, E-, and F-layers taking into account the effects of the Coriolis force as well.

Our paper is organized in the following fashion: In Section 2 the basic equations are formulated. The linear propagation of AGWs in the weakly ionized conductive ionosphere is investigated in Section 3. In Section 4 the reduced nonlinear equations for the IG waves accounting for the conductivity of the Earth's ionosphere are obtained. The evolution of nonlinear solitary vortex structures in the conductive ionosphere is investigated in Section 5. Our discussions and conclusions can be found in Section 6.

2. Basic equations

Let us introduce a local Cartesian system of coordinates (x, y, z) with the x-axis directed from the west to the east, the y-axis from the south to the north, and the z-axis along the local vertical. We are primarily interested in the dynamics at high latitudes in the northern hemisphere. Thus we assume that the geomagnetic field $\mathbf{B}_0 = -B_0\mathbf{e}_z$ is vertical and downward. Analogously we assume that the Earth's angular velocity at these latitudes has only a vertical component, i.e. $\mathbf{\Omega}_0 = (0, 0, \Omega_0)$. Furthermore, for the AGWs we consider $\mathbf{\Omega}_0$ and $\mathbf{B}_0$ to be uniform.

The dynamics of the electrically conducting ionospheric plasma can be described with the help of the momentum equation

$$\frac{d\mathbf{u}}{dt} \equiv \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla p}{\varrho} + \frac{\mathbf{j}\times\mathbf{B}_0}{\varrho} - 2\mathbf{\Omega}_0 \times \mathbf{u} + \mathbf{g}, \tag{1}$$

where $\mathbf{j}$ is the electric current, $\mathbf{u}$ is the bulk (neutral) velocity, p and ϱ are the pressure and mass density of the medium, and $\mathbf{g} = (0, 0, -g)$ is the gravitational acceleration.

The main purpose of the present study is to find the Ampere force $\mathbf{j}\times\mathbf{B}_0$, since it significantly determines the specific character of the ionospheric motions (especially at high altitudes). The most important impact of this force is on the inductive damping of the conductive medium (Cowling, 1976). For sufficiently large-scale motions we can in Eq. (1) neglect the viscous force relative to the Ampere force (Dokuchaev, 1959).

We consider the ionospheric plasma as quasi-neutral and neglect the inner electrostatic electric fields ($\mathbf{E} = -\nabla\varphi = 0$). Using the so-called noninductive approximation (Dokuchaev, 1959), it is sufficient to consider the currents arising in the gas whereas the vortex part of the self-generated electromagnetic field can be ignored. Assuming the ion and electron pressures to be small as compared with that of the neutrals one finds that the effective electric field in the generalized Ohm's law is equal to the dynamo field, i.e.,

$$\mathbf{j} = \sigma_\parallel \mathbf{E}_{d\parallel} + \sigma_\perp \mathbf{E}_{d\perp} + \frac{\sigma_H}{B_0}\mathbf{B}_0 \times \mathbf{E}_d, \tag{2}$$

where

$$\sigma_\parallel = \frac{ne^2}{m_e \nu_e}, \tag{3}$$

$$\sigma_p = \sigma_\perp = \frac{ne^2 \nu_{in}(\nu_e \nu_{in} + \omega_{ce}\omega_{ci})}{m_e(\omega_{ce}^2 \nu_{in}^2 + \nu_e^2 \nu_{in}^2 + \omega_{ce}^2 \omega_{ci}^2)} \tag{4}$$

and

$$\sigma_H = \frac{ne^2 \nu_{in}^2 \omega_{ce}}{m_e(\omega_{ce}^2 \nu_{in}^2 + \nu_e^2 \nu_{in}^2 + \omega_{ce}^2 \omega_{ci}^2)}. \tag{5}$$

In Eqs. (2)–(5) $\sigma_\parallel$, $\sigma_\perp$, and σ_H are the parallel, perpendicular (Pedersen), and Hall conductivities, respectively. The subscripts $\parallel$ and $\perp$ denote in Eq. (2) the components parallel and perpendicular to the external magnetic field. The quantities ν_{en} and ν_{in} are the effective collisional frequencies of the neutral particles with the electrons and ions, e is the electron charge, $\nu_e = \nu_{ei} + \nu_{en}$, $\omega_{ce} = eB_0/m_e$ and $\omega_{ci} = ZeB_0/m_i$ are the cyclotron frequencies of electrons and ions, m_e and m_i are the electron and ion masses, and Z is the ion charge state. The electric field $\mathbf{E}_d = \mathbf{u}\times\mathbf{B}_0$ stands for the so-called dynamo-field.

Eq. (1) should be supplemented by the continuity equation and the equation of state, i.e.

$$\frac{\partial \varrho}{\partial t} + (\mathbf{u}\cdot\nabla)\varrho + \varrho\nabla\cdot\mathbf{u} = 0, \tag{6}$$

$$\frac{\partial p}{\partial t} + (\mathbf{u}\cdot\nabla)p - c_s^2\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\varrho = 0, \tag{7}$$

where c_s is the sound speed.

Eqs. (1)–(7) constitute a full set of equations for describing the dynamics of AGWs in the weakly ionized ionosphere. The background pressure and mass density are stratified by the gravitational field. In an isothermal ionosphere they vary as $[p_0(z), \varrho_0(z)] = [p_0(0), \varrho(0)]\exp(-z/H)$, where $H = c_s^2/\gamma g$ stands for the reduced atmospheric height, and γ is the ratio of specific heats.

3. Linear propagation

Linearizing Eqs. (1)–(7) one obtains

$$\varrho_0\frac{\partial \mathbf{u}}{\partial t} + \nabla p - \mathbf{j}\times\mathbf{B}_0 + 2\varrho_0\mathbf{\Omega}_0\times\mathbf{u} - \varrho\mathbf{g} = 0, \tag{8}$$

$$\frac{\partial \varrho}{\partial t} + \mathbf{u}\cdot\nabla\varrho_0 + \varrho_0\nabla\cdot\mathbf{u} = 0, \tag{9}$$

$$\frac{\partial p}{\partial t} + \mathbf{u}\cdot\nabla p_0 - c_s^2\left(\frac{\partial \varrho}{\partial t} + \mathbf{u}\cdot\nabla\varrho_0\right) = 0, \tag{10}$$

which can be used to consider AGWs in different ionospheric layers.

The D-layer: The ionospheric *D*-layer is in the region between 50 and 80 km from the Earth's surface. One can there assume the frequency hierarchy (e.g., Kaladze and Tsamalashvili, 1997) $\nu_{en} \gg \nu_{ei}$, $\nu_{in}\nu_{en} \gg \omega_{ce}\omega_{ci}$, $\nu_{in} \gg \omega_{ci}$ and $\omega_{ce} \gg \nu_{en}$. Then, using Eqs. (3)–(5), we have the reduced conductivities

$$\sigma_\parallel = \frac{ne^2}{m_e\nu_{en}}, \quad \sigma_p = \sigma_\perp = \frac{\nu_{en}^2}{\omega_{ce}^2}\sigma_\parallel$$

and

$$\sigma_{\mathrm{H}} = \frac{\nu_{en}}{\omega_{ce}}\sigma_\parallel. \tag{11}$$

For numerical estimates we use the typical values $\Omega_0 = 7.3\times10^{-5}\,\mathrm{s}^{-1}$, $B_0 = 0.5\times10^{-4}\,\mathrm{T}$, $n/N \sim 10^{-12} - 10^{-8}$, $\nu_{ei} \sim 10^2\,\mathrm{s}^{-1}$, $\nu_{en} \sim 10^6\,\mathrm{s}^{-1}$, $\nu_{in} \sim 10^5\,\mathrm{s}^{-1}$, $\omega_{ce} \sim 10^7\,\mathrm{s}^{-1}$ and $\omega_{ci} \sim 3\times10^2\,\mathrm{s}^{-1}$. One then finds that the terms containing σ_{H} and $\sigma_\perp$ in Eq. (8) (see Eq. (2)) are negligibly small relative to the term with Ω_0. Thus, in the momentum equation (8) at the *D*-layer heights one can neglect the contribution of the Ampere force $\mathbf{j}\times\mathbf{B}_0$, i.e.

$$\varrho_0\frac{\partial \mathbf{u}}{\partial t} + \nabla p + 2\varrho_0\mathbf{\Omega}_0\times\mathbf{u} - \varrho\mathbf{g} = 0. \tag{12}$$

Combining Eqs. (9), (10) and (12) we find

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} + g\nabla u_z - c_s^2\nabla(\nabla\cdot\mathbf{u}) + 2\mathbf{\Omega}_0\times\frac{\partial \mathbf{u}}{\partial t} + (1-\gamma)\mathbf{g}\nabla\cdot\mathbf{u} = 0, \tag{13}$$

where $\mathbf{u} = (u_x, u_y, u_z)$, i.e. all velocity components have been taken into account.

Standard calculations for AGWs in a stratified atmosphere (cf. Gershman, 1974) then lead to the dispersion equation

$$\omega^4 - \omega^2 c_s^2\left(k^2 + \frac{1}{4H^2} + 4\frac{\Omega_0^2}{c_s^2}\right) + g^2(\gamma-1)k_\perp^2 + 4\Omega_0^2 c_s^2\left(k_z^2 + \frac{1}{4H^2}\right) = 0, \tag{14}$$

where $k^2 = k_x^2 + k_y^2 + k_z^2 = k_\perp^2 + k_z^2$.

Eq. (14) includes the influence of the Coriolis force and all the three components of the wave vector $\mathbf{k}$. In this sense it is a general equation for a neutral rotating fluid. As is seen from Eq. (8), when the equilibrium density profile ϱ_0 is exponential, amplitudes of the velocity components increase with height, but that the pressure and density decrease, i.e.

$$(u_x, u_y, u_z) \propto \exp\left(\frac{z}{2H}\right)$$

and

$$(\varrho, p) \propto \exp\left(-\frac{z}{2H}\right). \tag{15}$$

Our solutions for the Fourier components are not uniform with regard to the variable z. For a fixed ω and fixed propagation direction defined by the ratio $k_\perp/k_z$ Eq. (14) has a unique solution. But we have two frequency branches, as the solution of Eq. (14) is

$$\omega^2 = \frac{1}{2}c_s^2\left(k^2 + \frac{1}{4H^2} + 4\frac{\Omega_0^2}{c_s^2}\right) \pm \frac{1}{2}c_s^2\left\{\left(k^2 + \frac{1}{4H^2} + 4\frac{\Omega_0^2}{c_s^2}\right)^2 - 4\left[\frac{g^2(\gamma-1)k_\perp^2}{c_s^4} + 4\frac{\Omega_0^2}{c_s^2}\left(k_z^2 + \frac{1}{4H^2}\right)\right]\right\}^{1/2}. \tag{16}$$

One sees that the incorporation of the Coriolis force leads to the coupling of the AGWs and the inertial waves which results in the appearance of so-called inertio-acoustic-gravity (IAG) waves (cf. Kaladze et al., 2007).

Let us now consider the special cases of waves propagating along the vertical and horizontal directions.

When $k_\perp = 0$ (vertical propagation) the acoustic waves and the inertial waves are decoupled, i.e. Eq. (16) reduces to the two independent dispersion relations

$$\omega_1^2 = c_s^2\left(k_z^2 + \frac{1}{4H^2}\right) = \omega_a^2 + k_z^2 c_s^2 \tag{17}$$

and

$$\omega_2^2 = 4\Omega_0^2, \tag{18}$$

where $\omega_a = c_s/2H$ stands for the acoustic cut-off frequency.

The acoustic waves described by the dispersion relation (17) are always supersonic, i.e. their phase velocity $v_p = \omega/k_z$ exceeds c_s.

In the other limiting case $k_z = 0$ (horizontal propagation) one obtains from Eq. (14)

$$k_\perp^2 = \frac{(\omega^2 - \omega_a^2)(\omega^2 - \omega_i^2)}{c_s^2(\omega^2 - \omega_g^2)}, \tag{19}$$

where ω_g is Brunt-Väisälä frequency given by

$$\omega_g^2 = \frac{(\gamma - 1)g^2}{c_s^2} = \frac{(\gamma - 1)g}{\gamma H}, \tag{20}$$

and $\omega_i = 2\Omega_0$ stands for the inertial cut-off frequency. One sees that $\omega_a > \omega_g > \Omega_0$.

The curves, that bound the shaded regions in Fig. 1, characterize the dependence $\omega(k_\perp)$ for the horizontal propagation directions, when $k_z = 0$ (see Eq. (19)). The mode $\omega > \omega_a$ corresponds to acoustic type waves and the mode for which $\omega_i < \omega < \omega_g$ corresponds to gravitational type waves. When $\omega_g < \omega < \omega_a$ and $\omega < \omega_i$ the propagation of traveling waves becomes impossible. The shaded regions I and II in Fig. 1 define the zones where the propagation of gravitational and acoustic atmospheric waves is possible. There the conditions $k_\perp^2 > 0$ and $k_z^2 > 0$ are satisfied.

Eq. (14) may be re-written similar to Eq. (19), i.e.

$$k_\perp^2 = \frac{(\omega^2 - \omega_a^2 - k_z^2 c_s^2)(\omega^2 - \omega_i^2)}{c_s^2(\omega^2 - \omega_g^2)}. \tag{21}$$

If $\omega_g < \omega < \omega_a$ or $\omega < \omega_i$ then the condition $k_\perp^2 > 0$ is satisfied only when $k_z^2 < 0$. Thus, we come to the conclusion on the presence of forbidden zones for wave propagation. When $k_z \to 0$ these zones occupy the frequency intervals $(0, \omega_i)$ and (ω_g, ω_a). For $k_z^2 > 0$ the lower interval remains the same as for $k_z^2 = 0$. However, the upper limit is defined by a frequency ω_a^* larger than ω_a. It is given by

$$\omega_a^* = \sqrt{\omega_a^2 + k_z^2 c_s^2}. \tag{22}$$

Propagation of supersonic (acoustic) waves is possible if $\omega > \omega_a^*$. Such waves have sufficiently short periods. For subsonic (gravitational) waves, according to the condition

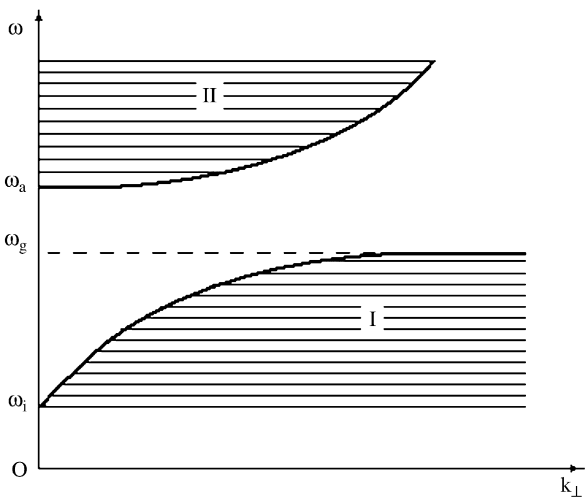

Fig. 1. A plot of the possible propagation regions for AGWs.

$\omega_i < \omega < \omega_g$, we obtain the following limitation on the period τ of the gravitational waves

$$\frac{\pi}{\Omega_0} = \tau_i > \tau > \tau_g = 2\pi\left(\frac{\gamma H}{(\gamma - 1)g}\right)^{1/2}, \tag{23}$$

where $\tau = 2\pi/\omega$.

The relations (14) and (21) can be substantially simplified if $k_z^2 \gg k_\perp^2$. For the gravitational type waves this is equivalent to the limitation

$$\omega^2 \ll \omega_g^2. \tag{24}$$

Using the condition (24), we obtain from Eq. (14) the dispersion relation (cf. Stenflo, 1991)

$$\omega^2 = \frac{k_\perp^2 \omega_g^2}{k_z^2 + 1/4H^2} + \omega_i^2, \tag{25}$$

that describes so-called inertio-gravity waves.

From Eq. (14) we can obtain the phase velocity $\mathbf{v}_p = \omega \mathbf{k}/k^2$ and group velocity $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$. The x-component of the phase velocity v_{px} characterizes the motion of the phase front in the azimuthal direction. Such formulae are in general quite complex. Thus, we limit our study to low frequencies. Taking derivatives of expression (25) we thus find the group velocity components

$$v_{gx,y} = \frac{\partial\omega}{\partial k_{x,y}} = \frac{\omega_g^2 k_{x,y}}{\omega(k_z^2 + 1/4H^2)}, \tag{26}$$

$$v_{gz} = \frac{\partial\omega}{\partial k_z} = \frac{\omega_g^2 k_\perp^2 k_z}{\omega(k_z^2 + 1/4H^2)^2}. \tag{27}$$

We note that for the region of applicability of these equations ($k_z^2 \gg k_\perp^2$) we have $|v_{gx,y}| \gg |v_{gz}|$. In addition owing to the existence of Ω_0 we have $v_{gx,y} \to 0$, when $k_{x,y} \to 0$ and $v_{gz} \to 0$, when $k_\perp \to 0$ or $k_z \to 0$.

For the phase velocities we have

$$v_{px,y} = \frac{\omega k_{x,y}}{k_z^2}, \quad v_{pz} = \frac{\omega}{k_z}. \tag{28}$$

I.e., when $k_\perp^2 \ll k_z^2$ we have $|v_{px,y}| \ll |v_{pz}|$.

The group velocities (26) and (27) are smaller than the corresponding values without Ω_0. But the phase velocities (28) are larger in the same comparison. Thus, incorporation of the Earth's rotation provides a noticeable change in the propagation dynamics of low-frequency AGWs.

The E-layer: Let us now consider the ionospheric E-layer that is situated at heights 100–150 km above the Earth's surface. The plasma conditions in this region ($v_e \approx v_{en}, v_{in} v_{en} \ll \omega_{ce}\omega_{ci}$ and $v_{in} \gg \omega_{ci}$) allow us to simplify the expression for the inductive electric current. First, since $\omega_{ci} \ll v_{in}$ the ions can be considered as unmagnetized. It is well-known that the ion velocity across the magnetic field coincides with the wind velocity (Kaladze et al., 2003, 2004), i.e. $\mathbf{v}_i = \mathbf{v}$ and thus the ions are completely dragged by the ionospheric winds. In this limiting case the Hall conductivity is $\sigma_H \approx en/B_0$, whereas the Pedersen conductivity is small, i.e., $\sigma_p \approx \sigma_H \omega_{ci}/v_{in} \ll \sigma_H$ (see Eqs. (4) and (5)). This allows us to neglect the ion friction caused by the Pedersen conductivity in the E-layer. However, the electrons are magnetized, $\omega_{ce} \gg \nu_{en}$, and thus they are frozen in the external magnetic field. For a numerical estimate we use the typical values $n/N \sim 10^{-8}-10^{-6}$, $\nu_{ei} \sim 10^3\,\mathrm{s}^{-1}$, $\nu_{en} \sim 10^4\,\mathrm{s}^{-1}$, $\nu_{in} \sim 10^3\,\mathrm{s}^{-1}$, $\omega_{ce} \sim 10^7\,\mathrm{s}^{-1}$, $\omega_{ci} \sim 3 \times 10^2\,\mathrm{s}^{-1}$, and $\sigma_{\mathrm{H}} \approx 3\times\ 10^{-4}\,\mathrm{S/m}$. Thus, according to Eqs. (2) and (8) for the E-region the appropriate momentum equation is

$$\varrho_0 \frac{\partial \mathbf{u}}{\partial t} + \nabla p + 2\varrho_0 \mathbf{\Omega}_0 \times \mathbf{u} + \sigma_{\mathrm{H}} B_0 \mathbf{B}_0 \times \mathbf{u} - \varrho \mathbf{g} = 0. \tag{29}$$

Analogously to Eq. (13) we have

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} + g\nabla u_z - c_s^2 \nabla(\nabla \cdot \mathbf{u}) + 2\mathbf{\Omega}_0 \times \frac{\partial \mathbf{u}}{\partial t} + \frac{en}{\varrho_0}\mathbf{B}_0 \times \frac{\partial \mathbf{u}}{\partial t} + (1-\gamma)\mathbf{g}\nabla \cdot \mathbf{u} = 0. \tag{30}$$

Eq. (30) shows that the effect of the geomagnetic field corresponds to a replacement of the planetary angular rotation vector according to

$$2\mathbf{\Omega}_0 \to 2\mathbf{\Omega}_0 + \frac{en}{\varrho_0}\mathbf{B}_0. \tag{31}$$

The ratio of the magnetic term to the Coriolis term depends on the degree of ionization $n/N = mn/\varrho_0$. For the E-layer the value $enB_0/\varrho_0 \approx (n/N)\omega_{ci} \approx 10^{-4}\,\mathrm{s}^{-1}$ is comparable to $2\Omega_0 \sim 10^{-4}\,\mathrm{rad/s}$. In addition we consider the ratio n/ϱ_0 to be independent on height z (Gershman, 1974).

Thus, we conclude that all results in the previous section remain the same if

$$\omega_i \longrightarrow \omega_i - \frac{enB_0}{\varrho_0}. \tag{32}$$

The F-layer: Let us finally consider the ionospheric F-layer (150–400 km above the Earth's surface). In this region $\omega_{ce}\omega_{ci} \gg \nu_e \nu_{in}$ and $\omega_{ci} \gg \nu_{in}$. From Eqs. (3)–(5) we then have, respectively,

$$\sigma_\parallel \approx \frac{ne^2}{m_e \nu_e}, \quad \sigma_\perp = \sigma_p \approx \frac{\nu_e \nu_{in}}{\omega_{ci}\omega_{ce}} \sigma_\parallel$$

and

$$\sigma_{\mathrm{H}} \approx \frac{\nu_{in}}{\omega_{ci}} \sigma_p. \tag{33}$$

With typical numerical values we have $n/N \sim 10^{-5}-10^{-3}$, $\nu_e \sim \nu_{ei} \sim 10^3\,\mathrm{s}^{-1}$, $\nu_{en} \sim 10^2\,\mathrm{s}^{-1}$, $\nu_{in} \leqslant 10\,\mathrm{s}^{-1}$, $\omega_{ce} \sim 10^7\,\mathrm{s}^{-1}$ and $\omega_{ci} \sim 3 \times 10^2\,\mathrm{s}^{-1}$. Thus, in this region the contribution of Hall conductivity in Eq. (2) can be neglected. As a result we have

$$\mathbf{j} \times \mathbf{B}_0 = -\sigma_p B_0^2 \mathbf{u}_\perp. \tag{34}$$

Thus, similar to Eq. (8) we have the equation

$$\varrho_0 \frac{\partial \mathbf{u}}{\partial t} + \nabla p + \sigma_p B_0^2 \mathbf{u}_\perp + 2\varrho_0 \mathbf{\Omega}_0 \times \mathbf{u} - \varrho \mathbf{g} = 0. \tag{35}$$

For the ratio of the magnetic term to the Coriolis term we have $\sigma_p B_0^2/\varrho_0 = \nu_{in}(n/N) \sim 10^{-3}$ and we can thus neglect the influence of the Coriolis force in Eq. (35). At these heights the geostrophic character of the motions is finally lost and the motion remains the same as in a viscous medium, where the viscosity appears in the inductive (magnetic) inhibition (Cowling, 1976). Thus, in the F-layer, analogously to Eqs. (13) and (30), we have

$$\frac{\partial^2 \mathbf{u}}{\partial t^2} + g\nabla u_z - c_s^2 \nabla(\nabla \cdot \mathbf{u}) + \frac{\sigma_p B_0^2}{\varrho_0} \frac{\partial \mathbf{u}_\perp}{\partial t} + (1-\gamma)\mathbf{g}\nabla \cdot \mathbf{u} = 0. \tag{36}$$

As in previous sections we can then obtain the dispersion equation

$$F(\omega) = \left[\omega^4 - \omega^2 c_s^2 \left(k^2 + \frac{1}{4H^2}\right) + k_\perp^2(\gamma - 1)g^2\right] \times \left(\omega + \mathrm{i}\frac{\sigma_p B_0^2}{\varrho_0}\right) + \omega\left(\frac{\sigma_p B_0^2}{\varrho_0}\right)^2 \times \left[c_s^2\left(k_z^2 + \frac{1}{4H^2}\right) - \omega^2\right] = 0. \tag{37}$$

When obtaining this dispersion equation we have supposed that the ratio $\sigma_p B_0^2/\varrho_0 = \nu_{in}(n/N)$ is conserved and that it does not depend on height, i.e. on the z-coordinate (Gershman, 1974). Decomposing the frequency into its real and imaginary parts $\omega = \omega_0 + \mathrm{i}\Gamma$ (with $|\Gamma| \ll |\omega_0|$), we obtain from Eq. (37)

$$\mathrm{i}\Gamma \frac{\partial F}{\partial \omega}\bigg|_{\omega=\omega_0} = -F(\omega_0), \tag{38}$$

where ω_0 is the AGW eigenfrequency when $\Omega_0 = 0$ (see Eq. (14)) and satisfies the equation

$$\omega_0^4 - \omega_0^2 c_s^2 (k^2 + 1/4H^2) + k_\perp^2(\gamma - 1)g^2 = 0. \tag{39}$$

The corresponding zones for possible propagation are shown in Fig. 1 for $\omega_i = 0$. From Eq. (38) we then find

$$\Gamma = -\frac{1}{4}\frac{\sigma_p B_0^2}{\varrho_0} \frac{\omega_0^2 - c_s^2(k_z^2 + 1/4H^2)}{\omega_0^2 - c_s^2(k^2 + 1/4H^2)}. \tag{40}$$

With eigenfrequencies ω_0 given by Eq. (39), this expression is always negative and defines the damping rate of AGWs in the ionospheric F-layer. The magnitude of Eq. (40) is

$$|\Gamma| \sim \frac{\sigma_p B_0^2}{\varrho_0}. \tag{41}$$

4. Nonlinear propagation. Basic equations for IG waves

The nonlinear properties of small amplitude AGWs have been considered by many authors (e.g., Yeh and Liu, 1981; Miropol'sky, 1981; Weinstock, 1984). Reduced nonlinear equations describing the dynamics of propagation of AGW solitary structures have also been obtained by Stenflo (1987, 1990, 1996). Although such nonlinear equations with vector nonlinearities are useful in the theory of neutral atmospheric motion, they have to be improved for the conductive ionospheric motion because they do not take into account the influence of electromagnetic forces. Investigations of the charged particle influence that makes the atmosphere electrically conducting have therefore been carried out to study solitary vortical motions caused by Rossby waves (e.g., Kaladze and Tsamalashvili, 1997; Kaladze, 1998, 1999; Kaladze et al., 2004).

In what follows we will obtain reduced nonlinear equations for the IG waves taking into account the conductivity of the Earth's ionosphere.

It is common knowledge that the density variations due to the IG waves do not exceed 3–4% (Miropol'sky, 1981). In this case the ratio of the density perturbations to the unperturbed density is $\tilde{\varrho}/\varrho_0 \simeq (1-4)\times 10^{-2}$. In the momentum equation (1) we can therefore neglect $\tilde{\varrho}$ as compared with $\varrho_0(z)$ for the inertial and Coriolis force terms and use the equation

$$\varrho_0(z)\left(\frac{d\mathbf{u}}{dt} + 2\mathbf{\Omega}_0\times\mathbf{u}\right) = -\nabla p + \varrho\mathbf{g} + \sigma_H B_0 \mathbf{u}\times\mathbf{B}_0 - \sigma_P B_0^2 \mathbf{u}_\perp. \quad (42)$$

This is known as the Boussinesq approximation. The other equation concerns the total density ($\varrho = \varrho_0 + \tilde{\varrho}$), i.e.

$$\frac{d\varrho}{dt} = 0, \quad (43)$$

where $d/dt = \partial/\partial t + \mathbf{u}\cdot\nabla$.

To exclude the high-frequency acoustic mode we make use of the incompressibility condition

$$\nabla\cdot\mathbf{u} = 0. \quad (44)$$

Eqs. (42)–(44) constitute a full set of equations necessary for studies of vortex motions for low-frequency incompressible IG waves. We next consider two-dimensional motion in the $(x-z)$ plane, assuming $\partial/\partial y = 0$, $\mathbf{u} = (u, 0, w)$, $\mathbf{\Omega}_0 = (0, 0, \Omega_0)$ and $\mathbf{B}_0 = (0, 0, -B_0)$. The coordinate system is the same as in previous sections. From Eq. (42) one then obtains the equations

$$\varrho_0(z)\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} - \sigma_P B_0^2 u, \quad (45)$$

$$\varrho_0(z)\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} - \varrho g. \quad (46)$$

One sees that for the latitudes under consideration the IG waves are not influenced by the Coriolis force and Hall conductivity. After differentiating Eq. (45) with respect to z and Eq. (46) with respect to x, and using Eq. (44) one obtains

$$\varrho_0\left(\frac{\partial\zeta}{\partial t} + u\frac{\partial\zeta}{\partial x} + w\frac{\partial\zeta}{\partial z}\right) + \frac{\partial\varrho_0}{\partial z}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = g\frac{\partial\varrho}{\partial x} - u\frac{\partial}{\partial z}(\sigma_P B_0^2) - \sigma_P B_0^2\frac{\partial u}{\partial z}, \quad (47)$$

where the y-component of the vorticity ζ is

$$\zeta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}. \quad (48)$$

Condition (44) allows us to introduce the stream function ψ from

$$u = -\frac{\partial\psi}{\partial z}, \quad w = \frac{\partial\psi}{\partial x}. \quad (49)$$

The vorticity (48) can then be written as $\zeta = -(\partial^2/\partial x^2 + \partial^2/\partial z^2)\psi = -\nabla^2\psi$. Thus, our basic equations (43) and (47) in terms of ψ are

$$\varrho_0\left[\frac{\partial}{\partial t}\nabla^2\psi + J(\psi,\nabla^2\psi)\right] = -g\frac{\partial\tilde{\varrho}}{\partial x} - \frac{d\varrho_0}{dz}\left[\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial z}\right) + J\left(\psi,\frac{\partial\psi}{\partial z}\right)\right] - \frac{\partial}{\partial z}(\sigma_P B_0^2)\frac{\partial\psi}{\partial z} - (\sigma_P B_0^2)\frac{\partial^2\psi}{\partial z^2}, \quad (50)$$

$$\frac{\partial\tilde{\varrho}}{\partial t} + \frac{\partial\psi}{\partial x}\frac{d\varrho_0}{dz} + J(\psi,\tilde{\varrho}) = 0, \quad (51)$$

where $J(a,b) = (\partial a/\partial x)(\partial b/\partial z) - (\partial a/\partial z)(\partial b/\partial x)$ denotes the Jacobian. As in the previous section we look for the solutions of Eqs. (50) and (51) in the form

$$\psi = e^{z/2H}\bar{\psi}, \quad \tilde{\varrho} = e^{-z/2H}\bar{\varrho}. \quad (52)$$

From Eqs. (50) and (51) one obtains

$$\frac{\partial\nabla^2\bar{\psi}}{\partial t} - \frac{1}{4H^2}\frac{\partial\bar{\psi}}{\partial t} + e^{z/2H}\times\left[\frac{1}{2H}\nabla^2\bar{\psi}\frac{\partial\bar{\psi}}{\partial x} - \frac{1}{2H}\bar{\psi}\frac{\partial\triangle\bar{\psi}}{\partial x} + J(\bar{\psi},\nabla^2\bar{\psi})\right] = -e^{-z/2H}\frac{g}{\varrho_0}\frac{\partial\bar{\varrho}}{\partial x} - \frac{\sigma_P B_0^2}{\varrho_0}\times\left(\frac{\bar{\psi}}{4H^2} + \frac{1}{H}\frac{\partial\bar{\psi}}{\partial z} + \frac{\partial^2\bar{\psi}}{\partial z^2}\right) - \frac{1}{\varrho_0}\frac{\partial}{\partial z}(\sigma_P B_0^2)\left(\frac{\bar{\psi}}{2H} + \frac{\partial\bar{\psi}}{\partial z}\right), \quad (53)$$

$$\frac{\partial\bar{\varrho}}{\partial t} + e^{z/2H}\left[J(\bar{\psi},\bar{\varrho}) - \frac{1}{2H}\bar{\varrho}\frac{\partial\bar{\psi}}{\partial x} - \frac{1}{2H}\bar{\psi}\frac{\partial\bar{\varrho}}{\partial x}\right] = e^{z/H}\frac{\varrho_0}{H}\frac{\partial\bar{\psi}}{\partial x}. \quad (54)$$

For the nonlinear terms it is reasonable to assume that $\exp(z/2H)\approx 1$, i.e. $k_z \gg 1/2H$ (the sufficiently short wavelength limit). That means that the Jacobian is more essential than the other terms. Thus, we obtain instead of Eqs. (53) and (54)

$$\frac{\partial}{\partial t}\left(\nabla^2\bar{\psi} - \frac{1}{4H^2}\bar{\psi}\right) + J(\bar{\psi},\nabla^2\bar{\psi}) = -e^{-z/2H}\frac{g}{\varrho_0}\frac{\partial\bar{\varrho}}{\partial x} - \frac{\sigma_P B_0^2}{\varrho_0}\left(\frac{\bar{\psi}}{4H^2} + \frac{1}{H}\frac{\partial\bar{\psi}}{\partial z} + \frac{\partial^2\bar{\psi}}{\partial z^2}\right) - \frac{1}{\varrho_0}\frac{\partial}{\partial z}(\sigma_P B_0^2)\left(\frac{\bar{\psi}}{2H} + \frac{\partial\bar{\psi}}{\partial z}\right) \quad (55)$$

and

$$\frac{\partial\bar{\varrho}}{\partial t} + J(\bar{\psi},\bar{\varrho}) = e^{z/H}\frac{\varrho_0}{H}\frac{\partial\bar{\psi}}{\partial x}. \quad (56)$$

Furthermore we use here the $\varrho_0(z) = \varrho_0(0)\exp(-z/H)$ distribution and introduce the new variable $\chi = g\bar{\varrho}/\varrho_0(0)$. Assuming that $\sigma_P B_0^2/\varrho_0$ is constant along the z-axis

(Gershman, 1974), we obtain

$$\frac{\partial}{\partial t}\left(\nabla^2\bar{\psi} - \frac{1}{4H^2}\bar{\psi}\right) + J(\bar{\psi}, \nabla^2\bar{\psi}) = -\frac{\partial\chi}{\partial x} - \frac{\sigma_p B_0^2}{\varrho_0}\left(\frac{\partial^2\bar{\psi}}{\partial z^2} - \frac{1}{4H^2}\bar{\psi}\right), \quad (57)$$

$$\frac{\partial\chi}{\partial t} + J(\bar{\psi}, \chi) = \omega_g^2\frac{\partial\bar{\psi}}{\partial x}. \quad (58)$$

Here $\omega_g = (g/H)^{1/2}$ is the Brunt-Väisälä frequency for the incompressible fluid (cf. with Eq. (20)).

The system of Eqs. (57) and (58) describes the dynamics of nonlinear solitary vortices for low-frequency IG waves in our stable stratified ($\omega_g^2 > 0$) ionosphere for sufficiently high latitudes. For the neutral *D*-layer ($\sigma_p = 0$) in the linear regime we obtain the dispersion relation for the IG waves

$$\omega^2 = \frac{k_x^2\omega_g^2}{k_x^2 + k_z^2 + 1/4H^2}, \quad (59)$$

propagating along the *x*-axis (the parallels).

For the upper *E*- and *F*-layers of the ionosphere we find from Eqs. (57) and (58) that these waves decay with the damping rate

$$\Gamma = -\frac{1}{2}\frac{\sigma_p B_0^2}{\varrho_0}\frac{k_z^2 + 1/4H^2}{k_x^2 + k_z^2 + 1/4H^2}, \quad (60)$$

which by the order of magnitude is the same as (40).

5. Evolution of solitary vortex structures

The nonlinear behavior of the low-frequency acoustic-gravity perturbations is dominated by the presence of the convective derivative, and the corresponding vector-product nonlinearity can thus produce various coherent localized vortex structures for a broad range of background configurations. The forms of such vortices are strongly dependent on the spatial profile of the unperturbed medium. In a quiescent atmosphere with exponential density and pressure profiles, the standard traveling dipolar vortices (also called acoustic-gravity modons) have been found with the transverse dimensions either much smaller (Stenflo, 1987), or comparable with the density scale length (Stenflo and Stepanyants, 1995). We note that Stenflo and Stepanyants (1995) have considered IG waves in a stable stratified atmosphere. Stenflo (1987) and Stepanyants (1989, 1991) have constructed solutions in the form of acoustic-gravity modons in an unstable stratified atmosphere (where IG waves do not exist). Alternatively to the dipolar vortices (with similar exponential density and pressure profiles) an acoustic vortex chain analogous to the Kelvin–Stewart cat's eyes was found under conditions that exist in the Earth's atmosphere at 4–6 km altitude (Stenflo, 1994). The nonlinear equations presented in Section 4 have been derived under the assumption of an exponential density profile. Analogous equations are however applicable for other density profiles also. Dipolar vortices will thus in general be formed. If the density gradient is small, tripolar vortices may also be formed. Such fundamental vortex structures described by the Euler equation may appear in two-dimensional incompressible flows (Van Heijst et al., 1991; Carton and Legras, 1994). Jovanovič et al. (2001) have generalized the equations for strongly nonlinear low-frequency acoustic-gravity phenomena to allow for complicated profiles of the density, the pressure and the flow velocity, as well as for the presence of horizontal shear flows. For a class of parabolic profiles of the pressure and density a fully nonlinear solution in the form of a tripolar vortex was constructed. Apart from the known Kelvin-Stewart cat's eyes, dipolar and tripolar structures, new solutions having the form of a row of counter-rotating vortices, and several weakly two-dimensional vortex chains were also investigated (Jovanovič et al., 2002).

From Eq. (59) we conclude that the phase velocity $v_{\mathrm{ph}} = \omega/k_x$ is bounded by the interval

$$-v_{\max} \leqslant v_{\mathrm{ph}} \leqslant v_{\max}, \quad (61)$$

where in the case of an incompressible atmosphere $v_{\max} = 2H\omega_g = 2(gH)^{1/2}$. Thus when a source is moving in the *x*-direction with a velocity larger than $v_{\max}$, there is no resonance with IG waves. This means that such waves will not be generated by the source, i.e. there are no energy losses (Stepanyants and Fabrikant, 1992). Therefore one can obtain a stationary solution for the localized nonlinear formation of a pulse propagating horizontally with a velocity $|V| > v_{\max}$. Such solutions of the nonlinear Eqs. (57) and (58) in the form of spatially localized dipole-vortex solutions (modons) propagating in the neutral ($\sigma_p = 0$) *D*-layer have been found by Stenflo and Stepanyants (1995). We note that if we take $H \approx 6$ km as an estimate we get $v_{\max} \approx 500$ m/s. Thus the formed nonlinear solitary vortex structure should be supersonic and do not decay due to the generation of linear waves in the region $|V| < v_{\max}$.

By multiplying Eq. (57) by $-\bar{\psi}$ and integrating over *x* and *z*, we get

$$\frac{1}{2}\frac{\partial}{\partial t}\int \mathrm{d}x\,\mathrm{d}z[(\nabla\bar{\psi})^2 + \bar{\psi}^2/4H^2] = -\int \chi\frac{\partial\bar{\psi}}{\partial x}\,\mathrm{d}x\,\mathrm{d}z - \frac{\sigma_p B_0^2}{\varrho_0} \times \int\left[\left(\frac{\partial\bar{\psi}}{\partial z}\right)^2 + \bar{\psi}^2/4H^2\right]\mathrm{d}x\,\mathrm{d}z. \quad (62)$$

If we multiply Eq. (58) by χ, we obtain

$$\chi\frac{\partial\bar{\psi}}{\partial x} = \frac{1}{\omega_g^2}\left[\frac{1}{2}\frac{\partial\chi^2}{\partial t} + \chi J(\bar{\psi}, \chi)\right]. \quad (63)$$

Integration of this equation over *x* and *z* gives

$$\int \chi\frac{\partial\bar{\psi}}{\partial x}\,\mathrm{d}x\,\mathrm{d}z = \frac{1}{2\omega_g^2}\frac{\partial}{\partial t}\int \chi^2\,\mathrm{d}x\,\mathrm{d}z. \quad (64)$$

The combination of Eqs. (62) and (64) gives the energy dynamic law

$$\frac{\partial E}{\partial t} = -\frac{\sigma_p B_0^2}{\varrho_0}\int\left[\left(\frac{\partial\bar{\psi}}{\partial z}\right)^2 + \frac{\bar{\psi}^2}{4H^2}\right]\mathrm{d}x\,\mathrm{d}z, \quad (65)$$

where

$$E = \int \mathrm{d}x\,\mathrm{d}z\left[\frac{1}{2}(\nabla\bar{\psi})^2 + \frac{\bar{\psi}^2}{8H^2} + \frac{\chi^2}{2\omega_g^2}\right], \quad (66)$$

is the energy of the nonlinear solitary vortical dipole structure of the IG waves.

Thus, we conclude that the energy of the solitary vortex dipole structure of IG waves is decreasing owing to Ohm losses. The damping rate of such solitary vortex structures remains typically the same as for linear waves (see Eq. (41)).

6. Discussion

The purpose of the present paper is to consider the propagation peculiarities of AGWs in the Earth's weakly ionized conductive ionosphere. According to observations AGWs exist in a large range of heights extending from the troposphere to $z \leqslant 500$ km. At such heights the Earth's ionosphere is conductive due to presence of charged particles. The basic equations that incorporate both electromagnetic and Coriolis forces are discussed in Section 2. Using the so-called noninductive approximation we consider the current arising in the gas but neglect the vortex parts of the self-generated electromagnetic field. Thus, only the dynamo electric field is taken into account. In addition we are primarily interested in the wave dynamics at high latitudes in the northern hemisphere, i.e. we assume that the geomagnetic field $\mathbf{B}_0 = -B_0\mathbf{e}_z$ is vertical and directed downward. Analogously we define the Earth's rotation angular velocity as $\mathbf{\Omega}_0 = \Omega_0\mathbf{e}_z$. The ionospheric gas is vertically stratified, and we consider an isothermal atmosphere for the adiabatically propagating AGW perturbations.

In Section 3 with the help of generalized Ohm's law the linear propagation of AGWs in different ionospheric layers is investigated. It is shown that in the *D*-layer (50–80 km) the influence of the electromagnetic (ponderomotive) force is negligibly small. A general dispersion equation is obtained and analyzed (see Eqs. (14) and (16)). It is shown that all perturbed velocity components are increasing with height, while the pressure and density are decreasing. Incorporation of the Coriolis force leads to the coupling of AGWs with the inertial waves. This results in the appearance of inertio-acoustic-gravity (IAG) waves. When $k_\perp = 0$ the acoustic and inertial waves propagate differently (see Eqs. (17) and (18)). In the other limiting case of horizontal propagation ($k_z = 0$) we have found (in addition to the known acoustic cut-off frequency ω_a) the inertial cut-off frequency at $\omega_i = 2\Omega_0$. Possible propagation regions for AGWs when the influence of the Coriolis force is taken into account are depicted in Fig. 1. It is found that the acoustic waves are supersonic.

In conclusion we note that the propagation of acoustic waves is not influenced by the Coriolis force. Relating to subsonic (gravitational) waves they represent the low-frequency branch of the AGWs (we call them IG waves, see Fig. 1) having the frequency range $\omega_i < \omega < \omega_g$. For numerical estimations we use $\gamma = 1.4$, $H = 10$ km to obtain $10^{-4}\,\mathrm{s}^{-1} < \omega < 1.7\times10^{-2}\,\mathrm{s}^{-1}$. Considering intermediate values of the IG wavelengths ($k \sim 1/H$, $\omega \sim \omega_g$) we find that the group and phase velocities are of the same order $v_g \sim v_p \sim \omega_g H \sim 10^{-2}\ \mathrm{s}^{-1} \times 10^4\,\mathrm{m} \sim 10^2\,\mathrm{m/s}$. This estimation agrees with existing observations. We note that the values of the obtained group velocity are smaller than the corresponding values without accounting for Ω_0. However, the phase velocities are increasing in the same way. Thus, the incorporation of the Earth's rotation provides a substantial change in the propagation dynamics of low-frequency IG waves.

An analogous investigation for the ionospheric *E*-layer (100–150 km) is also carried out in Section 3. It is shown that in this layer the influence of the electromagnetic force is essential (the Hall conductivity predominates) along with the Coriolis force and that the effect of the geomagnetic field is to replace the planetary angular rotation vector as shown by Eq. (31). We note that the angular velocity vector $\mathbf{\Omega}_0$ and the geomagnetic force $\mathbf{B}_0$ provide opposite effects.

Linear propagation of AGW perturbations in the *F*-layer (150–400 km) is also considered in Section 3. It is shown that at such heights the influence of the Coriolis force can be neglected and that the action of the electromagnetic forces is defined by the Pedersen conductivity (see Eqs. (35) and (36)). In this case the dispersion equation is obtained (see Eq. (37)) and solved with respect to the frequency. It is shown that AGWs are damped and that the corresponding damping rate can be found (see Eq. (40)). It is easy to see that this damping rate is of the same order for both the high-frequency and low-frequency branches of AGWs (see Eq. (41)). For typical *F*-layer values $\sigma_p \cong 3\times10^{-5}$ S/m, $\varrho_0 = 10^{-10}\,\mathrm{kg/m^3}$, $B_0 = 0.5\times10^{-4}$ T, we get for the damping rate $\Gamma \sim 10^{-3}\,\mathrm{s}^{-1}$. Thus linear AGWs freely propagating through the atmospheric *D* and *E* layers undergo strong damping due to Joule losses in the *F*-layer.

In Section 4 we have investigated the nonlinear propagation of AGWs in the conductive Earth's ionosphere. The main purpose of that investigation is to find the influence of the conductivity of the Earth's ionosphere on the solitary vortical formations induced by external generation (e.g., earthquakes, volcanic eruptions, etc.). In particular, nonlinear solitary vortical structures formed by the IG waves (the low-frequency branch of AGWs) are considered. Vortical structures deserve attention because they carry trapped particles and contribute essentially to transport phenomena. Basic nonlinear equations for the two-dimensional (in the x–z plane) IG wave motions taking into account the Pedersen conductivity of the ionosphere is obtained. We note that for the comparatively high latitudes under consideration nonlinear IG waves are not influenced by the Coriolis force and Hall conductivity. In the absence of the Ohm's conductivity the obtained nonlinear equations describe the propagation of solitary dipole vortex structures. It is shown that the necessary condition to obtain such stationary solution is $|V| > v_{\max}$, where V is the propagation velocity of the solitary structure and $v_{\max}$ is the maximum phase velocity of linear IG waves. We note that such two-dimensional solitary dipole vortex structures are significantly different from those related to atmospheric Rossby modons (e.g., Larichev and Reznik, 1976; Petviashvili and Pokhotelov, 1992). A main difference is that the restriction on the velocity in our case is completely symmetrical, i.e. the modons can propagate faster than the maximum phase velocity $v_{\max}$ of the linear perturbations in any horizontal direction. According to estimates for a model atmosphere with constant equilibrium temperature we have $v_{\max} = 2H\omega_g$. For the numerical values given above ($H = 10\,\text{km}$ and $\omega_g = 1.7 \times 10^{-2}\,\text{s}^{-1}$) we have $v_{\max} \approx 3.4 \times 10^2\,\text{m/s} \approx c_s$ (where c_s is the sound speed). For smaller velocities ($|V| < v_{\max}$), it is obvious that our vortex solution will decay due to the generation of linear waves. The vortex lifetime can, however, be relatively large, and the vortex solution above can thus be of interest also in that case. In addition, we should note that the temperature is not constant in the Earth's atmosphere, and that the factor $\gamma - 1$ in Eq. (20) therefore must be replaced by $\gamma - 1 + HT^{-1}d_zT$, where T is the equilibrium temperature. With temperature gradients close to the instability threshold (Stenflo, 1994) one can therefore have very suitable conditions, namely $v_{\max} < |V| \ll c_s \approx 330\,\text{m/s}$, for the occurrence of stationary modons. In Section 5 the evolution (dynamic) equation for the energy of solitary vortical structures on IG waves is obtained (see Eq. (65)). It is shown that the energy of the solitary vortical dipole structure of IG waves is decreasing owing to Joule losses. The damping rate of such solitary vortical structures remains essentially the same as for linear waves (see Eq. (41)).

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