

## Dust-lower-hybrid waves in quantum plasmas

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### Abstract

The dispersion relation of the dust-lower-hybrid wave has been derived using the quantum hydrodynamic model of plasmas in an ultracold Fermi dusty plasma in the presence of a uniform external magnetic field. The dust dynamics, electron Fermi temperature effect, and the quantum corrections give rise to significant effects on the dust-lower-hybrid wave of the magnetized quantum dusty plasmas.

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There has been a considerable interest on the quantum mechanical effects in some specific areas of plasma physics, viz. microelectronics [1,2], dense astrophysical systems [3–5], and in laser-produced plasmas [6]. When a plasma is cooled to an extremely low temperature, the de Broglie wavelengths of the plasma particles could be comparable to the dimensions of the systems. In such plasmas, the ultracold dense plasma would behave as a Fermi gas and quantum mechanical effects might play a vital role in the behavior of the charge carriers of these plasmas under these extreme conditions. In microelectronics and very large integrated circuit fabrications, the systems may develop contaminants due to etching, implantations, etc. which might lead to new properties. The laser-produced plasmas and plasmas in high density astrophysical objects may also be contaminated by a number of reasons. Thus, these ultracold plasma systems may behave as dusty plasmas [7–10] where quantum mechanical effects could lead to new properties of these systems.

Waves and instabilities play a vital role in these ultracold and superdense plasma systems, particularly in diagnostics of

charged grain impurities in microelectronics. Recently, using the magnetohydrodynamic model for the magnetized quantum plasmas developed by Haas [11] and others, Shukla et al. [12] have studied the nonlinear interactions in quantum magneto-plasmas including the usual high-frequency ( $\omega \simeq \sqrt{\omega_{ce}\omega_{ci}}$ ) electrostatic lower-hybrid wave where electrons are assumed magnetized and ions unmagnetized. We present here the quantum effects on the low-frequency dust-lower-hybrid waves below the ion-cyclotron frequency, where dust dynamics plays the vital role. Shukla and Ali [9] have also studied the modification of dust-acoustic waves in unmagnetized dusty quantum plasmas. Haas et al. [1] studied the linear ion-acoustic wave in an unmagnetized quantum plasma at these super conditions. The counterpart of dust-acoustic wave in a magnetized dusty plasma involving the dust dynamics is the existence of the dust-lower-hybrid (DLH) mode which was pointed out in the literature [13–15]. In this Letter, we investigate the modification of the DLH wave in a quantum plasma in the presence of a uniform external magnetic field.

We consider a collisionless supercooled and dense dusty plasma consisting of electrons, ions, and relatively highly charged and massive dust particles in the presence of a uniform external magnetic field, ( $\mathbf{B}_0 \parallel \hat{z}$ ). We assume that the electrons and ions possess significant quantum mechanical effects. However, we neglect the quantum effects on dust dynamics because

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of the high mass which gives rise to an insignificant de Broglie wavelength.

The governing equations in the quantum hydrodynamic model for electrons and ions of the dusty plasma in the presence of a DLH perturbation and the external magnetic field  $\mathbf{B}_0$  are

$$\frac{\partial n_j}{\partial t} + n_{j0} \nabla \cdot \mathbf{v}_j = 0, \quad (1)$$

$$\frac{\partial}{\partial t} \mathbf{v}_j = -\frac{q_j}{m_j} \nabla \phi + \frac{q_j}{m_j c} \mathbf{v}_j \times \mathbf{B}_0 + \frac{\hbar^2}{4m_j^2 n_{j0}} \nabla (\nabla^2 n_{j1}), \quad (2)$$

where  $j = e, i$ ,  $\phi$  is the electrostatic potential of the DLH mode,  $\omega_{cj} = q_j B_0 / m_j c$ ;  $q_j$ ,  $m_j$ ,  $n_{j0}$ , and  $c$  are charge, mass, equilibrium particle density of the  $j$ th species respectively, and the velocity of light in a vacuum.

Following the standard techniques [13–16], the dielectric susceptibility of the plasma can be obtained by solving Eqs. (1) and (2) to obtain

$$\chi_j = \frac{k_{\perp}^2}{k^2} \frac{\omega_{pj}^2 f_j}{\omega_{cj}^2 - \omega^2 f_j} - \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pj}^2}{\omega^2} \frac{1}{f_j}, \quad (3)$$

where the quantum correction factor is given by

$$f_j = 1 - \frac{\hbar^2 k^4}{4m_j^2 \omega^2}. \quad (4)$$

Here, both the electrons and ions are considered cold and magnetized. The susceptibility for the unmagnetized and cold dust particles is obtained as

$$\chi_d = -\frac{\omega_{pd}^2}{\omega^2}, \quad (5)$$

where the dust plasma frequency,  $\omega_{pd} = (4\pi q_d^2 n_{d0} / m_d)^{1/2}$  and the symbol  $d$  refers to dust grains. Substituting the density perturbations,  $n_s = -\chi_s k^2 \phi / 4\pi q_s$  with  $s = e, i, d$  in the Poisson's equation, one can easily obtain the dispersion relation for the dust-lower-hybrid mode from [13–15]

$$\epsilon(\omega, \mathbf{k}) = 1 + \chi_e + \chi_i + \chi_d = 0, \quad (6)$$

as

$$\omega^2 = \frac{\omega_{pd}^2 \left[ 1 + \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pe}^2}{\omega_{pd}^2} \frac{1}{f_e} \left( 1 + \frac{\omega_{pi}^2}{\omega_{pe}^2} \frac{f_e}{f_i} \right) \right]}{1 + \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pi}^2 f_i}{\omega_{ci}^2} \left( 1 + \frac{\omega_{pe}^2}{\omega_{pi}^2} \frac{\omega_{ci}^2}{\omega_{ce}^2} \frac{f_e}{f_i} \right)}. \quad (7)$$

Assuming  $k_{\perp}^2 \gg k_{\parallel}^2$  to take into account the maximum effect of the external magnetic field and  $\omega_{pi}^2 \gg \omega_{ci}^2$  for a high-density plasma, the electrostatic DLH frequency turns out to be

$$\omega^2 = \frac{\omega_{dlh}^2}{f_i} \left( 1 + \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pe}^2}{\omega_{pd}^2} \frac{1}{f_e} \right), \quad (8)$$

where  $\omega_{dlh} = \omega_{pd} \omega_{ci} / \omega_{pi}$  is the DLH frequency and  $f_e$  is defined in Eq. (4). For nearly perpendicular propagation and small quantum mechanical correction, we can substitute  $\omega = \omega_{dlh}$  in

$f_e$  and  $f_i$ . Thus, the DLH mode-frequency with quantum correction is given by

$$\omega^2 = \frac{\omega_{dlh}^2}{1 - \hbar^2 k^4 / 4m_i^2 \omega_{dlh}^2} \times \left( 1 + \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pe}^2}{\omega_{pd}^2} \frac{1}{1 - \hbar^2 k^4 / 4m_e^2 \omega_{dlh}^2} \right). \quad (9)$$

In absence of the quantum correction, Eq. (9) represents the dust-lower-hybrid oscillation in a cold dusty magnetoplasma [13–15]. However, for an ultracold dusty quantum magnetoplasma, Eq. (9) gives rise to a propagating wave mode with finite group velocity.

Next, we include the temperature correction to the DLH wave. For this, we assume that the electrons and ions behave as zero-temperature Fermi gases which satisfy the pressure law [1, 17,18]

$$p_j = m_j V_{Fj}^2 n_j^3 / 3n_{j0}^2, \quad j = e, i, \quad (10)$$

where  $n_j$  is the total number density of the  $j$ th species,

$$V_{Fj} = (2k_B T_{Fj} / m_j)^{1/2}, \quad (11)$$

is the Fermi speed,  $k_B$  is the Boltzmann constant, and  $T_{Fj}$  is the Fermi temperature. Taking  $-\nabla p_j$  force in the equation of motion, Eq. (2) and following the same procedure, one can easily derive the susceptibility given by Eq. (3) where  $f_j$  is replaced by

$$F_j = 1 - \frac{k^2 V_{Fj}^2}{\omega^2} - \frac{\hbar^2 k^4}{4m_j^2 \omega^2}, \quad (12)$$

which contains the temperature effect.

Let us now consider a supercooled Fermi dusty plasma where electrons are considered hot at the Fermi temperature, ions are cold and magnetized, and dust particles are unmagnetized and cold. Then

$$\begin{aligned} \chi_e &= \frac{1}{k^2 \lambda_{Fe}^2}, \\ \chi_i &= \frac{k_{\perp}^2}{k^2} \frac{\omega_{pi}^2}{\omega_{ci}^2} f_i - \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pi}^2}{\omega^2} \frac{1}{f_i}, \\ \chi_d &= -\frac{\omega_{pd}^2}{\omega^2}, \end{aligned} \quad (13)$$

where  $\lambda_{Fe} = V_{Fe} / \sqrt{2} \omega_{pe}$  is the Debye length of electrons at the Fermi temperature,  $T_{Fe}$  and  $f_i$  is defined in Eq. (4). Here, we have assumed the low-frequency electrostatic wave of the dusty magnetoplasma with  $\omega^2 \ll \omega_{ci}^2$  and  $\omega^2 \ll k^2 V_{Fe}^2$ .

Taking  $k_{\perp}^2 \gg k_{\parallel}^2$  and  $\omega_{pi}^2 \gg \omega_{ci}^2$ , we write the dielectric function of the quantum dusty plasma under consideration with quantum correction as

$$\begin{aligned} \epsilon(\omega, \mathbf{k}) &= 1 + \frac{1}{k^2 \lambda_{Fe}^2} + \frac{k_{\perp}^2}{k^2} \frac{\omega_{pi}^2}{\omega_{ci}^2} F_i' \\ &\quad - \frac{k_{\parallel}^2}{k^2} \frac{\omega_{pi}^2}{\omega^2} F_i' - \frac{\omega_{pd}^2}{\omega^2}, \end{aligned} \quad (14)$$

where

$$F'_i = 1 - \frac{\hbar^2 k^4}{4m_i^2 \omega_{dlh}^2}. \quad (15)$$

For  $\omega_{pi}^2/\omega_{ci}^2 \gg 1/k^2 \lambda_{Fe}^2 \gg 1$ , we finally obtain the dispersion relation of the dust-lower-hybrid wave as

$$\omega^2 = \frac{\omega_{dlh}^2}{1 - \hbar^2 k^4 / 4m_i^2 \omega_{dlh}^2} \left( 1 + \frac{k_{\parallel}^2 \omega_{pi}^2}{k^2 \omega_{pd}^2} \frac{1}{1 - \hbar^2 k^4 / 4m_i^2 \omega_{dlh}^2} \right) \times \left( 1 - \frac{1}{k_{\perp}^2 \rho_{Fs}^2} \right), \quad (16)$$

where

$$\rho_{Fs}^2 = \frac{C_{Fs}^2}{\omega_{ci}^2} F'_i, \quad C_{Fs}^2 = \omega_{pi}^2 \lambda_{Fe}^2. \quad (17)$$

Here,  $\rho_{Fs}$  is the ion Larmor radius at the electron Fermi temperature with quantum correction for ions. Eqs. (9) and (16) are the main results of this Letter. The DLH mode frequency in a cold Fermi dusty magnetoplasma is given by Eq. (9) where the Fermi temperature effect is neglected. This involves the dust dynamics and the motion of magnetized electrons and ions. The DLH frequency is seen to be significantly modified by the quantum effect [cf. Eq. (9)]. The dispersion relation for the DLH wave in the magnetized Fermi dusty plasma is given by Eq. (16) where the electrons at the Fermi temperature drive the wave.

To summarize, we have derived the dispersion relation for the dust-lower-hybrid waves in an ultracold and uniformly magnetized Fermi dusty plasma by employing the quantum hydrodynamic model of a plasma with quantum and thermal correc-

tions. It is found that the dispersion relation of the dust-lower-hybrid wave is significantly affected by the quantum correction. This is a fundamental mode of a quantum dusty plasma in the presence of a uniform external magnetic field. These modes will find applications in diagnosing the charged dust impurities in microelectronics and wave-particles interactions in the dusty quantum magnetoplasmas.

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