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Drift wave instability in a nonuniform quantum dusty magnetoplasma

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Using the quantum hydrodynamic model of plasmas and with quantum effects arising through the Bohm potential and the Fermi degenerate pressure, the possible drift waves and their instabilities have been investigated in considerable detail in a nonuniform dusty magnetoplasma. It is found that in the presence of a nonuniform ambient magnetic field, the drift waves grow in amplitude by taking energy from the streaming ions and density inhomogeneity. The implication of the drift wave instability for nonthermal electrostatic fluctuations to laboratory and astrophysical environments is also pointed out. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3086861]

In recent years, there has been a growing enthusiasm in quantum plasmas because of their importance in microelectronics and electronic devices with nanoelectronic components,[1,2] dense astrophysical systems,[3–5] and in laser-produced plasmas.[6–9] When a plasma is cooled to an extremely low temperature, the de Broglie wavelengths of the plasma particles could be at least comparable to the scale lengths, such as Debye length or Larmor radius, in the system. In such plasmas, the ultracold dense plasma would behave as a Fermi gas and quantum mechanical effects might play a vital role in the behavior of the charge carriers of these plasmas under extreme conditions.

We can visualize that the density and static ambient magnetic field in a quantum dusty magnetoplasma can be nonuniform with finite scale lengths. The density inhomogeneity causes the presence of drift waves due to diamagnetic drifts, and the magnetic field inhomogeneity causes a uniform stream of ions. In this Brief Communication, we study the possible drift waves and their instabilities due to the free energy sources of density nonuniformity and a uniform beam of ions due to the ambient magnetic field gradient.

We consider an infinitely extended inhomogeneous high density dusty magnetoplasma containing electrons, ions, and charged dust grains in the presence of an inhomogeneous static ambient magnetic field $\mathbf{B}_0(x) \parallel \hat{\mathbf{z}}$. At equilibrium, we assume that the charge quasineutrality condition is satisfied, that is, $n_{i0}+(q_d/e)n_{d0}=n_{e0}$, where $n_{j0}(x)$ is the equilibrium number density of the jth species (j=electrons, ions, or dust), q_d is the average charge on a dust grain, and e is the electronic charge. We analyze the stability properties of the system against electrostatic perturbations including the effects of the heavier species.

The governing equations in the quantum hydrodynamic (QHD) model[10–14] for electrons, ions, and charged dust grains ($j=e,i,d$) in the presence of the ambient magnetic field $\mathbf{B}_0$ are

$$m_j n_{j0}\frac{\partial}{\partial t}\mathbf{v}_j = -n_{j0}q_j\nabla\phi + n_{j0}\frac{q_j}{c}\mathbf{v}_j\times\mathbf{B}_0 - \nabla p_{Fj} + \frac{\hbar^2}{4m_j}\nabla(\nabla^2 n_{j1}), \tag{1}$$

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j\mathbf{v}_j) = 0, \tag{2}$$

where $n_j=n_{j0}+n_{j1}$, $\hbar$ is Planck's constant divided by 2π, $\phi(\mathbf{r},t)$ is the electrostatic potential in the quantum magnetoplasma, and q_j, m_j, n_j, and c are the charge, mass, total equilibrium number density with equilibrium value n_{j0} of the jth species, and the velocity of light in a vacuum, respectively. Here, we take into account the quantum effects of all the species when they are considered extremely cold. In Eq. (1), we assume that the plasma particles in a one-dimensional zero-temperature Fermi gas satisfies the pressure law,[11,12] $p_j=m_jV_{Fj}^2n_j/3n_{j0}^2$, where $V_{Fj}=(2k_BT_{Fj}/m_j)^{1/2}$ is the Fermi speed, and k_B, T_{Fj}, and n_j are the Boltzmann constant, Fermi temperature, and the total number density with its equilibrium value n_{j0}, respectively.

Further, Poisson's equation satisfying the electrostatic potential ϕ of the electrostatic perturbation is

$$\nabla^2\phi = 4\pi e\left(n_{e1} - n_{i1} - \frac{q_d}{e}n_{d1}\right). \tag{3}$$

In the presence of the density inhomogeneities in the x-direction and the ambient magnetic field, $\mathbf{B}_0=\hat{\mathbf{z}}B_0(x)$, we assume the presence of drift waves propagating in the YZ-plane, proportional to $\exp[-i(\omega t-k_yy-k_zz)]$, where $k_y^2 \gg k_z^2$. Here, ω and $\mathbf{k}$ are the angular frequency and wavenumber vector, respectively. Using Eqs. (1)–(3) and after some straightforward calculations, we obtain the dielectric susceptibility for the jth species where $j=e,i,d$ as

$$\chi_j = -\frac{\omega_{pj}^2\left[\dfrac{k_z^2}{\omega^2} + \dfrac{k_y^2}{\omega^2-\omega_{cj}^2}\left(1-\dfrac{\omega_{cj}}{k_yL_j\omega}\right)\right]}{k^2 - k^2V_{Fj}'^2\left[\dfrac{k_z^2}{\omega^2} + \dfrac{k_y^2}{\omega^2-\omega_{cj}^2}\left(1-\dfrac{\omega_{cj}}{k_yL_j\omega}\right)\right]}, \tag{4}$$

where $\omega_{pj}=(4\pi n_{j0}q_j^2/m_j)^{1/2}$ and $\omega_{cj}=q_jB_0/m_jc$ are the plasma frequency and the cyclotron frequency of the jth spe-

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cies. In Eq. (4), $V'_{Fj}=V_{Fj}(1+\gamma_j)^{1/2}$, where $\gamma_j=\hbar^2k^2/8m_jT_{Fj}$ and the scale length of inhomogeneity $L_j=-n_{j0}/n'_{j0}$ where $n'_{j0}=\partial n_{j0}(x)/\partial x$. We use Eq. (4) to find the general dielectric response function of the nonuniform quantum dusty magnetized plasma under various possible conditions from

$$\epsilon(\omega,\mathbf{k}) = 1 + \chi_e(\omega,\mathbf{k}) + \chi_i(\omega,\mathbf{k}) + \chi_d(\omega,\mathbf{k}). \quad (5)$$

To study the possible drift waves and their instability in a nonuniform quantum dusty plasma in the presence of a static ambient magnetic field with a gradient, we assume

$$\omega \le \omega_i^* \ll \omega_{ci}, \quad k_y^2 \gg k_z^2,$$

$$kV'_{Fe} \gg \omega \gg kV'_{Fi}. \quad (6)$$

Then, the dielectric response function becomes

$$\epsilon(\omega,\mathbf{k}) = 1 + \frac{\omega_{pe}^2}{k_y^2V_{Fe}'^2} + f_i - \frac{\omega_i^*}{\omega} - \frac{\omega_{pd}^2}{\omega^2}, \quad (7)$$

where $\omega_i^*=\omega_{pi}^2/k_yL_i\omega_{ci}$ is the ion-drift frequency and $f_i=\omega_{pi}^2/\omega_{ci}^2$. Here, the quantum mechanical effect is taken through the motion of electrons, which is neglected for ions and dust grains because of their heavier masses. The dispersion relations of the electrostatic drift waves are given by $\epsilon=0$, that is,

$$\omega = \frac{\omega_i^*}{1+f_i+\omega_{pe}^2/k_y^2V_{Fe}'^2}, \quad -\frac{\omega_{pd}^2}{\omega_i^*}. \quad (8)$$

Here, we incorporate the quantum mechanical effect through the electron dynamics only. Ions are considered cold, inhomogeneous, and magnetized. However, the relatively massive and charged grains are logically taken as cold and unmagnetized.

In the absence of the dust component, that is, in an electron-ion plasma, if we assume that $\omega_{pe}^2/k^2V_{Fe}'^2 \gg f_i \gg 1$, then first of Eq. (8) reduces to

$$\omega = \frac{\omega_i^*}{\omega_{pe}^2}\left(k^2V_{Fe}^2 + \frac{\omega_{qe}^2}{4}\right), \quad (9)$$

where the quantum angular frequency is given by $\omega_{qe}=\hbar k^2/m_e$. For a quantum plasma (e.g., nanoscale microelectronics, semiconductor plasmas, laser-produced plasmas, etc.), the ratio of the Bohm term to degenerate term in Eq. (1) becomes

$$\frac{k^2}{38n^{2/3}} > 1 \quad (10)$$

for a given electron density and short wavelength perturbation ($\lambda \ge \lambda_B$) in a supercooled plasma ($\lambda_B \gg L$). Thus, the quantum effect arising through the Bohm potential effect is larger than that due to Fermi degenerate pressure effect. Thus, Eq. (9) reduces to

$$\omega \simeq \omega_i^*\left(\frac{\hbar^2k_y^4}{4m_e^2\omega_{pe}^2}\right). \quad (11)$$

It may be mentioned here that the new drift wave with quantum effect found by Shukla and Stenflo[15] reduces to the ion-drift wave given by Eq. (11). However, in a complete degenerate plasma, the drift wave is given by

$$\omega = \left(\frac{k^2V_{Fe}^2}{\omega_{pe}^2}\right)\omega_i^*, \quad (12)$$

where $V_{Fe}=(2k_BT_{Fe}/m_e)^{1/2}$ is the Fermi velocity of electrons and $k^2V_{Fe}^2 > \omega_{qe}^2/4$.

For cold nonuniform electron-ion plasmas neglecting the presence of the dust component, one can easily obtain the dispersion relation of the drift waves from Eqs. (4), (6), and (7) as

$$\omega = \frac{\omega_e^*}{f_i}\left(1 + \frac{L_e}{L_i}\right), \quad (13)$$

where $k_y^2 \gg k_z^2$ and $f_i \gg f_e \gg 1$ are satisfied. Here, $f_e=\omega_{pe}^2/\omega_{ce}^2$.

On account of the density inhomogeneities, the electrons and ions acquire diamagnetic drift frequencies. However, because of the inhomogeneity of the ambient static magnetic field in the dusty plasma with a gradient in the x-direction, ions can have an additional drift velocity[16]

$$\mathbf{V}_0 = -\frac{c\,\partial B_0(x)/\partial x}{4\pi q_{d0}n_{d0}}\hat{\mathbf{y}} \quad (14)$$

In the presence of the density nonuniformity and the continuous ion streaming because of the magnetic field nonuniformity, we can study the instability of the drift wave from[17]

$$1 + \frac{\omega_{pe}^2}{k_y^2V_{Fe}'^2} + f_i - \frac{\omega_i^*}{\omega - k_yV_0} - \frac{\omega_{pd}^2}{\omega^2} = 0 \quad (15)$$

or

$$1 - \frac{\omega_i^*/F}{\omega - k_yV_0} - \frac{\omega_{pd}^2/F}{\omega^2} = 0, \quad (16)$$

where

$$F = 1 + \frac{\omega_{pe}^2}{k_y^2V_{Fe}'^2} + f_i. \quad (17)$$

Letting $\omega=k_yV_0+\delta$ where $\delta \ll k_yV_0$, Eq. (16) becomes

$$1 - \frac{\omega_i^*/F}{\delta} - \frac{\omega_{pd}^2/F}{k_y^2V_0^2}\left(1 - \frac{2\delta}{k_yV_0}\right) = 0. \quad (18)$$

For $k_yV_0 \sim \omega_{pd}/\sqrt{F}$, the growth rate of the drift wave is given by $(\omega=\omega_r+i\ \gamma)$,

$$\gamma \simeq \frac{\omega_{pi}(k_yV_0)^{3/2}}{\sqrt{2}\omega_{pd}\sqrt{k_y|L_i|\omega_{ci}}}. \quad (19)$$

Equations (11), (12), and (19) are the main results of the present paper. We notice here from Eq. (19) that the drift wave instability growth rate increases with increasing ion drift speed V_0 due to the sharp magnetic field gradient but decreases with the ion inhomogeneity scale length L_i. However, in the presence of collisions of the charge particles with the neutral atoms/molecules, the growth rate of the drift wave will saturate, which is beyond the scope of the paper. At large enough neutral density, the instability may even be suppressed.

In summary, we present a rigorous study of possible drift waves and their instability in a nonuniform quantum dusty magnetoplasma in a dense astrophysical object and its environments. QHD equations for the plasma particles in the presence of a nonuniform static ambient magnetic field and Poisson's equations for the electrostatic potential have been employed to derive a general expression for the dielectric response function of the plasma. Linear dispersion relations for possible drift waves with or without the quantum effects arising through the Fermi degenerate pressure and the Bohm potential have been derived for possible cases of interest. In the absence of quantum effects via the dynamics of cold electrons, we obtain the electron and ion drift waves. Drift waves in the dusty plasma approximations are presented with or without quantum effects. It is found that in the presence of uniform ion streaming due to the inhomogeneous static ambient magnetic field, the drift wave in a dusty plasma suffers an instability. This instability growth rate increases with the ion-streaming velocity and decreases with the scale length of ion inhomogeneity in the nonuniform quantum dusty magnetoplasma.

In laboratory plasmas, ion streaming can occur in the presence of externally applied static crossed electric and magnetic fields. Hence, the drift waves can be excited, whose properties can be investigated in the laboratory. In astrophysical situations, the drift waves might generate sufficient electromagnetic noise due to parametric cascading of electromagnetic signals, which should be taken into account in the telescopic observations. Thus, the drift wave excitation can contribute to better understanding of the cross-field energy and charged particle transport in quantum plasmas.

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