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M. Salimullah, M. Jamil, H. A. Shah, and G. Murtaza

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# Jeans instability in a quantum dusty magnetoplasma

M. Salimullah,[1,2,a)] M. Jamil,[1] H. A. Shah,[1] and G. Murtaza[2]
[1]Department of Physics, GC University, Lahore-54000, Pakistan
[2]Salam Chair in Physics, GC University, Lahore-54000, Pakistan

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Jeans instability in a homogeneous cold quantum dusty plasma in the presence of the ambient magnetic field and the quantum effect arising through the Bohm potential has been examined using the quantum magnetohydrodynamic model. It is found that the Jeans instability is significantly reduced by the presence of the dust-lower-hybrid wave and the ion quantum effect. The minimum wavenumber for Jeans stability depends clearly on ion quantum effect and the dust-lower-hybrid frequency also. © 2009 American Institute of Physics. [DOI: 10.1063/1.3070664]

In recent years, there has been a growing enthusiasm in quantum plasmas because of their importance in microelectronics and electronic devices with nanoelectronic components,[1,2] dense astrophysical systems,[3–5] and in laser-produced plasmas.[6–9] When a plasma is cooled to an extremely low temperature, the de Broglie wavelengths of the plasma particles could be at least comparable to the scale lengths, such as, Debye length or Larmor radius, etc., in the systems. In such plasmas, the ultracold dense plasma would behave as a Fermi gas and quantum mechanical effects might play a vital role in the behavior of the charge carriers of these plasmas under these extreme conditions. Shukla and Stenflo[10] have pointed out the importance and the range of validity of quantum effect on Jeans instabilities of self-gravitating astrophysical quantum dusty plasma systems where the electromagnetic and gravitational forces on plasma charge carriers become comparable.

In the dense astrophysical situations, such as, the interior of white dwarfs and neutron stars, the plasma density may be very large. Here, the interparticle distance may be comparable to the de Broglie wavelength of the cold plasma particles. Hence, the quantum effect arising through the Bohm potential might become important. However, the Jeans instability is a relatively fast process for the gravitational collapse.[11–16] The presence of ambient magnetic field and the quantum effect might affect the Jeans instability in a quantum dusty magnetoplasma. In this brief communication, we examine the role of the ambient magnetic field in comparison to the quantum effect on the Jeans instability in any astrophysical cold quantum dusty plasma system.

We consider an infinitely extended homogeneous high density dusty magnetoplasma containing electrons, ions, and charged dust grains in the presence of a homogeneous static ambient magnetic field $\mathbf{B}_0 \| \hat{\mathbf{z}}$. At equilibrium, we assume that the charge quasineutrality condition is satisfied, that is $n_{i0}+(q_d/e)n_{d0}=n_{e0}$, where $n_{j0}$ is the equilibrium number density of the $j$th species ($j$=electrons, ions or dust), $q_d$ is the average charge on a dust grain, and $e$ is the electronic charge. We analyze the stability properties of the system against the electrostatic perturbations including the self-gravitational effects among the heavier species.

The governing equations in the quantum hydrodynamic (QHD) model[17–21] for the supercooled electrons, ions, and charged dust grains in the presence of the ambient magnetic field $\mathbf{B}_0$ are

$$\frac{\partial}{\partial t}\mathbf{v}_j = -\frac{q_j}{m_j}\nabla\phi + \frac{q_j}{m_j c}\mathbf{v}_j \times \mathbf{B}_0 + \frac{\hbar^2}{4m_j^2 n_{j0}}\nabla(\nabla^2 n_{j1}), \tag{1}$$

$$\frac{\partial n_j}{\partial t} + n_{j0}\nabla\cdot\mathbf{v}_j = 0, \tag{2}$$

where $\hbar$ is the Planck's constant divided by $2\pi$, $\phi(\mathbf{r},t)$ is the electrostatic potential in the quantum magnetoplasma, and $q_j$, $m_j$, $n_{j0}$, and $c$ are the charge, mass, equilibrium number density of the $j$th species, and the velocity of light in a vacuum, respectively. Here, we take into account the quantum effects of all the species when they are considered extremely cold.

Further, the Poisson's equations satisfying the electrostatic potential $\phi$ of the perturbation and the gravitation potential $\psi$ for the relatively heavier dust grains are

$$\nabla^2\phi = 4\pi e\left(n_{e1} - n_{i1} - \frac{q_d}{e}n_{d1}\right) \tag{3}$$

and

$$\nabla^2\psi = 4\pi G m_{d0} n_{d1}, \tag{4}$$

where $G$ is the universal gravitational constant and the subscript 1 indicates the perturbed quantities in the presence of the perturbed potential $\phi$ and $\psi$.

Following Ref. 22 for the cold and quantum magnetoplasma, we can easily derive the dielectric susceptibilities of magnetized electrons and ions as

$$\chi_\alpha = \frac{k_\perp^2}{k^2}\frac{\omega_{p\alpha}^2 f_\alpha}{\omega_{c\alpha}^2 - \omega^2 f_\alpha^2} - \frac{k_\|^2}{k^2}\frac{\omega_{p\alpha}^2}{\omega^2}\frac{1}{f_\alpha}, \tag{5}$$

where $\alpha=e,i$ and the quantum factor arising due to the Bohm potential is given by

[a)]Permanent address: Department of Physics, Jahangirnagar University, Savar, Dhaka-1342, Bangladesh. Electronic mail: msu_49@yahoo.com.

 

$$f_\alpha = 1 - \frac{\hbar^2 k^4}{4m_\alpha^2\omega^2}. \quad (6)$$

For $k_\perp^2 \gg k_\parallel^2$ for maximum magnetic field effect and $\omega_{c\alpha}^2 \gg \omega^2 f_\alpha^2$ for low-frequency perturbation, the above susceptibilities for electrons/ions reduce to

$$\chi_\alpha \simeq \sigma_\alpha f_\alpha, \quad (7)$$

where $\sigma_\alpha = \omega_{p\alpha}^2/\omega_{c\alpha}^2$ and

$$f_\alpha = 1 - \frac{\omega_{Q\alpha}^2}{4\omega_{d\mathrm{lh}}^2}, \quad (8)$$

with $\omega_{Q\alpha} = \hbar k^2/m_\alpha$ and $\omega_{d\mathrm{lh}} = \omega_{pd}\omega_{ci}/\omega_{pi}$ being the dust-lower-hybrid frequency.[23,24] For nearly perpendicular propagation and neglecting small quantum mechanical correction, we have substituted[22] $\omega \simeq \omega_{d\mathrm{lh}}$ in $f_e$ and $f_i$ given by Eq. (8).

Considering the relatively massive dust grains to be unmagnetized but mobile and including the self-gravitational and quantum effects, we can obtain

$$\frac{\partial \mathbf{v}_{d1}}{\partial t} = -\frac{q_d}{m_d}\nabla\phi - \nabla\psi + \frac{\hbar^2}{4m_d^2 n_{d0}}\nabla\nabla^2 n_{d1}, \quad (9)$$

$$n_{d1} = n_{d0}\mathbf{k}\cdot\mathbf{v}_{d1}/\omega. \quad (10)$$

Writing $n_{d1} = -\chi_d k^2\phi/4\pi q_d$, we obtain the dielectric susceptibility for dust-plasma as

$$\chi_d = -\frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2 - \omega_{Qd}^2/4}, \quad (11)$$

where $\omega_{Qd} = \hbar k^2/m_d$ and $\omega_{Jd} = \sqrt{4\pi G m_d n_{d0}}$ is the dust Jeans frequency. Using Eqs. (7) and (11) in the Poisson's equation, Eq. (3), we obtain the dispersion relation for $(\omega, \mathbf{k})$ from

$$1 + \sigma_e f_e + \sigma_i f_i - \frac{\omega_{pd}^2}{\omega^2 + \omega_{Jd}^2 - \omega_{Qd}^2/4} = 0. \quad (12)$$

Since $\omega_{pi}^2 f_i/\omega_{ci}^2 \gg \omega_{pe}^2 f_e/\omega_{ce}^2$ for the high density quantum magnetoplasma, the above dispersion relation yields

$$\omega^2 = -\omega_{Jd}^2 + \frac{\omega_{d\mathrm{lh}}^2}{f_i} + \frac{\omega_{Qd}^2}{4}. \quad (13)$$

Thus, in the presence of the ambient magnetic field and the quantum effect on the cold dust grains, the growth rate of the modified Jeans instability is given by

$$\gamma = \left(\omega_{Jd}^2 - \frac{\omega_{d\mathrm{lh}}^2}{f_i} - \frac{\omega_{Qd}^2}{4}\right)^{1/2}. \quad (14)$$

In the absence of the ambient magnetic field and the quantum effect arising through the Bohm diffraction term of the dust component, the well known Jeans instability is obtained. Clearly, both the magnetic field $\mathbf{B}_0$ and the ion and dust quantum effects reduce the growth rate. Therefore, they stabilize the Jeans instability. Obviously, the quantum effect on Jeans instability will be significant for large values of the wavenumber $k$. In the absence of the ambient magnetic field, the limiting value of $k$ was obtained in Ref. 10. However, in the presence of the magnetic field, this value of $k$ is significantly modified.

To have some numerical appreciation of our analytical results, we have calculated the Jeans frequency and the dust-lower-hybrid frequency for the following typical parameters of an astrophysical plasma environment, $n_{e0} \simeq n_{i0} = 10^{30}\ \mathrm{cm}^{-3}$, $n_{d0} = 10^{15}\ \mathrm{cm}^{-3}$, $m_d = 10^{13} m_i$, $m_i = 12 m_p$ ($m_p$ is the proton mass), $Z_d = 10^2$, and $B_0 = 10^{10}$ G. The numerical values of $\omega_{Jd}$ and $\omega_{d\mathrm{lh}}$ turn out to be $4.10\times 10^{-1}\ \mathrm{s}^{-1}$ and $7.98\times 10^{-2}\ \mathrm{s}^{-1}$, respectively. The Jeans instability growth rate is further reduced by the quantum effect on ions for large $k$ [cf. Eqs. (8) and (14)] when the dust quantum effect is neglected because of their heavier mass. It is noted that the Jeans instability growth rate is a sensitive function of the strength of the ambient magnetic field because of the presence of the dust-lower-hybrid wave.

The stability condition for the quantum dusty magnetoplasma can be obtained from $\gamma = 0$. Neglecting the product of ion and dust quantum corrections, we obtain the stability condition of the Jeans instability as

$$k \geqslant \sqrt{\frac{2m_i}{\hbar}}\left[\frac{\omega_{Jd}^2 - \omega_{d\mathrm{lh}}^2}{\omega_{Jd}^2/\omega_{d\mathrm{lh}}^2 + m_i^2/m_d^2}\right]^{1/4}. \quad (15)$$

We note here that the ion quantum correction stabilizes the Jeans instability in the presence of the ambient static magnetic field when the above condition on $k$ in Eq. (15) is satisfied. Usually, $\omega_{Jd} > \omega_{d\mathrm{lh}}$ since the dust-lower-hybrid wave is an ultralow frequency wave in a magnetized plasma. Thus, the Jeans stability condition on $k$ in a quantum dusty magnetoplasma reduces to

$$k \geqslant \sqrt{\frac{2m_i\omega_{d\mathrm{lh}}}{\hbar}}. \quad (16)$$

Hence, the ion quantum correction gives rise to the limit of the wavenumber which increases with the strength of the ambient static magnetic field. Both the magnetic field effect and the ion quantum effect arising through the Bohm diffraction term contribute to the minimum wavenumber which stabilizes the Jeans instability.

In summary, we have examined the Jeans instability in the presence of the ambient static magnetic field and the quantum effects of the plasma species arising through the Bohm potential in a cold quantum dusty magnetoplasma. It is noticed that the Jeans instability is drastically reduced by the ion quantum effect and the dust-lower-hybrid frequency resulting from the dynamics of unmagnetized dust grains and the magnetized ions. The electron contribution to this reduction is insignificant. It may be added here that the nonuniformities in the plasma density and the ambient magnetic field might play important roles on the Jeans instability and the work in these lines is in progress.

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