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# Linear and nonlinear properties of an obliquely propagating dust magnetosonic wave

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**Abstract.** Linear and nonlinear properties of the two-dimensional obliquely propagating dust magnetosonic wave are studied in a three-component dusty plasma. The dispersion relations in the linear and Kadomstev–Petviashvili (KP) equation in the nonlinear regime are derived for small-amplitude perturbations. It is shown that the linear dispersion properties of the low-frequency dust magnetosonic wave depend on the angle $\theta$ that the magnetic field makes with the $x$-axis, the ratio of ion to electron concentration, and the plasma beta. It is found that retaining the electron pressure term gives rise to novel features in the dust magnetosonic wave. The slow magnetosonic wave is found to be the damped mode and, therefore, the only propagating mode in our system is the fast magnetosonic mode. It is found that the KP equation admits compressive solitary structures. Finally, it is found that the amplitude of the soliton increases as the ratio of electron to ion concentration, $p$, angle $\theta$, and the plasma beta, $\beta$, is increased.

## 1. Introduction

Dusty plasmas have gained much attention in recent years [1–4]. A dusty plasma is defined as a normal electron plasma with an additional constituent of micrometre or submicrometre sized particles. The presence of this additional component enhances the complexity of the system and, therefore, dusty plasmas are often referred to as complex plasmas. Dusty plasmas are low-temperature, fully or partially ionized electrically conducting fluids composed mainly of electrons, ions, charged dust grains, and neutral atoms. The applications of dusty plasmas include space, astrophysical, and laboratory plasmas [5–8].

Fast and slow magnetosonic modes are the fundamental modes of a magnetized plasma from the magnetohydrodynamic (MHD) stand point. The fast magnetosonic waves propagate perpendicular to the ambient magnetic field. One of the primary examples of such a wave is the Earth's bow shock that is formed due to the interaction between the solar wind and the obstacle of the Earth's magnetic field. Although it is a nonlinear wave, it is termed as a fast magnetosonic wave in the MHD picture.

A vast amount of literature is found on the linear and nonlinear studies of magnetosonic waves [9–13]. Maruyama et al. [14] studied the interactions of nonthermal energetic ions with nonlinear magnetosonic waves by means of a one-dimensional (one space coordinate and three velocity components), relativistic, electromagnetic particle simulation code with full ion and electron dynamics and found that in a plasma with strong MHD turbulence, some of the ions could be repeatedly accelerated to higher energies by many nonlinear pulses. De Juli and Schneider [15] analyzed the absorption of the magnetosonic wave due to the presence of dust particles with variable charge and the modification of this absorption due to finite-Larmor-radius effects. Brodin et al. [16] studied the nonlinear interaction between the Alfvén and slow magnetosonic wave and showed that the efficiency of the process can be significantly enhanced if the Alfvén wave propagation directions are appropriately chosen. The authors discussed the relevance of their study to investigate the nonlinear excitation of MHD waves in space plasmas. Mushtaq and Shah [17] studied the linear and nonlinear two-dimensional magnetosonic waves in electron–positron–ion (e–p–i) plasma and derived the Kadomstev–Petviashvili (KP) soliton equation using the reductive perturbative scheme for fast and slow magnetosonic modes in the nonlinear regime. The authors found that the propagation properties of the solitary waves depended on positron concentration, the angle $\theta$ (that the external magnetic field makes with the $x$-axis), and the plasma beta. Shukla and Rahman [18] studied the magnetohydrodynamics of dusty plasmas (assuming the wave frequency to be much smaller than the ion gyrofrequency) and showed the linear coupling between dust-Alfvén, dust-magnetosonic, and dust whistler waves. The authors also found the dust inertia to play a major role in the wave dynamics. Alam et al. [19] studied the obliquely propagating waves in a magnetized plasma with stationary dust for both low and high frequencies of the ion cyclotron frequency and found that the fluctuations in grain charges due to liberation of additional electrons and protons induce momentum change giving rise to a change in the stability criterion of the electrostatic and electromagnetic waves in dusty plasmas. Recently, Marklund et al. [20] studied the ion magnetosonic solitons in dusty plasmas and found that the small number of dust particles led to the formation of static nonlinear structures instead of shocklets that are normally formed in usual magnetoplasmas.

In this paper, we investigate the obliquely propagating dust magnetosonic wave both in the linear and nonlinear regimes. Using a reductive perturbative technique, we derive the nonlinear KP equation for both fast and slow dust magnetosonic waves. The effects of electron concentration, $p$, the angle $\theta$ which the magnetic field makes with the $x$-axis, and the plasma beta (the ratio of plasma kinetic to magnetic energy) on the dynamics of the dust magnetosonic waves are studied both analytically and numerically. The paper is organized as follows. In Sec. 2, we present the basic set of equations and outline the reductive perturbative scheme used to obtain the linear and nonlinear sets of equations. In Sec. 3, we derive the KP equation along with its stationary solution for the obliquely propagating dust magnetosonic wave. In Sec. 4, we recapitulate and conclude the findings of our paper.

## 2. Governing equations

We present here linear and nonlinear investigations of a three-component e–d–i magnetoplasma. We consider a Cartesian system in which the background magnetic field $B_0$ lies in the $(x, y)$ plane making a small angle $\theta$ with the $x$-axis, and the propagation in the nonlinear regime is considered in the $(x, z)$ plane. The governing equations used in this paper are the effective one-fluid isothermal MHD equations. We develop the effective one-fluid model for the e–d–i plasma by writing down the usual fluid equations for electrons, dust, and ions. The electron and ion inertia is ignored as we are interested in studying the very-low-frequency dust wave. We also ignore the displacement current term in Ampère's law owing to the same reason. The effective one-fluid model for a dusty plasma developed here closely follows the work of Rao [21] and the basic set of equations reads as follows.

The inertialess electron and ion momentum equations are given by

$$0 = -n_\mathrm{e} e\mathbf{E} - \frac{en_\mathrm{e}}{c}(v_\mathrm{e} \times \mathbf{B}) - \nabla p_\mathrm{e}, \tag{1}$$

$$0 = n_\mathrm{i} e\mathbf{E} + \frac{en_\mathrm{i}}{c}(v_\mathrm{i} \times \mathbf{B}) - \nabla p_\mathrm{i}. \tag{2}$$

The inertial cold dust momentum equation is given by

$$m_\mathrm{d} n_\mathrm{d} \frac{d\mathbf{v}_\mathrm{d}}{dt} = -en_\mathrm{d}^* \mathbf{E} - \frac{en_\mathrm{d}^*}{c}(\mathbf{v}_\mathrm{d} \times \mathbf{B}). \tag{3}$$

The continuity equation for species $\alpha$ (where $\alpha = \mathrm{e}, \mathrm{i}, \mathrm{d}$) is given by

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha v_\alpha) = 0. \tag{4}$$

Maxwell's equations are given by

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{5}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}, \tag{6}$$

where the current density $j$ is defined as

$$\mathbf{j} = en_\mathrm{i}\mathbf{v}_\mathrm{i} - en_\mathrm{e}\mathbf{v}_\mathrm{e} - en_\mathrm{d}^*\mathbf{v}_\mathrm{d} \tag{7}$$

where $n_\mathrm{d}^* = z_\mathrm{d} n_\mathrm{d}$. The quantities $\mathbf{v}_\alpha, n_\alpha (n_{\alpha 0})$, and $p_\alpha$ are the fluid velocities, particle densities, and thermal pressures of species $\alpha$, respectively, which is given by the relation $p_\alpha = n_\alpha k_\mathrm{B} T_\alpha$, $T_\alpha$ being the temperature of the species $\alpha$ (the dust temperature is ignored here). $m_\mathrm{d}$ is the dust mass, $e$ is the charge of species $\alpha$, and $z_\mathrm{d}$ is the charge number associated with the dust. $c$ is the velocity of light, $\mathbf{E}$ is the electric field vector, $\mathbf{B}$ is the magnetic field vector, and $d/dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ is the convective fluid derivative.

In order to arrive at the governing set of equations for an e–d–i plasma, we substitute for $v_\mathrm{e}$ in (1) to get

$$\mathbf{E} = -\frac{1}{en_\mathrm{e}c}\left(en_\mathrm{i}\mathbf{v}_\mathrm{i} - en_\mathrm{d}^*\mathbf{v}_\mathrm{d} - \frac{c}{4\pi}\nabla \times \mathbf{B}\right) \times \mathbf{B} - \frac{1}{en_\mathrm{e}}\nabla p_\mathrm{e}. \tag{8}$$

Using (2), (3), and (7), we obtain the following momentum equations for ion and dust species, respectively:

$$0 = -\frac{en_\mathrm{i}n_\mathrm{d}}{n_\mathrm{e}c}\mathbf{v}_\mathrm{i} \times B + \frac{en_\mathrm{i}n_\mathrm{d}}{n_\mathrm{e}c}\mathbf{v}_\mathrm{d} \times \mathbf{B} + \frac{1}{4\pi}\frac{n_\mathrm{i}}{n_\mathrm{e}}(\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{n_\mathrm{i}}{n_\mathrm{e}}\nabla p_\mathrm{e} - \nabla p_\mathrm{i}, \tag{9}$$

$$m_{\rm d}n_{\rm d}\frac{d\mathbf{v}_{\rm d}}{dt} = \frac{en_{\rm i}n_{\rm d}}{n_{\rm e}c}\mathbf{v}_{\rm i}\times\mathbf{B} - \frac{en_{\rm i}n_{\rm d}}{n_{\rm e}c}\mathbf{v}_{\rm d}\times\mathbf{B} - \frac{1}{4\pi}\frac{n_{\rm d}}{n_{\rm e}}(\nabla\times\mathbf{B})\times\mathbf{B} + \frac{n_{\rm d}}{n_{\rm e}}\nabla p_{\rm e}. \tag{10}$$

Using the quasi-neutrality condition $n_{\rm i} = n_{\rm e} + n_{\rm d}^*$ (since we are treating long-wavelength MHD waves here), and adding (9) and (10), we obtain

$$n_{\rm d}\frac{d\mathbf{v}_{\rm d}}{dt} = \frac{n_{0\rm d}v_{\rm Ad}^2}{B_0^2}(\nabla\times\mathbf{B})\times\mathbf{B} - c_{\rm sd}^2(1+\sigma)\nabla n_{\rm i} + c_{\rm sd}^2\nabla n_{\rm d}^*, \tag{11}$$

where $\sigma = T_{\rm i}/T_{\rm e}$ is the ion to electron temperature ratio, $c_{\rm sd} = \sqrt{k_{\rm B}T_{\rm e}/m_{\rm d}}$ is the dust acoustic speed, and $v_{\rm Ad} = B_0/\sqrt{4\pi m_{\rm d}n_{\rm d}}$ is the dust Alfvén velocity. Eliminating $E$ between (3) and (8), the magnetic induction so obtained is given as follows:

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}_{\rm d}\times\mathbf{B}) + \frac{B_0}{\Omega_{\rm cd}^*}\nabla\times\frac{d\mathbf{v}_{\rm d}}{dt}, \tag{12}$$

where $\Omega_{\rm cd}^* = z_{\rm d}\Omega_{\rm cd}$ ($\Omega_{\rm cd} = eB_0/m_{\rm d}c$ is the dust cyclotron frequency). From (9), we obtain the expression for the perpendicular ion velocity component, $v_{\perp\rm i}$, as

$$\begin{aligned} v_{\perp\rm i} = {} & \frac{1}{B^2}\mathbf{B}\times(\mathbf{v}_{\rm d}\times\mathbf{B}) + \frac{c}{4\pi e n_{\rm d}^* B^2}\mathbf{B}\times(\nabla\times\mathbf{B})\times\mathbf{B} \\ & - \frac{c}{e n_{\rm d}^* B^2}\mathbf{B}\times\nabla p_{\rm e} - \frac{c n_{\rm e}}{e n_{\rm i} n_{\rm d}^* B^2}\mathbf{B}\times\nabla p_{\rm i}. \end{aligned} \tag{13}$$

Neglecting the ion fluid velocity component parallel to the magnetic field using the arguments given in Ref. [18], and using Eq. (13) in the ion continuity equation given by Eq. (4), we arrive at the following expression.

$$\begin{aligned} \frac{\partial n_{\rm i}}{\partial t} + \nabla\cdot\frac{n_{\rm i}}{B^2}\Bigg(&\mathbf{B}\times(\mathbf{v}_{\rm d}\times\mathbf{B}) + \frac{c}{4\pi e n_{\rm d}^*}\mathbf{B}\times(\nabla\times\mathbf{B})\times\mathbf{B} \\ & - \frac{c}{e n_{\rm d}^*}\mathbf{B}\times\nabla p_{\rm e} - \frac{c n_{\rm e}}{e n_{\rm d}^* n_{\rm i}}\mathbf{B}\times\nabla p_{\rm i}\Bigg) = 0. \end{aligned} \tag{14}$$

Equations (11) and (14) together with the dust continuity equation (4) and the magnetic induction equation (12) form the basic set of equations of the effective one-fluid model for a dusty plasma.

Using the reductive perturbative technique, we can reduce the above set of equations into a simpler form so that the linear and nonlinear analyses become possible. The reductive perturbative technique is generally employed to investigate waves whose wavelengths are longer when compared with the typical length scale of the system. It has been extensively used to study small-amplitude waves in plasma physics [22–28]. The technique enables us to rescale both space and time variables, thereby facilitating the analysis of long-wavelength phenomena. It may be noted that the reductive perturbative technique is a special form of multiple scale analysis [29]. Following [30], we expand the variables in the following manner:

$$
\begin{aligned}
n_{\mathrm{i}} &= n_{\mathrm{i,d0}} + \epsilon n_{\mathrm{i,d1}} + \epsilon^2 n_{\mathrm{i,d2}} + \cdots ,\\
v_{\mathrm{d}x} &= \epsilon u_{\mathrm{d1}} + \epsilon^2 u_{\mathrm{d2}} + \cdots ,\\
v_{\mathrm{d}y} &= \epsilon v_{\mathrm{d1}} + \epsilon^2 v_{\mathrm{d2}} + \cdots ,\\
v_{\mathrm{d}z} &= \epsilon^{3/2} w_{\mathrm{d1}} + \epsilon^{5/2} w_{\mathrm{d2}} + \cdots ,\\
B_x &= B_0 \cos\theta,\\
B_y &= B_0 \sin\theta + \epsilon B_{y1} + \epsilon^2 B_{y2} + \cdots ,\\
B_z &= \epsilon^{3/2} B_{z1} + \epsilon^{5/2} B_{z2} + \cdots .
\end{aligned}
\tag{15}
$$

Note that the anisotropy in the velocity components is due to the influence of a strong magnetic field. In this expansion, the fluid gyromotion is treated as a higher-order effect. It should also be noted that the background magnetic field lies in the $(x, y)$ plane making an angle $\theta$ with the $x$-axis and the wave vector $k$ in the $(x, z)$ plane makes an angle with the $x$-axis that has a direct relation to the angle $\theta$. Thus what is done is that if the wave vector $k$ is taken in the $(x, y)$ plane then it can be rotated in such a way that it lies along the $x$-axis and the magnetic field which was originally in the $x$-direction now lies in the $(x, y)$ plane. The obliqueness in the $z$-direction occurs only in the higher-order (nonlinear) terms and appears via the stretched variable $\eta$ given below. This is done for mathematical ease and here we have followed the works of De Vito and Pantano [29] and Shah and Bruno [31], where the same scheme was used to study nonlinear magnetosonic waves in cold and hot electron ion plasmas via the KP equation, respectively.

It should be noted that all the perturbed quantities are functions of $x$, $z$, and $t$, and $\epsilon$ is a small parameter such that $\epsilon < 1$. Employing standard procedures, we stretch the variables as follows:

$$
\begin{aligned}
\xi &= \epsilon^{1/2}(x - v_{\mathrm{ph}} t),\\
\eta &= \epsilon z,\\
\tau &= \epsilon^{3/2} t,
\end{aligned}
\tag{16}
$$

where $v_{\mathrm{ph}}$ is the phase velocity of the wave whose exact expression is calculated below. The significance of the variable stretching procedure lies in its ability to introduce the new variables in such a way that the slowness of the coordinate dependence and the smallness of some of the physical variables are taken out in a systematic manner.

Normalizing (4) and (11)–(14), using (15) and (16) and collecting terms of the lowest order, i.e. ($\epsilon^{3/2}$), we obtain

$$
\lambda \frac{\partial}{\partial \xi} n_{\mathrm{d1}} = \frac{\partial}{\partial \xi} u_{\mathrm{d1}}, \tag{17}
$$

$$
\lambda \frac{\partial}{\partial \xi} u_{\mathrm{d1}} = \sin\theta \frac{\partial}{\partial \xi} B_{y1} + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p} \frac{\partial}{\partial \xi} n_{\mathrm{i1}} - z_{\mathrm{d}}\beta \frac{\partial}{\partial \xi} n_{\mathrm{d1}}, \tag{18}
$$

$$\lambda\frac{\partial}{\partial\xi}v_{\mathrm{d}1} = -\cos\theta\frac{\partial}{\partial\xi}B_{y1}, \tag{19}$$

$$\lambda\frac{\partial}{\partial\xi}B_{y1} = \sin\theta\frac{\partial}{\partial\xi}u_{\mathrm{d}1} - \cos\theta\frac{\partial}{\partial\xi}v_{\mathrm{d}1}, \tag{20}$$

$$\lambda\frac{\partial}{\partial\xi}n_{\mathrm{i}1} = \sin^2\theta\frac{\partial}{\partial\xi}u_{\mathrm{d}1} - \cos\theta\sin\theta\frac{\partial}{\partial\xi}v_{\mathrm{d}1}, \tag{21}$$

where $\beta = c_{\mathrm{sd}}^2/v_{\mathrm{Ad}}^2$ and $\lambda$ is the normalized phase velocity given by $\lambda = v_{\mathrm{ph}}/v_{\mathrm{Ad}}$. The perturbed number densities of the species $\alpha$ have been normalized by their respective background counterparts, the velocities have been normalized by the dust Alfvén velocity, the perturbed magnetic field components are divided by the ambient magnetic field $B_0$ and $p = n_{\mathrm{e}0}/n_{\mathrm{i}0}$. Using (17)–(21), the fluctuating variables $u_{\mathrm{d}1}, v_{\mathrm{d}1}, B_{y1}$, and $n_{\mathrm{i}1}$ can be expressed in terms of $n_{\mathrm{d}1}$ as

$$u_{\mathrm{d}1} = \lambda n_{\mathrm{d}1}, \tag{22}$$

$$v_{\mathrm{d}1} = -\frac{\lambda\cos\theta}{\sin\theta}\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)n_{\mathrm{d}1}, \tag{23}$$

where $\beta' = z_{\mathrm{d}}c_{\mathrm{sd}}^2/v_{\mathrm{Ad}}^2$,

$$B_{y1} = \frac{\lambda^2}{\sin\theta}\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)n_{\mathrm{d}1}, \tag{24}$$

$$n_{\mathrm{i}1} = \left(\sin^2\theta+\cos^2\theta\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)\right)n_{\mathrm{d}1}. \tag{25}$$

Using (17)–(25), we obtain the dispersion relation which reads as

$$\lambda^2 = \frac{1}{2}\left[\left(1+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\sin^2\theta-\beta'\right)\right.$$
$$\left.\pm\sqrt{\left(1+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\sin^2\theta-\beta'\right)^2+4\cos^2\theta\beta'}\right]. \tag{26}$$

Equation (26) is the dispersion relation for obliquely propagating two-dimensional dust magnetosonic waves propagating in a dusty plasma. The dispersion relation shows that the propagation of the low-frequency dust magnetosonic wave depends on the angle $\theta$, the ratio of the ion to electron concentration, the plasma beta (the ratio of kinetic to magnetic energy) given by $\beta$ here, the charge number associated with the dust, and the relative concentration of electrons $p$. The upper (positive) and lower (negative) signs represent the fast and slow dust magnetosonic modes, respectively. We have selected a range of $p$, $\theta$, and $\beta$ values and the slow magnetosonic wave has been found to be a damped mode for all of them. We therefore plot the fast magnetosonic wave against the different plasma parameters to investigate the linear and nonlinear behavior in this study.

Figure 1 shows the variation of the phase velocity of a fast dust magnetosonic wave with the electron concentration $p$. It indicates an increase in the phase velocity with increasing values of $p$ in a three-component dusty plasma. It is also found that the phase velocity of the wave increases with increase of the angle $\theta$ (the angle the

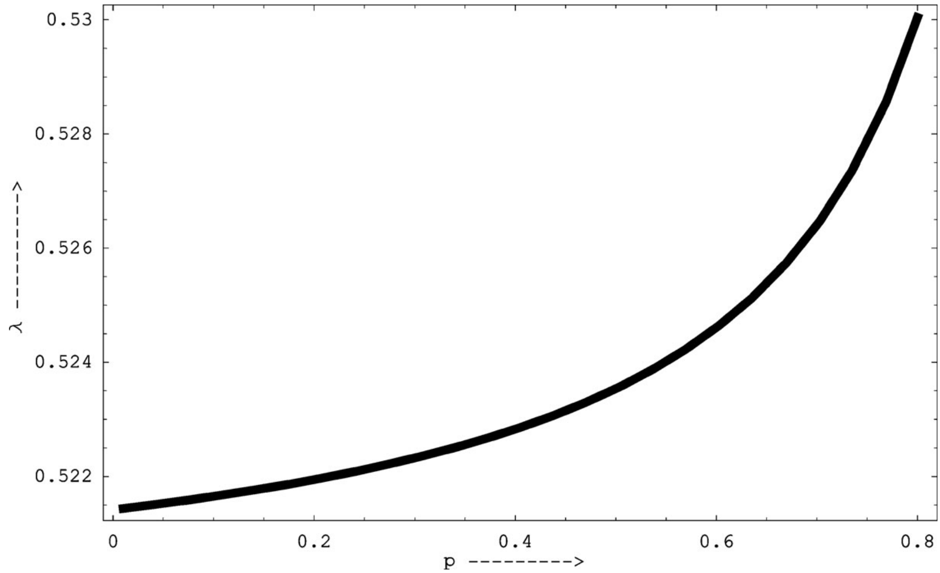


**Figure 1.** Variation of the phase velocity $\lambda$ of a fast dust magnetosonic wave with respect to $p$ $(=n_{e0}/n_{i0})$. Other physical parameters are taken arbitrarily as $\sigma = 0.02$, $\theta = 15°$, $\beta = 0.1$, and $z_d = 100$.

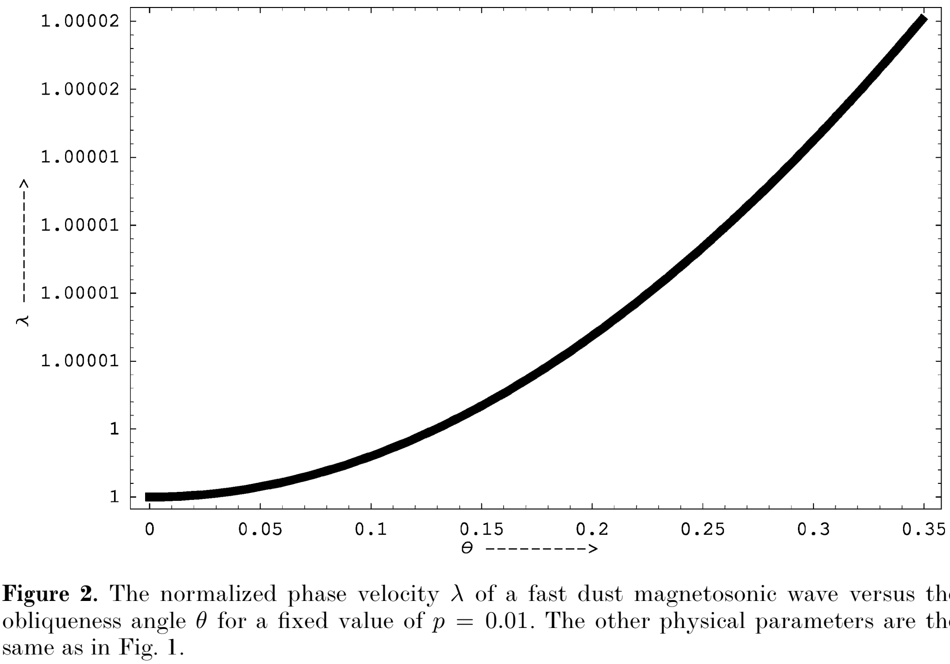


**Figure 2.** The normalized phase velocity $\lambda$ of a fast dust magnetosonic wave versus the obliqueness angle $\theta$ for a fixed value of $p = 0.01$. The other physical parameters are the same as in Fig. 1.

ambient magnetic field makes with the $x$-axis) (see Fig. 2). However, it should be noted that the results are valid only for small values of angle $\theta$.

Figure 3 shows that the phase velocity of the wave increases with increasing $\beta$ meaning thereby that the propagation properties of the dust magnetosonic wave

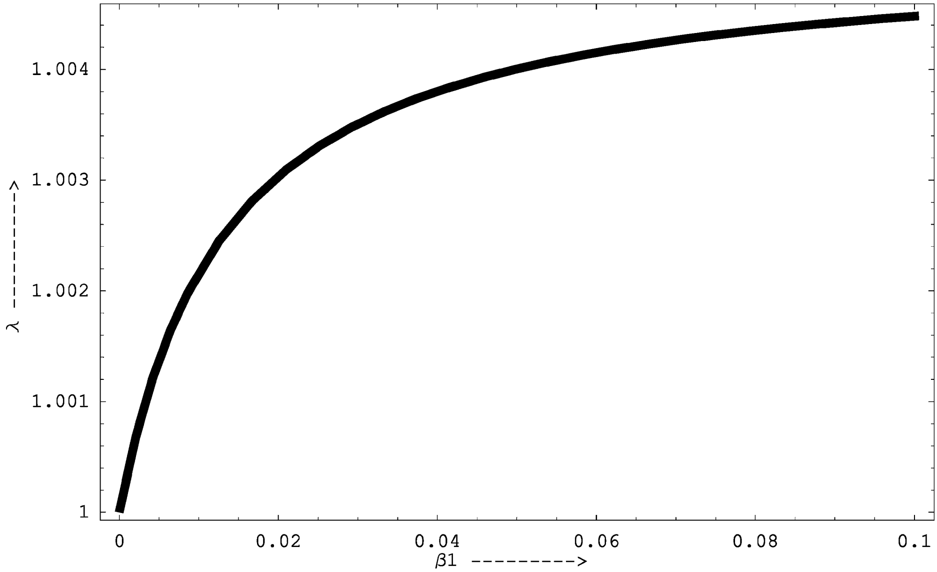


**Figure 3.** Variation of the linear phase velocity $\lambda$ with increasing values of $\beta$ ($=c_{\mathrm{sd}}^2/v_{\mathrm{Ad}}^2$). The other parameters are the same as in Figs 1 and 2.

become significant for high-beta plasmas. It is worth noting here that all the above-mentioned features occur due to the minus sign appearing in the equations due to negatively charged dust. This leads to a competition between the terms $\beta'$ and $[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta$ which give rise to all the interesting features that we observe both in the linear and nonlinear regimes. It should be mentioned that the slow magnetosonic wave is not a damped mode in the electron–ion [31] or electron–positron–ion [17] plasma but it is for a dusty plasma with negatively charged dust owing to the reason described above. If we set $n_{\mathrm{i}} \longrightarrow 0$, $m_{\mathrm{d}} \longrightarrow m_{\mathrm{i}}$, $n_{\mathrm{d}} \longrightarrow n_{\mathrm{i}}$, and $-ez_{\mathrm{d}} \longrightarrow e$, we retrieve the dispersion relation for an electron–ion plasma given in [32].

The dispersion relation given by (26) differs from the one derived in [18] because the authors there ignored the pressure contribution due to electrons by assuming that $n_{\mathrm{e}}(T_{\mathrm{e}}+T_{\mathrm{i}}) \ll n_{\mathrm{d}}(T_{\mathrm{d}}+T_{\mathrm{i}})$. Note that we consider here cold magnetized dust as opposed to the hot magnetized dust assumed by Shukla and Rahman [18]. It turns out that the above-mentioned assumption hid the interesting information introduced by retaining the electron pressure term, i.e. the competition between the terms $\beta'$ and $[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta$ giving rise to novel features of the $\theta$ and $\beta$ dependences of the dust magnetosonic wave. It is also worth mentioning that if we take positive dust then one can get both the slow and fast modes of the magnetosonic wave as given in [17] for electron–positron–ion plasmas.

## 3. Nonlinear analysis

We develop the nonlinear KP soliton for the fast two-dimensional magnetosonic modes (the slow mode is a damped mode in the present study) in a dusty plasma by normalizing (4) and (11)–(14), using (15) and (16) and collecting terms of the order $\epsilon^2$ and $\epsilon^{5/2}$. In order $\epsilon^2$, we have the $z$-component of the momentum equation,

$$\lambda \frac{\partial w_{\mathrm{d1}}}{\partial \xi} = \sin\theta \frac{\partial B_{y1}}{\partial \eta} - \cos\theta \frac{\partial B_{z1}}{\partial \xi} + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\frac{\partial n_{\mathrm{i1}}}{\partial \eta} - \beta \frac{\partial n^*_{\mathrm{d1}}}{\partial \eta} \tag{27}$$

and the $z$-component of the magnetic induction equation,

$$\lambda \frac{\partial B_{z1}}{\partial \xi} = -\cos\theta \frac{\partial w_{\mathrm{d1}}}{\partial \xi} + \frac{\lambda v_{\mathrm{Ad}}}{\Omega^*_{\mathrm{cd}}}\frac{\partial^2 v_{\mathrm{d1}}}{\partial \xi^2}. \tag{28}$$

Using (23)–(25), we obtain from (27) and (28)

$$\frac{\partial w_{\mathrm{d1}}}{\partial \xi} = \frac{\lambda\Theta_1}{(\lambda^2-\cos^2\theta)}\frac{\partial n_{\mathrm{d1}}}{\partial \eta} + \frac{\Theta_2\lambda^2 v_{\mathrm{Ad}}\cos^2\theta}{\Omega^*_{\mathrm{cd}}\sin\theta(\lambda^2-\cos^2\theta)}\frac{\partial^2 n_{\mathrm{d1}}}{\partial \xi^2}, \tag{29}$$

where

$$\begin{aligned}\Theta_1 = &\left(\frac{1}{\lambda^2 + [z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)\\ &\times\left(\lambda^2\left(\lambda^2+\beta' - \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\sin^2\theta\right) + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\left(\lambda^2 + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right.\\ &\times\left[\sin^2\theta + \cos^2\theta\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)\right]\\ &\left.-\beta'\left(\lambda^2 + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right),\end{aligned}$$

$$\Theta_2 = \frac{(\lambda^2+\beta'-[\beta(1+\sigma)/(1-p)]\sin^2\theta)}{(\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta)},$$

and

$$\frac{\partial B_{z1}}{\partial \xi} = -\frac{\Theta_1\cos\theta}{(\lambda^2-\cos^2\theta)}\frac{\partial n_{\mathrm{d1}}}{\partial \eta} - \frac{\Theta_2\lambda v_{\mathrm{Ad}}\cos\theta}{\Omega^*_{\mathrm{cd}}\sin\theta}\frac{(1+\cos^2\theta)}{(\lambda^2-\cos^2\theta)}\frac{\partial^2 n_{\mathrm{d1}}}{\partial \xi^2}. \tag{30}$$

Collection of $\epsilon^{5/2}$ terms give us the following set of equations:

(i)

$$-\lambda\frac{\partial}{\partial\xi}n_{\mathrm{d2}} + \frac{\partial}{\partial\xi}u_{\mathrm{d2}} = f_1 \tag{31}$$

where

(ii)

$$f_1 = -\frac{1}{v_{\mathrm{Ad}}}\frac{\partial}{\partial\tau}n_{\mathrm{d1}} - \frac{\partial}{\partial\xi}(n_{\mathrm{d1}}u_{\mathrm{d1}}) - \frac{\partial}{\partial\eta}w_{\mathrm{d1}},$$

$$-\lambda\frac{\partial}{\partial\xi}u_{\mathrm{d2}} + \sin\theta\frac{\partial}{\partial\xi}B_{y2} + \frac{\beta(1+\sigma)}{1-p}\frac{\partial}{\partial\xi}n_{\mathrm{i2}} - \beta\frac{\partial}{\partial\xi}n_{\mathrm{d2}} = f_2 \tag{32}$$

where

(iii)

$$f_2 = -\frac{1}{v_{\mathrm{Ad}}}\frac{\partial}{\partial \tau}u_{\mathrm{d1}} + \lambda n_{\mathrm{d1}}\frac{\partial}{\partial \xi}u_{\mathrm{d1}} - B_{y1}\frac{\partial}{\partial \xi}B_{y1} - u_{\mathrm{d1}}\frac{\partial}{\partial \xi}u_{\mathrm{d1}},$$

$$-\lambda\frac{\partial}{\partial \xi}v_{\mathrm{d2}} - \cos\theta\frac{\partial}{\partial \xi}B_{y2} = f_3 \tag{33}$$

where

(iv)

$$f_3 = -\frac{1}{v_{\mathrm{Ad}}}\frac{\partial}{\partial \tau}v_{\mathrm{d1}} + \lambda n_{\mathrm{d1}}\frac{\partial}{\partial \xi}v_{\mathrm{d1}} - u_{\mathrm{d1}}\frac{\partial}{\partial \xi}v_{\mathrm{d1}},$$

$$-\lambda\frac{\partial}{\partial \xi}B_{y2} - \cos\theta\frac{\partial}{\partial \xi}v_{\mathrm{d2}} + \sin\theta\frac{\partial}{\partial \xi}u_{\mathrm{d2}} = f_4 \tag{34}$$

where

(v)

$$f_4 = -\frac{1}{v_{\mathrm{Ad}}}\frac{\partial}{\partial \tau}B_{y1} - \frac{\partial}{\partial \xi}(u_{\mathrm{d1}}B_{y1}) - \frac{\lambda v_{\mathrm{Ad}}}{\Omega^*_{\mathrm{cd}}}\frac{\partial^2}{\partial\eta\partial\xi}u_{\mathrm{d1}} + \frac{\lambda v_{\mathrm{Ad}}}{\Omega^*_{\mathrm{cd}}}\frac{\partial^2}{\partial\xi^2}w_{\mathrm{d1}} - \sin\theta\frac{\partial}{\partial\eta}w_{\mathrm{d1}},$$

$$-\lambda\frac{\partial}{\partial \xi}n_{\mathrm{i2}} + \sin^2\theta\frac{\partial}{\partial \xi}u_{\mathrm{d2}} - \cos\theta\sin\theta\frac{\partial}{\partial \xi}v_{\mathrm{d2}} = f_5 \tag{35}$$

where

(vi)

$$f_5 = -\frac{1}{v_{\mathrm{Ad}}}\frac{\partial}{\partial \tau}n_{\mathrm{i1}} - 2\sin\theta\cos^2\theta\frac{\partial}{\partial \xi}(u_{\mathrm{d1}}B_{y1}) - \sin^2\theta\frac{\partial}{\partial \xi}(n_{\mathrm{i1}}u_{\mathrm{d1}})$$

$$+\cos\theta\sin\theta\frac{\partial}{\partial \xi}(n_{\mathrm{i1}}v_{\mathrm{d1}}),$$

$$\cos^3\theta\frac{\partial}{\partial \xi}(v_{\mathrm{d1}}B_{y1}) - \frac{v_{\mathrm{Ad}}\cos\theta\sin\theta}{\Omega_{\mathrm{cd}}}\frac{\partial^2}{\partial\xi^2}B_{z1} - \frac{\partial}{\partial\eta}w_{\mathrm{d1}} - \frac{v_{\mathrm{Ad}}}{\Omega_{\mathrm{cd}}}\cos^2\theta\frac{\partial^2}{\partial\eta\partial\xi}B_{y1}$$

$$-\cos\theta\frac{\partial w_{\mathrm{d1}}}{\partial\eta} + \frac{\lambda v_{\mathrm{Ad}}}{\Omega^*_{\mathrm{cd}}}\frac{\partial^2 v_{\mathrm{d1}}}{\partial\eta\partial\xi} = 0. \tag{36}$$

Solving (29)–(33) and using the dispersion relation (given by (26)), we arrive at the following expression:

$$Vf_1 + Wf_2 + Xf_3 + Yf_4 + Zf_5 = 0, \tag{37}$$

where

$$V = \left(-\lambda\sin^2\theta\left(\lambda^2 + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right)$$
$$-\left((-\lambda^2 + \cos^2\theta)\left(\lambda^2 - \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\sin^2\theta\right)\right),$$

$$W = -\lambda^2(-\lambda^2 + \cos^2\theta),$$

$$X = \left(\cos\theta\sin\theta\left(\lambda^2 + \frac{z_{\rm d}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right) + \frac{z_{\rm d}\beta(1+\sigma)}{1-p}\cos\theta\sin\theta,$$

$$Y = \lambda\sin\theta\left(\lambda^2 + \frac{z_{\rm d}\beta(1+\sigma)}{1-p}\cos^2\theta\right) \quad \text{and} \quad Z = -\lambda\frac{z_{\rm d}\beta(1+\sigma)}{1-p}.$$

Now differentiating (35) with respect to $\xi$, using equations of the order $\epsilon^2$ and $\epsilon^{5/2}$, and expressing all the quantities in terms of one variable $\phi$ (assuming $n_{\rm d1}/n_{\rm d0} = \phi$), we get after some algebraic manipulation the desired KP equation:

$$\frac{\partial}{\partial\xi}\left(\frac{\partial}{\partial\tau}\phi + A\frac{\partial}{\partial\xi}\phi^2 + C\frac{\partial^3}{\partial\xi^3}\phi\right) + D\frac{\partial^2}{\partial\eta^2}\phi = 0. \tag{38}$$

The coefficients read as $A = A_1/A_0$, $C = C_1/A_0$, and $D = D_1/A_0$, where

$$A_0 = \frac{-V\sin\theta\Gamma_2 - W\Gamma_3^*\sin\theta\Gamma_2 + X\lambda\cos\theta\Gamma_1 - Y\lambda^2\Gamma_1 - Z\Gamma_2\Gamma_3\sin\theta}{v_{\rm Ad}\sin\theta\Gamma_2},$$

$$\begin{aligned} A_1 = {} & \left(\frac{1}{2\sin^2\theta\Gamma_2^2}\right)(-2V\Gamma_3^*\sin^2\theta\Gamma_2^2 + W\lambda\Gamma_3^*\sin^2\theta\Gamma_2^2 + W\lambda^4\Gamma_1^2 \\ & + X\lambda\cos\theta\sin^2\theta\Gamma_3^*\Gamma_1\Gamma_2 - X\lambda^2\cos\theta\sin\theta\Gamma_1\Gamma_2 - 2Y\lambda^2\Gamma_3^*\Gamma_1\Gamma_2\sin^2\theta \\ & - 4Z\lambda^2\Gamma_3^*\Gamma_2\cos^2\theta\sin^2\theta - 2Z\sin^4\theta\Gamma_2^2\Gamma_3^*\Gamma_3 \\ & - 2Z\lambda\cos^2\theta\sin^2\theta\Gamma_1\Gamma_2\Gamma_3 - 2Z\lambda^3\Gamma_1^2\cos^4\theta),\end{aligned}$$

$$C_1 = \frac{Y\lambda v_{\rm Ad}\Omega_{\rm cd}\Gamma_9 - Zv_{\rm Ad}\Omega_{\rm cd}^*\Gamma_7\sin\theta\cos^2\theta}{\Omega_{\rm cd}\Omega_{\rm cd}^*},$$

$$\begin{aligned} D_1 = {} & \left(\frac{1}{\Omega_{\rm cd}\Omega_{\rm cd}^*\Gamma_2(\Gamma_{10}-\Gamma_5)\sin\theta}\right)(-V\Omega_{\rm cd}\Omega_{\rm cd}^*\Gamma_2\Gamma_4\Gamma_{10}\sin\theta \\ & - Yv_{\rm Ad}\Omega_{\rm cd}\Gamma_2\Gamma_3^*\Gamma_4\sin\theta + Y\lambda v_{\rm Ad}\Omega_{\rm cd}\Gamma_2\Gamma_4\Gamma_8\sin\theta - Y\Omega_{\rm cd}\Omega_{\rm cd}^*\Gamma_2\Gamma_4\Gamma_{10}\sin^2\theta \\ & - Zv_{\rm Ad}\Omega_{\rm cd}^*\Gamma_2\Gamma_4\Gamma_6\cos^2\theta\sin^2\theta - Z\Omega_{\rm cd}\Omega_{\rm cd}^*\Gamma_2\Gamma_4\Gamma_{10}\sin\theta \\ & - Z\lambda^2 v_{\rm Ad}\Omega_{\rm cd}^*\Gamma_1\Gamma_4\cos^2\theta),\end{aligned}$$

with

$$\Gamma_1 = \lambda^2 + \beta' - \frac{z_{\rm d}\beta(1+\sigma)}{1-p}\sin^2\theta,$$

$$\Gamma_2 = \lambda^2 + \frac{z_{\rm d}\beta(1+\sigma)}{1-p}\cos^2\theta, \quad \Gamma_3 = \frac{\sin^2\theta\Gamma_2 + \cos^2\theta\Gamma_1}{\Gamma_2},$$

$$\Gamma_3^* = \frac{\lambda^2\Gamma_1 + [z_{\rm d}\beta(1+\sigma)/(1-p)]\Gamma_2\Gamma_3 - \beta'\Gamma_2}{\lambda\Gamma_2},$$

$$\Gamma_4 = \left(\frac{2(\lambda^2\Gamma_1 + [z_{\rm d}\beta(1+\sigma)/(1-p)]\Gamma_2\Gamma_3 - \beta'\Gamma_2)}{\lambda\Gamma_2}\right),$$

$$\Gamma_5 = \frac{v_{\mathrm{Ad}}\lambda^2\Gamma_1}{\Omega^*_{\mathrm{cd}}\Gamma_2\sin\theta},$$

$$\Gamma_6 = \frac{\lambda^2(-\lambda^2+\cos^2\theta)\Gamma_1 + [z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\Gamma_2\Gamma_3 - \beta'\Gamma_2}{(-\lambda^2+\cos^2\theta)\Gamma_2},$$

$$\Gamma_7 = \frac{v_{\mathrm{Ad}}\lambda^3\cos\theta}{\Omega^*_{\mathrm{cd}}(-\lambda^2+\cos^2\theta)\sin\theta}\frac{\Gamma_1}{\Gamma_2},$$

$$\Gamma_8 = -\lambda\Gamma_6, \quad \Gamma_9 = -\left(\lambda\Gamma_7 + \frac{v_{\mathrm{Ad}}\lambda^2\Gamma_1}{\Omega^*_{\mathrm{cd}}\Gamma_2\sin\theta}\right), \quad \Gamma_{10} = -\frac{v_{\mathrm{Ad}}\lambda^2\Gamma_1}{\Omega^*_{\mathrm{cd}}\Gamma_2\sin\theta}.$$

In order to find the steady-state solution of the KP equation, i.e. (3), we have transformed the coordinates $\xi$, $\eta$, and $\tau$ into a new frame that moves with the soliton, i.e. $\chi = (k_\xi\xi + k_\eta\eta - V_0\tau)/\triangle$, where $V_0$ is the constant speed of the wave frame. Following [30], the steady-state solution of (3) can be written as

$$\phi = \phi_{\mathrm{m}}\, sech^2\chi, \tag{39}$$

where $\phi_{\mathrm{m}}$ is the maximum amplitude of an obliquely propagating dust magnetosonic soliton given by $\phi_{\mathrm{m}} = 3(V_0 - D)/2A$ and $\Delta$ is the width of the soliton which bears the relation $\Delta = [4C/(V_0 - D)]^{1/2}$. The subtle balance between the coefficients $A$, $C$, and $D$ results in the formation of the soliton in the KP equation as seen from the relations of maximum amplitude and width of the soliton.

Using (29), (30), and (39), we get expressions for the $z$-components of the velocity and magnetic field:

$$w_{\mathrm{d}1} = \left(\frac{\lambda\Theta_1}{(\lambda^2-\cos^2\theta)} - \frac{2\tanh\boldsymbol{\chi}\cos^2\theta v_{\mathrm{Ad}}\lambda^2\Theta_2}{\sin\theta\Omega^*_{\mathrm{cd}}\triangle(\lambda^2-\cos^2\theta)}\right)\phi, \tag{40}$$

$$B_{z1} = \left(-\frac{\Theta_1\cos\theta}{(\lambda^2-\cos^2\theta)} + \frac{2\Theta_2\lambda v_{\mathrm{Ad}}\cos\theta\tanh\boldsymbol{\chi}}{\Omega^*_{\mathrm{cd}}\triangle\sin\theta}\frac{(1+\cos^2\theta)}{(\lambda^2-\cos^2\theta)}\right)\phi. \tag{41}$$

Equations (40) and (41) give us useful information regarding variations in the transverse direction (perpendicular to the plane of the ambient magnetic field), on the basis of which one can predict the stability of the solitary structure, which is beyond the scope of this work.

Using (8), (13), (26), and (39), we find the expressions for the normalized electric field components of a two-dimensional obliquely propagating dust magnetosonic wave in e–d–i plasmas,

$$\frac{cE_x}{B_0} = -\frac{(1-p)\sin\theta}{pv_{\mathrm{Ad}}}w_{\mathrm{d}1} + \left(\frac{\sin\theta}{pv_{\mathrm{Ad}}}\Theta_3 + \Theta_4\right)\phi, \tag{42}$$

$$\frac{cE_y}{B_0} = \frac{(1-p)\cos\theta}{pv_{\mathrm{Ad}}}w_{\mathrm{d}1} + \left(-\frac{\cos\theta}{pv_{\mathrm{Ad}}}\Theta_3 + \Theta_5\right)\phi, \tag{43}$$

$$\frac{cE_z}{B_0} = \left(\frac{(2-p)}{p}\lambda\sin\theta - \frac{\lambda\cos^2\theta(\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta)}{\sin\theta(\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta)}\right)\phi, \tag{44}$$

where $\Theta_3 = \Pi_1 - \Pi_2 - \Pi_3 + \Pi_4$

$$\Pi_1 = \left\{\lambda\left(\lambda^2 + \left[z_{\mathrm{d}}\beta(1+\sigma)\left(\lambda^2 + \frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right.\right.\right.$$

$$
\times \left(\sin^2\theta + \cos^2\theta\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)\right)\Bigg](1-p)^{-1}
$$

$$
\times \left[-\beta'\left(\lambda^2+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right]\Bigg\}
$$

$$
\times \left[(\lambda^2-\cos^2\theta)\left(\lambda^2+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right]^{-1},
$$

$$
\Pi_2 = \left[2\tanh\boldsymbol{\chi}\cos^2\theta v_{\mathrm{Ad}}\left(\lambda^2+\beta'-\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\sin^2\theta\right)\right]
$$

$$
\times \left[\sin\theta\Omega^*_{\mathrm{cd}}\triangle(\lambda^2-\cos^2\theta)\left(\lambda^2+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right]^{-1}\lambda^2,
$$

$$
\Pi_3 = \left[2v_{\mathrm{Ad}}\tanh\boldsymbol{\chi}\left(\lambda^2+\beta'-\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\sin^2\theta\right)\lambda^2\right]
$$

$$
\times \left[\sin\theta\Omega^*_{\mathrm{cd}}\triangle\left(\lambda^2+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right]^{-1},
$$

$$
\Pi_4 = -\Bigg[2v_{\mathrm{Ad}}\beta(1+p\sigma)\tanh\boldsymbol{\chi}\sin\theta
$$

$$
\times \left(\sin^2\theta + \cos^2\theta\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)\right)\Bigg]
$$

$$
\times [\Omega_{\mathrm{cd}}(1-p)\triangle]^{-1} + \frac{2v_{\mathrm{Ad}}\beta'\tanh\boldsymbol{\chi}}{\Omega_{\mathrm{cd}}\triangle},
$$

$$
\Theta_4 = \frac{(1-p)}{p}\,\frac{2\tanh\boldsymbol{\chi}(\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta)\lambda^2}{\Omega^*_{\mathrm{cd}}(\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta)\triangle}
$$

$$
+ \left[2\beta\tanh\boldsymbol{\chi}\left(\sin^2\theta+\cos^2\theta\left(\frac{\lambda^2+\beta'-[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta}{\lambda^2+[z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta}\right)\right)\right]
$$

$$
\times (p\Omega_{\mathrm{cd}}\triangle)^{-1} - \frac{2(1-p)\beta\tanh\boldsymbol{\chi}}{p\Omega_{\mathrm{cd}}\triangle},
$$

and

$$
\Theta_5 = \frac{(1-p)}{p}\left[2\tanh\boldsymbol{\chi}\cos\theta\left(\lambda^2+\beta'-\frac{\beta(1+\sigma)}{1-p}\sin^2\theta\right)\lambda^2\right]
$$

$$
\times \left[\triangle p\sin\theta\left(\lambda^2+\frac{z_{\mathrm{d}}\beta(1+\sigma)}{1-p}\cos^2\theta\right)\right]^{-1}.
$$

It should be noted that (42)–(44) are useful for analysis of acceleration of the dust particle via $v_p \times B$ acceleration [33–35], where the dust particle can be accelerated up to $v \sim cE/B_0$, by the fast dust magnetosonic waves with effects of finite $\beta$, concentration and temperature of the species, and obliqueness angle $\theta$. The calculation and discussion on particle acceleration and consequently plasma heating

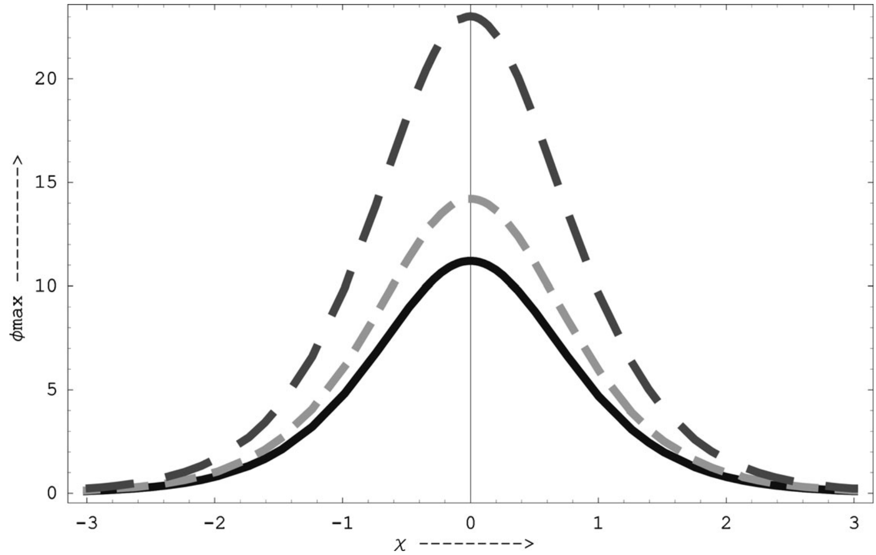


**Figure 4.** Variation of the maximum amplitude of the fast dust magnetosonic soliton as a function of $\chi$ for different values of the ratio of electron to ion concentration, $p$, in an e–d–i plasma. It is found that the maximum amplitude decreases with increasing values of $p$. The other parameters are $\sigma = 0.02$, $\theta = 15°$, $\beta = 0.01$, and $z_{\mathrm{d}} = 100$.

by the slow and fast modes of magnetosonic waves are given in general in the above-mentioned references.

Finally, with the application of (8), (26), (39), (40), and (41), we find the expressions for the normalized current density components of two-dimensional obliquely propagating dust magnetosonic waves in three-component dusty plasmas,

$$j_x = \left(\frac{2v_{\mathrm{Ad}}^2 \tanh \boldsymbol{\chi}(\lambda^2 + \beta' - [z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta)\lambda^2}{\triangle\Omega_{\mathrm{cd}}\sin\theta(\lambda^2 + [z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta)}\right)\phi, \tag{45}$$

$$j_y = \left(\frac{2v_{\mathrm{Ad}}^2 \tanh \boldsymbol{\chi}\Theta_2}{\triangle\Omega_{\mathrm{cd}}}\right)\phi, \tag{46}$$

$$j_z = -\left(\frac{2v_{\mathrm{Ad}}^2 \tanh \boldsymbol{\chi}(\lambda^2 + \beta' - [z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\sin^2\theta)\lambda^2}{\triangle\Omega_{\mathrm{cd}}\sin\theta(\lambda^2 + [z_{\mathrm{d}}\beta(1+\sigma)/(1-p)]\cos^2\theta)}\right)\phi. \tag{47}$$

Equations (45)–(47) are the parallel ($j_x$, $j_y$) and perpendicular ($j_z$) currents which are useful in the statistical spectrum for weak turbulence produced in the plasmas due to the finite incoherent fluctuations of magnetosonic solitary waves [31, 36, 37] and they have a strong influence on the transport properties of the plasmas. The perpendicular current $j_z$ appears because of perpendicular inertial dust motion, i.e. polarization drifts and $E \times B_0$ drift as well.

Figure 4 shows the variation of the maximum amplitude, $\phi_{\mathrm{m}}$, of the fast KP soliton as a function of $\chi$ for different values of the electron concentration $p$. We find that $\phi_{\mathrm{m}}$ decreases with increasing values of $p$. It should be noted that we have chosen an intermediate value of $\beta$ in order to clearly see the effect of electron concentration $p$ on the shape of the soliton. The relation of $\phi_{\mathrm{m}}$ with the angle $\theta$ that the magnetic field makes with the $x$-axis is also investigated. As the results

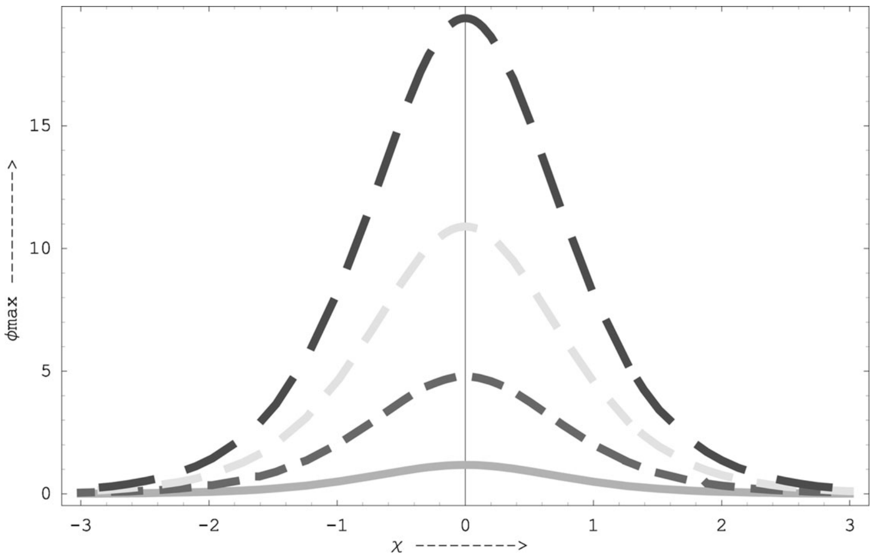


**Figure 5.** Variation of the maximum amplitude of the density as a function of $\chi$ for small values of angle $\theta$ which the ambient magnetic field makes with the $x$-axis. We find that the density increases as we increase the angle $\theta$ from $0°$ to $20°$.

are valid only for small values of angle $\theta$, we vary $\theta$ from $0°$ to $20°$. We find that the amplitude of the soliton increases as we increase the angle $\theta$ (see Fig. 5). It is worth mentioning that the amplitude of the soliton exhibits a compressive (hump) nature for the range of plasma parameters used here.

Figure 6 shows the relationship between the maximum amplitude ($\phi_{\mathrm{m}}$) of the KP soliton with increasing values of $\beta$. Again, the amplitude of the soliton exhibits a compressive nature. It is found that increasing the value of $\beta$ enhances the amplitude of the soliton as is evident from Fig. 6.

## 4. Summary and conclusions

In this paper, we have investigated the linear and nonlinear properties of an obliquely propagating magnetosonic wave in a three-component dusty plasma. We have ignored the electron and ion inertia to study the low-frequency wave related to dust dynamics. We have derived the linear and nonlinear relations for small-amplitude perturbations. The linear analysis shows that the dispersion properties of the low-frequency dust magnetosonic wave depend on the angle $\theta$ that the ambient magnetic field makes with the $x$-axis, the ratio of ion to electron concentration, the plasma beta (the ratio of kinetic to magnetic energy) given by $\beta$, the charge number associated with the dust, and the relative concentration of electrons $p$. It is found that retaining the electron pressure term gives rise to novel features in the dust magnetosonic wave. The slow dust magnetosonic wave is found to be the damped mode and, therefore, the only propagating mode in a dusty plasma is the fast magnetosonic mode. We also derive the nonlinear KP equation for small-amplitude perturbations and find that it admits compressive solitons. We find that the amplitude of the soliton enhances as we increase the values of $p$, $\theta$, and $\beta$.

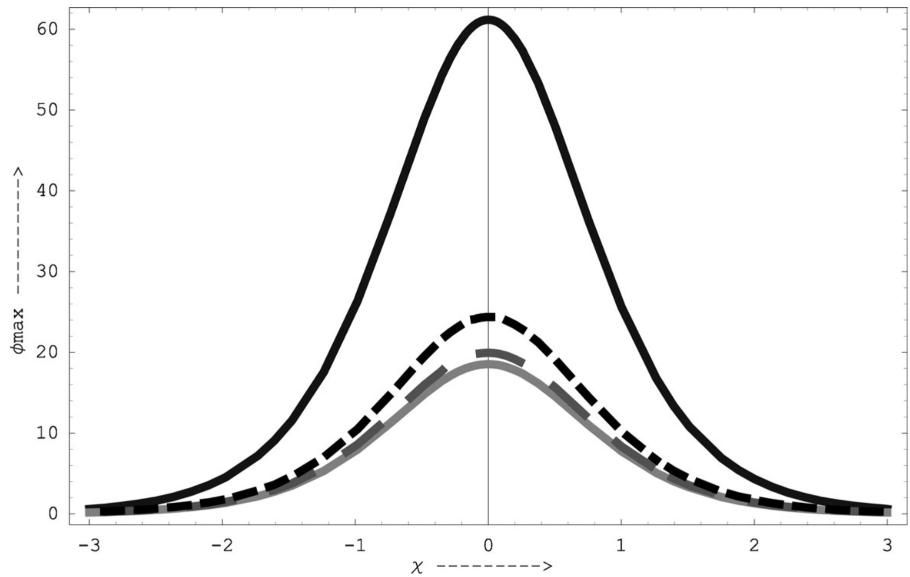


**Figure 6.** Variation of the maximum amplitude of the density as a function of $\chi$ for different values of $\beta$. We find that the amplitude of the compressive soliton increases as we increase the values of $\beta$.

## References


[1] Goertz, C. K. 1989 *Rev. Geophys.* **27**, 271.

[2] Verheest, F. 1996 *Space Sci. Rev.* **77**, 267
Shukla, P. K. 2001 *Phys. Plasmas* **8**, 1791.

[3] Shukla, P. K. and Mamun, A. A. 2002 *Introduction to Dusty Plasma Physics*. Bristol: Institute of Physics.

[4] Mushtaq, A., Shah, H. A., Rubab, N. and Murtaza, G. 2006 *Phys. Plasmas* **13**, 62903.

[5] Rao, N. N., Shukla, P. K. and Yu, M. Y. 1990 *Planet Space Sci.* **38**, 543.

[6] Melandso, F. 1996 *Phys. Plasmas* **3**, 3890.

[7] Misra, A. P. and Roychowdhury, A. 2006 *Phys. Plasmas* **13**, 062307
Salimullah, M. 1996 *Phys. Lett.* A **215**, 296.

[8] Shukla, P. K. and Mamun, A. A. 2003 *New J. Phys.* **5**, 1.

[9] Adlam, J. H. and Allen, J. E. 1958 *Philos. Mag.* **3**, 448.

[10] Davis, L., Lust, R. and Schuluter, A. 1958 *Z. Naturforsch. A* **13a**, 916.

[11] Gardener, C. S. and Morikawa, G. K. 1965 *Commun. Pure Appl. Math.* **18**, 35.

[12] Berezin, Y. A. and Karpman, V. I. 1964 *Sov. Phys. JETP* **46**, 1880.

[13] Kakutani, T. and Ono, H. 1969 *J. Phys. Soc. Japan* **26**, 1305.

[14] Maruyama, K., Bessho, N. and Oshawa, Y. 1998 *Phys. Plasma* **5**, 3257.

[15] De Juli, M. C. and Schneider, R. S. 2000 *J. Plasma Physics* **64**, 57.

[16] Brodin, G., Stenflo, L. and Shukla, P. K. 2003 *J. Plasma Physics* **69**, 183.

[17] Mushtaq, A. and Shah, H. A. 2005 *Phys. Plasma* **12**, 12301.

[18] Shukla, P. K. and Rahman, H. U. 1996 *Phys. Plasma* **03**, 430.

[19] Khurshid Alam, Md., Roy Chowdhury, A. and Paul, S. N. 1999 *Int. J. Theoret. Phys.* **38**, 757.

[20] Marklund, M., Stenflo, L. and Shukla, P. K. Magnetosonic solitons in a dusty plasma slab. *Phys. Plasma* (submitted).

[21] Rao, N. N. 1997 *Advances in Dusty Plasmas* (ed. P. K. Shukla, D. A. Mendis and T. Desai). Singapore: World Scientific, p. 41.

[22] Taniuti, T. and Wei, C. C. 1968 *J. Phys. Soc. Japan* **24**, 941.

[23] Taniuti, T. and Yajima, N. 1969 *J. Math. Phys.* **10**, 1369.

[24] Jeffrey, A. and Kakutani, T. 1972 *SIAM Rev.* **14**, 582.

[25] Kakutani, T. 1974 *Suppl. Prog. Theor. Phys.* **55**, 97.

[26] Taniuti, T. and Washima, H. 1969 *Phys. Rev. Lett.* **21**, 209.

[27] Jeffrey, A. and Kawahara, T. 1982 *Asymptotic Methods in Nonlinear Wave Theory*. Boston: Pitman.

[28] Shu, C. H. and Gardener, C. S. 1969 *J. Math. Phys.* **10**, 536.

[29] De Vito, M. and Pantano, P. 1984 *Lett. Nuovo Cimento Soc. Ital. Fis.* **40**, 58.

[30] Satsuma, J. 1976 *J. Phys. Soc. Japan* **40**, 286.

[31] Shah, H. A. and Bruno, R. 1987 *J. Plasma Physics* **37**, 143.

[32] Alexandrov, A. F., Bogdankevich, L. S. and Rukhadze, A. A. 1984 *Principles of Plasma Electrodynamics*. Berlin: Springer.

[33] Ohsawa, Y. 1986 *Phys. Fluids* **29**, 1844; 1985 *Phys. Fluids* **28**, 2130.

[34] Lembege, B. and Dawson, J. M. 1984 *Phys. Rev. Lett.* **53**, 1053
Lembege, B., Ratliff, S. T., Dawson, J. M. and Ohsawa, Y. 1984 *Phys. Rev. Lett.* **52**, 1500.

[35] Ohsawa, Y. 1985 *J. Phys. Soc. Japan* **54**, 4073.

[36] Jancel, R. and Stenflo, L. 1978 *Phys. Scr.* **17**, 533
Suzuki, T., Ito, A. and Yoshida, Z. 2003 *Fluid Dyn. Res.* **32**, 247.

[37] de Baar, M. R., Thyagaraja, A., Hogeweij, G. M. D., Knight, P. J. and Min, E. 2005 *Phys. Rev. Lett.* **94**, 35002.