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Modified Debye screening potential in a magnetized quantum plasma

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ABSTRACT

The effects of quantum mechanical influence and uniform static magnetic field on the Shukla–Nambu–Salimullah potential in an ultracold homogeneous electron–ion Fermi plasma have been examined in detail. It is noticed that the strong quantum effect arising through the Bohm potential and the ion polarization effect can give rise to a new oscillatory behavior of the screening potential beyond the shielding cloud which could explain a new type of possible robust ordered structure formation in the quantum magnetoplasma. However, the magnetic field enhances the Debye length perpendicular to the magnetic field in the weak quantum limit of the quantum plasma.

There has been a growing interest in quantum plasmas in recent years [1–3]. Particular emphasis has been given to microelectronics [4,5], astrophysical and cosmological objects [6,7], laser-produced plasmas [8], and in Fermi gas in general [9]. To study quantum effects in plasmas, the quantum hydrodynamic (QHD) model has been developed [9–12]. Using QHD model, a number of works on new electrostatic and electromagnetic waves in unmagnetized quantum plasmas and their modifications have been reported in the literature [13–16]. Some limited studies have been made in quantum magnetoplasmas [17–19].

Recently, the quantum effects on the Debye shielding problem in an ultracold unmagnetized plasma have been examined by a number of workers [20–23]. To our best knowledge, the effect of external magnetic field on the Debye shielding in quantum plasmas has not been reported in the literature. Basically the symmetric Debye–Hückel potential arises due to the quasi-neutrality in the unmagnetized plasma. However, in the presence of a static magnetic field strong anisotropy may cause an additional shielding potential known as the Shukla–Nambu–Salimullah (SNS) potential [24–26]. In this Letter, we examine the effect of static magnetic field on the fundamental problem of SNS potential in a general quantum magnetoplasma. The Debye length is found to be significantly modified in the quantum magnetoplasma in any direction perpendicular to the magnetic field.

Basically, if a plasma is cooled to an extremely low-temperature, the de Broglie wavelength of the charge carriers may be comparable to the various scale-lengths of the systems, viz., the Debye length of the plasma, Larmor radius, etc. In such situations, the ultracold plasma must behave as a Fermi gas and the quantum mechanical effects are expected to play a vital role in the behavior of the collective interactions of the charged particles. However, a plasma is a plasma if the Debye length is smaller than the size of the plasma systems. So, one could say that when the de Broglie wavelength of carriers is comparable to the Debye length, the quantum mechanical effect must be significant in the Debye shielding.

We consider a collisionless and homogeneous ultracold plasma consisting of electrons and ions in the presence of a uniform external magnetic field ($\mathbf{B}_0 \| \hat{z}$). We assume that the electrons and ions possess significant quantum mechanical effects in an ultracold Fermi gas, an electron–ion plasma. Fermi plasmas obey the pressure law [1,4,10]

$$p_j = m_j V_{Fj}^2 n_j^3 / 3n_{j0}^2, \tag{1}$$

where $j = e$ for electrons and $j = i$ for ions, m_j is the mass, $V_{Fj} = (2k_B T_{Fj}/m_j)^{1/2}$ is the Fermi speed, k_B is the Boltzmann constant, and T_{Fj} is the Fermi temperature. Here, n_j is the total number density with its equilibrium value n_{j0}. It may be mentioned

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that in a three-dimensional quantum plasma, the total pressure should be proportional to $n^{(N+2)/N}$ where $N = 3$. However, according to Manfredi [1] this choice is not good as the results of QHD model differ from Wigner–Poisson model. Therefore, the total pressure of the three-dimensional quantum plasma is described by Eq. (1). Similar pressure law was employed by Shukla and Ali [20] for three-dimensional quantum plasmas.

The linearized equation of motion with Bohm potential and Fermi temperature for the jth species in a homogeneous quantum plasma in the presence of an external magnetic field $\mathbf{B}_0$ is

$$\frac{\partial \mathbf{v}_{j1}}{\partial t} = -\frac{q_j}{m_j}\nabla\phi_1 + \frac{q_j}{m_j c}\mathbf{v}_{j1}\times\mathbf{B}_0 - \frac{\nabla p_{j1}}{m_j n_{j0}} + \frac{\hbar^2}{4m_j^2 n_{j0}}\nabla\left(\nabla^2 n_{j1}\right), \tag{2}$$

and the continuity equation is

$$\frac{\partial n_{j1}}{\partial t} + n_{j0}\nabla\cdot\mathbf{v}_{j1} = 0, \tag{3}$$

where q_j and m_j are the charge and mass of the jth species. Here, ϕ_1 is the perturbed potential of an electrostatic wave, $\hbar$ is the Planck's constant divided by 2π. The quantum effect in Eq. (2) appears through the Fermi temperature T_{Fj} and the last term known as the Bohm potential.

Following the standard techniques [19] and assuming the perturbations in the form $\propto e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}$, the dielectric susceptibility of the plasma can be obtained by solving Eqs. (2) and (3) to yield

$$\chi_j = \frac{k_\perp^2}{k^2}\frac{\omega_{pj}^2 f_j}{\omega_{cj}^2 - \omega^2 f_j^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pj}^2}{\omega^2}\frac{1}{f_j}, \tag{4}$$

where the quantum factor is given by

$$f_j = 1 - \frac{k^2 V_{Fj}^2}{\omega^2}(1+\gamma_j), \tag{5}$$

and

$$\gamma_j = \frac{\hbar^2 k^2}{8 m_j k_B T_{Fj}}. \tag{6}$$

Here, both the electrons and ions are considered thermal and magnetized. Taking $k^2 V_{Fe}^2(1+\gamma_e) \gg \omega^2$ for electrons, we obtain

$$\chi_e \simeq \frac{1}{k^2 \lambda_{Fe}'^2}, \tag{7}$$

where

$$\lambda_{Fe}' = \frac{V_{Fe}}{\omega_{pe}}\sqrt{1+\gamma_e}. \tag{8}$$

For ions, we assume $k^2 V_{Fi}^2(1+\gamma_i) \ll \omega^2 \ll \omega_{ci}^2$, and consequently, $f_i \approx 1$.

Thus, the dielectric function of the quantum magnetoplasma reduces to

$$\epsilon(\omega,\mathbf{k}) = 1 + \frac{1}{k^2\lambda_{Fe}'^2} + \frac{k_\perp^2}{k^2}\alpha_i - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega^2}, \tag{9}$$

where $\alpha_i = \omega_{pi}^2/\omega_{ci}^2$. For consistency, we analyze keeping quantum effects for both electrons and ions for an ultracold quantum plasma. However, for typical conditions $k^2 V_{Fe}^2 \gg \omega^2$ and $k^2 V_{Fi}^2(1+\gamma_i) \ll \omega^2 \ll \omega_{ci}^2$, we obtained the dielectric function of the quantum magnetoplasma given by Eq. (9).

Now, the electrostatic potential around a test charge in the presence of an electrostatic mode $(\omega,\mathbf{k})$ in a uniform plasma, whose dielectric response function is given by Eq. (9) is [27,28]

$$\Phi(\mathbf{x},t) = \int \frac{q_t}{2\pi^2 k^2}\frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega,\mathbf{k})}\exp(i\mathbf{k}\cdot\mathbf{r})\,d\mathbf{k}\,d\omega, \tag{10}$$

where $\mathbf{r} = \mathbf{x} - \mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of a test charge particle, and q_t is its charge. Substituting Eq. (9) into Eq. (10), we obtain the potential due to the test charge as

$$\Phi(\mathbf{x},t) = \frac{q_t}{2\pi^2}\int \frac{\delta(\omega - \mathbf{k}\cdot\mathbf{v}_t)e^{i\mathbf{k}\cdot\mathbf{r}}k_\perp\,dk_\perp\,d\theta\,dk_\parallel\,d\omega}{k_\parallel^2 + k_\perp^2(1+\alpha_i) + k_{Fe}'^2 - k_\parallel^2\omega_{pi}^2/\omega^2}, \tag{11}$$

where $k_{Fe}' \equiv 1/\lambda_{Fe}'$. Assuming $\mathbf{v}_t \parallel \hat{z}$ and evaluating ω- and θ-integrations, we obtain

$$\Phi(\rho,\xi) = \frac{q_t}{\pi}\int \frac{J_0(k_\perp\rho)e^{ik_\parallel\xi}k_\perp\,dk_\perp\,dk_\parallel}{k_\parallel^2 + k_\perp^2(1+\alpha_i) + k_{Fe}^2[1/(1+\gamma_e) - M^{-2}]}, \tag{12}$$

where $M = v_t/C_{Fs}$, $C_{Fs} = \omega_{pi}\lambda_{Fe}$ and $\lambda_{Fe} = V_{Fe}/\omega_{pe} = 1/k_{Fe}$. Now, we consider two specific cases of the quantum plasma, viz. the weak and the strong quantum considerations.

Case I: $\gamma_e \ll 1$.

This condition $\gamma_e \ll 1$ is equivalent to neglecting the Bohm potential term. The quantum effect is then taken through the Fermi temperature. Following Refs. [24,25], we finally obtain after performing the $k_\parallel$- and $k_\perp$-integrations

$$\Phi(\rho,\xi) = \frac{q_t}{\sqrt{1+\alpha_i}}\frac{e^{-\sqrt{(1-M^{-2})/(1+\alpha_i)}(\rho^2+\xi^2(1+\alpha_i))^{1/2}/\lambda_{Fe}}}{(\rho^2 + (1+\alpha_i)\xi^2)^{1/2}}. \tag{13}$$

We express Eq. (13) in the dimensionless normalized form as

$$\Phi'(\rho',\xi') \equiv \frac{\Phi(\rho',\xi')}{q_t/\lambda_{Fe}} = \frac{1}{\sqrt{1+\alpha_i}}\frac{\exp[-\sqrt{\frac{1-M^{-2}}{1+\alpha_i}}(\rho'^2 + (1+\alpha_i)\xi'^2)^{1/2}]}{(\rho'^2 + (1+\alpha_i)\xi'^2)^{1/2}}, \tag{14}$$

where

$$\rho' \equiv \frac{\rho}{\lambda_{Fe}}, \qquad \xi' \equiv \frac{\xi}{\lambda_{Fe}}. \tag{15}$$

Case II: $\gamma_e \gg 1$ ($\hbar^2 k^2 \gg 8 m_e k_B T_{Fe}$).

In this limit, the Bohm potential effect dominates over Fermi pressure effect. We can then write Eq. (12) as

$$\Phi(\rho,\xi) = \frac{q_t}{\pi}\int \frac{J_0(k_\perp\rho)e^{ik_\parallel\xi}k_\perp\,dk_\perp\,dk_\parallel}{k_\parallel^2 + k_\perp^2(1+\alpha_i) + 1/(\lambda_{Fe}^2 + \lambda_{qe}^4 k^2) - M^{-2}/\lambda_{Fe}^2}, \tag{16}$$

where the quantum effect associated with the electrons arises through Fermi–Debye length λ_{Fe} and the wavelength associated with the quantum effect is

$$\lambda_{qe} = \left(\frac{\hbar^2}{4m_e^2\omega_{pe}^2}\right)^{1/4}. \tag{17}$$

Introducing dimensionless quantities as $\rho' = \rho/\lambda_{qe}$, $\xi' = \xi/\lambda_{qe}$ and $\mathbf{K} = \mathbf{k}\lambda_{qe}$, Eq. (16) reduces to

$$\Phi'(\rho',\xi') \equiv \frac{\Phi(\rho',\xi')}{q_t/\lambda_{qe}} = \frac{1}{\pi}\int \frac{J_0(K_\perp\rho')e^{iK_\parallel\xi'}(K_q^2 + K^2)K_\perp\,dK_\perp\,dK_\parallel}{1 + (K_\parallel^2 + K_\perp^2 + K_q^2)[K_\parallel^2 + K_\perp^2(1+\alpha_i) - M_q^{-2}]}. \tag{18}$$

The denominator of Eq. (18) is biquadratic in $K_\parallel$, so we can write it as

$$\Phi'(\rho', \xi') = \frac{1}{\pi} \int \frac{J_0(K_\perp \rho') e^{iK_\parallel \xi'} (K_q^2 + K^2) K_\perp \, dK_\perp \, dK_\parallel}{(K_\parallel^2 + K_0^2)(K_\parallel^2 - K_1^2)}, \tag{19}$$

where

$$K_0^2 = K_q^2 + K_\perp^2 (1 + \alpha_i) - M_q^{-2},$$
$$K_1^2 = \frac{1 + K_q^2 (K_\perp^2 (1 + \alpha_i) - M_q^{-2})}{M_q^{-2} - K_q^2 - K_\perp^2 (1 + \alpha_i)}, \tag{20}$$

and $M_q = v_t / C_{qs}$, the quantum ion-acoustic velocity, $C_{qs} = \omega_{pi} \lambda_{qe}$ and $K_q = \lambda_{Fe} / \lambda_{qe}$. After performing the $K_\parallel$-integration and following Refs. [24–26], we can separate SNS potential from Eq. (19), which comes out to be

$$\Phi_D'(\rho', \xi') = -\int_{K_q}^{1} \left(\frac{K_\perp^2 - K_0^2 + K_q^2}{K_0 (K_0^2 + K_1^2)} \right) J_0(K_\perp \rho') \times e^{-K_0 \xi'} K_\perp \, dK_\perp. \tag{21}$$

For the strong quantum effect, we must have $k^2 \gg 4 m_e^2 V_{Fe}^2 / \hbar^2$, that is $K \gg K_q$. Therefore, we choose the lower limit of integration of Eq. (21) as K_q for the strong quantum consideration. The upper limit of integration in Eq. (21) is taken as 1 because waves with wavelength below the effective Debye length do not contribute to the Debye potential.

Numerical solution and discussion

To better understand the behavior of the potential of a test charge particle in a quantum plasma, we have numerically solved Eq. (21). We choose some typical values for laboratory quantum plasmas [20]: $T_{Fe} = 300$ K and $n_e \approx n_i = 1.88 \times 10^{19}$ cm^{-3} with arbitrary value of $\alpha_i = \omega_{pi}^2 / \omega_{ci}^2$. The results of our calculations are depicted in the form of curves in Figs. 1–4.

Figs. 1 and 2 show the normalized SNS potential as a function of ρ' and ξ' in the presence of the static magnetic field. Here, the quantum effect arises through the Fermi temperature only. Fig. 1 shows the normalized potential as a function of ρ' with $\xi' = 0$ for $M = 1.2$, $\alpha_i = 10$. In Fig. 2, all parameters are same as in Fig. 1 except $\rho' = 0$ and $\xi' \neq 0$. Both figures clearly show that the potential does not remain symmetric with the application of magnetic field. The screening length in the direction perpendicular to magnetic field is much increased as compared to that in the direction parallel to magnetic field. This result resembles the result of Nambu and Nitta [25].

Fig. 3 shows the normalized SNS potential with quantum effects via the Bohm potential and the magnetic field. If we neglect the magnetic field in our chosen quantum plasma system, we retrieve the result of Shukla and Eliasson [22]. The normalized potential of Shukla and Eliasson has been shown by dotted lines in Fig. 3. The solid line in Fig. 3 shows the effect of the magnetic field on the potential in the quantum magnetoplasma. We are interested to explore the effect of the magnetic field in the direction perpendicular to the magnetic field, so we have chosen $\xi' = 0$, $M_q = 0.8$, and $\alpha_i = 10$. We observe that with the application of the magnetic field not only the screening length is increased but also the potential becomes oscillatory in that direction perpendicular to the magnetic field.

In an unmagnetized ultracold degenerate plasma, the oscillatory behavior of the shielding potential of a static test charge was already mentioned in Ref. [29]. The oscillatory nature of the shielding potential pointed out in Ref. [29] is symmetric in all directions in space. Recently, Shukla and Eliasson [22] have shown Friedel oscillation-like behavior of the symmetric Debye–Hückel

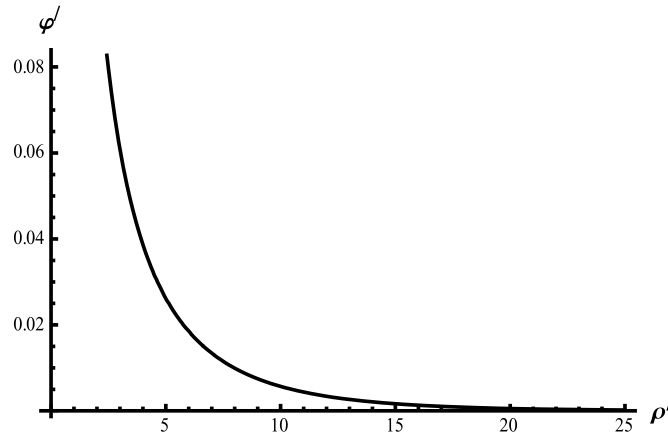

Fig. 1. The plot of the normalized SNS potential $\Phi'(\rho', \xi')$ without strong quantum effects (Eq. (14)) for $\xi' = 0$, $T_{Fe} = 300$ K, $n_e \approx n_i = 1.88 \times 10^{19}$ cm^{-3}, $M = 1.2$, $\alpha_i = 10$.

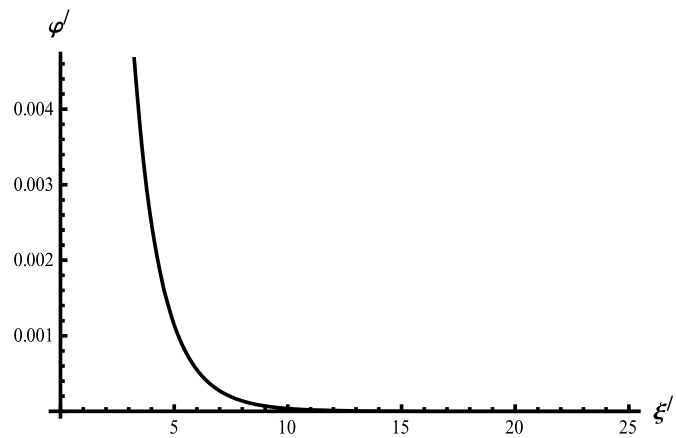

Fig. 2. The variation of the normalized SNS potential $\Phi'(\rho', \xi')$ without strong quantum effects with $\rho' = 0$. The other parameters are the same as in Fig. 1.

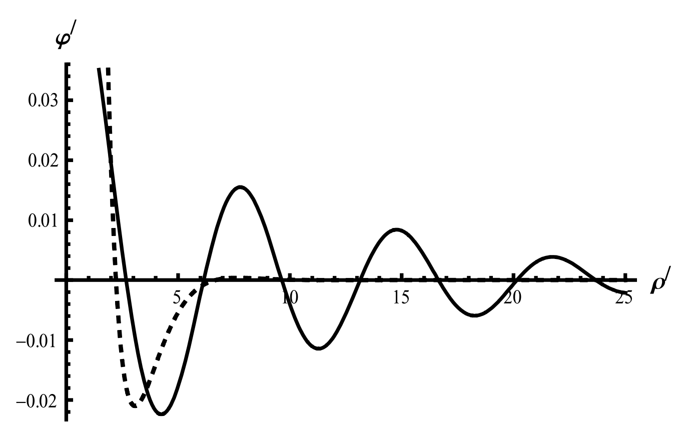

Fig. 3. The plot of $\Phi'(\rho', \xi')$ (solid line) with strong quantum effects for $M_q = 0.8$, $\alpha_i = 10$, and for $\xi' = 0$, $T_{Fe} = 300$ K, $n_e \approx n_i = 1.88 \times 10^{19}$ cm^{-3}, $\lambda_{qe} = 4.867 \times 10^{-8}$ cm. The dashed curve corresponds to Eq. (14) of Shukla and Eliasson [22].

potential of a slowly moving test charge in an unmagnetized quantum plasma ignoring the effect of Fermi temperature. However, when we take the effect of an external static magnetic field on the shielding of a slowly moving test charge in a plasma with strong quantum effect arising through the Bohm potential, we notice that the symmetry of the Debye–Hückel potential is broken. This is due to the ion polarization effect in the direction perpendicular to the

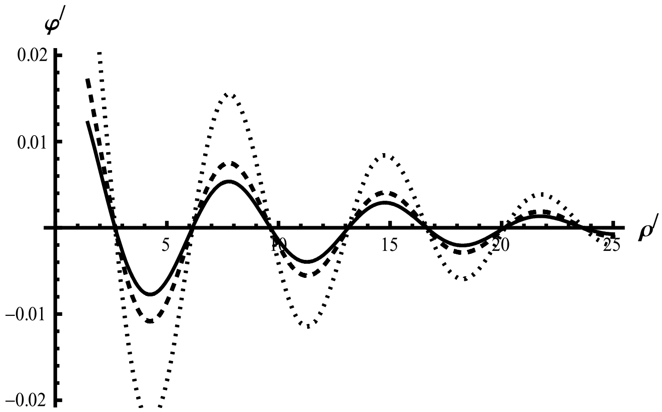

Fig. 4. The variation of $\Phi'(\rho', \xi')$ for different values of α_i. Solid line corresponds to $\alpha_i = 100$; dashed line, $\alpha_i = 50$ and $\alpha_i = 10$ for the dotted line. Other parameters are same as in Fig. 3.

magnetic field [24–26]. In the parallel direction, there is no such effect observed as the plasma is unaffected by the magnetic field in this direction. This 1D-oscillatory behavior is due to the ion polarization effect in the quantum magnetoplasma.

Fig. 4 shows the effect of the magnetic field on the normalized SNS potential in the quantum plasma via the parameter $\alpha_i = \omega_{pi}^2/\omega_{ci}^2$. From the solid, dashed and dotted curves, we notice that the amplitude of the oscillatory Debye–Hückel potential increases with decreasing value of α_i.

To summarize, we have explored the effect of the strong quantum limit and the externally applied static magnetic field on the Debye–Hückel shielding potential of a test particle by using the quantum magnetohydrodynamic model. The shielding effect through ion polarization drift also exists in an ultracold degenerate plasma similar to classical magnetoplasmas [24,25]. We find that with the application of magnetic field, the shielding length is much increased in the direction perpendicular to the direction of the magnetic field in a degenerate quantum plasma. When a strong magnetic field is applied, the quantum effect via the Bohm potential gives rise to an oscillating shielding behavior in the direction perpendicular to the magnetic field. No such effect is found in the direction parallel to the magnetic field. The shielding due to strong magnetic field and the quantum effect is independent of the test particle velocity. Thus, the strong magnetic field and quantum effects modify drastically the usual Debye shielding and show an oscillating behavior which might give rise to a robust crystal formation in the quantum magnetoplasma beyond the shielding cloud. This new oscillatory property of the SNS potential perpendicular to the static magnetic field in the quantum magnetoplasma is clearly due to the strong quantum effect and the ion polarization drift contribution.

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