Two types of lower-hybrid waves in dusty plasmas and cusp solitons

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A theoretical model is presented for the excitation of ultralow frequency dust-lower hybrid mode (DLH) oscillating at frequency $\Omega_{lhd}=\sqrt{(Z_d n_{do}/n_{io})\Omega_{ci}\Omega_{cd}}$ by employing the decay of a relatively high frequency dust-modified lower-hybrid (DMLH) wave into a relatively lower frequency DMLH and DLH based on three-wave resonant interaction. A coupled nonlinear Schrödinger (NLS) equation for the DMLH wave and Zakharov equation for the DLH wave are derived. The nonlinear contribution in the NLS equation comes from the DLH density fluctuations. Modulational instabilities of DMLH waves are investigated and its growth rates are studied. Additionally, one-dimensional nonlinear localized structures of bright solitons and nonlinear nonlocal structures like cusp solitons are obtained. It is shown that, when the phase velocity resonates with the dust sound speed ($\omega/\kappa \sim v_o$), the nonlocal nonlinearity leads to the generation of cusp solitons. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3072117]

I. INTRODUCTION

Lower hybrid waves (LH) in an electron-ion plasma is one of the low frequency modes which have been investigated for many years, both theoretically and experimentally.[1–6] This wave has received a great deal of attention due to its many applications in space and fusion plasmas.[5,6] In laboratory plasmas, lower hybrid (LH) waves are used for heating purposes, while in space they play an essential part in collisionless energy and momentum transport. This mode has also been described in two ion species plasmas.[7–12] Lower-hybrid waves are well known to admit nonlinear structures, such as ordinary solitons as well as envelope solitons. Such structures have been observed in the Earth's magnetosphere by the FREJA satellite,[13–17] and have been examined with and without an extra charged species in plasma.[12,18,19]

For a decade and a half, considerable attention has been given to dusty plasmas which are now known to exist in most plasma environments, i.e., astrophysical, space, and laboratory. Indeed, as is now known, such plasmas are rich in waves, especially low frequency modes like dust-acoustic and dust-ion acoustic waves. A great deal of theoretical and experimental research has been carried out in a wide range of problems in dusty plasmas.[20–25]

In laboratory experiments, it is usual to apply an external uniform magnetic field for the confinement and control of dusty plasmas, whereas in astrophysical or space environments, dusty plasmas are generally immersed in the ambient magnetic field.[26,27] An important example of low frequency waves in magnetized plasmas is the dust-lower hybrid (DLH) wave, which can also lie in the ultralow frequency range, i.e., below the ion cyclotron frequency. The dispersion relation for DLH waves was derived for the first time in Ref. 23, but it did not include the important dispersion effects which come from the ion terms. This mode can be easily excited as will be shown in this paper and its various properties can be explored in laboratory plasmas. DLH waves have been treated in different ways, for example, in Refs. 24 and 25 this mode has been excited in a plasma with and without opposite polarity of dust grains. Shukla *et al.*[28] showed that the ponderomotive force of large amplitude lower-hybrid waves can be used to generate space charge electric fields which lead to the acceleration of dust grains. Salimullah *et al.*[29] derived the dispersion relation and an expression for the damping of electrostatic dust-lower-hybrid mode using fluid and kinetic models where lighter species, i.e., electrons and ions were taken to be streaming and dust was taken to be unmagnetized. Amin *et al.*[30] examined the instabilities for the excitation of a dust-lower-hybrid wave in a uniform plasma where the dust grains are streaming. However, they had assumed $k_\parallel \gg k_\perp$, and under this assumption dust mass and density vanish and the contribution of dust disappears. Surprisingly, a large number of studies devoted to the consideration of LH waves and DLH waves have treated this problem by neglecting the spatial dispersion. We would like to note here that neglecting the spatial dispersion term can obscure processes which can lead to the excitation of new phenomenon like the one we shall discuss in this paper. However, the effect of the spatial term has been taken into account by Shukla *et al.* in 2003, where the authors have considered the effect of nonlinear coupling between large amplitude upper hybrid electrostatic waves and a low frequency modified Alfvén mode. Modulational and decay instabilities of a constant amplitude upper hybrid pump wave have been investigated.[31] Presently research on dust in tokamak plasmas has become a topic of growing interest and since lower hybrid waves are used as a source of auxiliary heating for the ions in the tokamak we expect that DLH waves will have an important role to play in these machines.

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Very recently, Shukla and Tsintsadze[32] investigated charge grain heating in the tokamak edge region, where it was shown that the normal component of the space charge electric field accelerates dust particles in the scrape-off layer close to the tokamak chamber wall. Dust also appears in the core of the tokamak plasma.[33,34] Thus it is reasonable to expect that dust and dusty plasmas are going to be crucial in the next generation ITER like fusion devices and may significantly effect heating processes in such machines. We may add here that DLH waves in tokamak plasmas may be used for diagnostic purposes also.[35]

In the present paper we consider in a dusty plasma, dust-modified lower hybrid (DMLH) waves with spatial dispersion. In order to show the importance of spatial dispersion, a three-wave resonant interaction is investigated showing that a DMLH wave can decay into another DMLH wave and to a DLH wave. We would like to state here that we have not calculated the growth rate associated with the three-wave resonant interaction as this is generally less than the growth associated with the modulational instability, which we subsequently investigate. We then consider the formation of DMLH wave solitons by deriving a set of nonlinear Schrödinger (NLS) and Zakharov equations with the nonlinear contribution coming from the DLH waves. Further, we investigate the modulational instability of the lower hybrid wave solitons and arrive at expressions for oscillatory instabilities in different limiting cases. We have also obtained soliton solutions from the set of coupled Zakharov and NLS equations in the quasistationary regime deriving ordinary soliton solution when nonlinearity is local and cusp soliton solution for nonlocal nonlinearity. We feel that such an approach to lower hybrid waves is important in dusty plasmas because cusp solitons may contribute significantly to the heating problem in tokamaks and to the interpretation of data from space and astrophysical plasmas. In addition, we would like to note that such a novel approach to the study of nonlinear lower hybrid waves has not been undertaken previously.

The paper is organized in the following manner: In Sec. II, the basic formulation of two types of dust-lower-hybrid waves is given and the respective dispersion relations are obtained. Section III deals with the mechanism of three-wave resonant interaction, and the subsequent derivation of the Zakharov and NLS equations is given. In Sec. IV, modulational instabilities are examined. Section V demonstrates the one-dimensional analytical solutions of ordinary and cusp solitons. Finally, conclusions are given in Sec. VI.

II. DESCRIPTION OF THE MODEL

We consider low frequency electrostatic lower-hybrid waves in a dusty plasma composed of electrons, ions and dust grains and have a constant negative charge.[26,36] The quasineutrality condition is given by

$$n_i = Z_d n_d + n_e, \tag{1}$$

where Z_d is the charge of dust grain, and $n_s = n_{s0} + \delta n_s + \delta n_s^L$, n_{s0} represents equilibrium density. Superscript L refers to the ultralow frequency and δn_s gives the density perturbation on the DLH time scale. s denotes the species dust, ions or electrons. While treating this problem, we shall first consider the fast time scale phenomenon followed by slow time (DLH) dynamics. In the former regime, dust dynamics is ignored while in the latter the dust species is activated and the electrons no longer participate in the dynamics.

We begin with the fundamental fluid equations for ions and electrons in the presence of a dc magnetic field, which govern the excitation of the plasma modes

$$n_s\left(\frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla\right)\mathbf{v}_s + \frac{T_s}{m_s}\nabla n_s = \frac{q_s}{m_s} n_s\left(\mathbf{E} + \frac{\mathbf{v}_s}{c} \times \mathbf{B}_o\right), \tag{2}$$

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{v_s}) = 0, \tag{3}$$

T_s is the temperature in energy units for s species. The constant magnetic field is applied in the $\mathbf{z}$-direction ($B_o\mathbf{z}$), the wave vector $\mathbf{k}$ is taken along the $\mathbf{x}$-axis, and c is the velocity of light, while s denotes ions ($q_i = e$) or electrons ($q_e = -e$). To find the dispersion relation for linear DMLH waves, we linearize Eqs. (1)–(3), and using a plane wave solution we get the general expressions for the perturbed densities of electrons and ions,

$$\frac{\delta n_e}{n_{oe}} = -\frac{\dfrac{e\varphi}{m_e}k^2}{(\omega^2 - \Omega_{ce}^2 - k^2 v_{te}^2)} \tag{4}$$

and

$$\frac{\delta n_i}{n_{oi}} = \frac{\dfrac{e\varphi}{m_i}k^2}{(\omega^2 - \Omega_{ci}^2 - k^2 v_{ti}^2)}. \tag{5}$$

Here, $v_{ts} = (T_s/m_s)^{1/2}$ is the thermal speed and $\Omega_{cs} = (eB_o/m_s c)$ is the gyrofrequency of charged particles of species s. Using the quasineutrality condition $\delta n_e = \delta n_i$, we obtain

$$\omega^2 = \Omega_{lh}^2 + k^2 v_o^2, \tag{6}$$

where $\Omega_{lh} = [(n_{io}/n_{eo})\Omega_{ce}\Omega_{ci}]^{1/2}$ is the dust-lower-hybrid frequency and v_o is the ion-sound speed given by $[(n_{io}/n_{eo})T_e/m_i]^{1/2}$. In the derivation of expression (6), we have assumed that only the electrons are magnetized and that $T_i \ll T_e$, dust dynamics is ignored and its presence is taken into account via the quasineutrality condition only. It is emphasized that Eq. (6) has a spatial dispersion term the contribution of which comes from the ions and, as we shall see later, the spatial term can play an effective role in the excitation of new modes. This propagation term also shows how the Debye length is modified for a magnetized plasma. If we ignore ω^2 on the left-hand side of Eq. (6), we obtain a modified expression for the Debye length given by $\lambda_D = v_{te}/\omega_{ce}$, which shows that as the strength of magnetic field increases, the Debye length decreases, and thus the particles in the Debye cloud remain mostly confined.

Now we derive a linear dispersion relation for the ultralow frequency lower-hybrid (DLH) waves whose excitation will be discussed in the next section. In this case, the dynamical effects of dust grains are included, while the electrons dynamics is ignored since the time with which velocity and density of electrons changes is much shorter than that of ions and dust, i.e.,

$$v_e\left(\frac{\partial v_e}{\partial t}\right)^{-1}, \quad n_e\left(\frac{\partial n_e}{\partial t}\right)^{-1} \ll t_i \sim \frac{1}{\omega_{pi}}, \quad t_d \sim \frac{1}{\omega_{pd}}, \tag{7}$$

where ω_{pi} and ω_{pd} are the Langmuir frequencies for ions and dust, respectively.

Proceeding along the lines of previous calculations for dispersion relation (6), we obtain a dispersion relation for the DLH wave given by

$$\Omega^2 = \Omega_{ulh}^2 + q^2 u_o^2, \tag{8}$$

where $\Omega_{ulh}^2=(Z_d n_{do}/n_{io})\Omega_{ci}\Omega_{cd}$, $\Omega_{cd}=Z_d e B_o/m_d c$ is the dust-gyro frequency, and u_o is the dust sound speed given by $[(Z_d^2 n_{do}/n_{io})T_i/m_d]^{1/2}$ which is much less than the sound speed obtained in the previous dispersion relation given by Eq. (6). q and Ω represent the wave number and frequency of the DLH wave, respectively. Here we have treated the dust to be cold and unmagnetized.

III. EXCITATION OF THE DLH MODE

Now, we consider the possible decay of a dust-modified lower hybrid wave with frequency ω and wave vector k into two waves, a DMLH wave having frequency ω' and wave number k' and a DLH wave with frequency Ω and wave number q. This simple physical picture can be obtained from Eqs. (6) and (8), provided the energy and momentum are conserved, i.e.,

$$\omega - \omega' = \Omega,$$

$$k - k' = q,$$

where the components of momentum k, k', and q are directed along the x-axis. From the above relations, we obtain

$$\omega - \omega' \simeq \frac{2v_o^2 k(k-k')}{\Omega_{lh}} \approx \left[\left(\frac{Z_d n_{do}}{n_{io}}\right)\Omega_{ci}\Omega_{cd}\right]^{1/2}. \tag{9}$$

We note here that we consider propagation in the x direction only. Momentum and energy conservation further lead to $k-k' \cong q^2/2k$, and $\omega-\omega'=(v_o q)^2/\Omega_{lh}$. Thus using this simple model, we have shown a possibility of the three-wave interactions, which leads to the generation of the DLH waves. We do not derive growth rates for this decay process because in the next section we consider the stronger instability which is the modulational instability.

On the basis of the aforesaid mechanism, we can excite the DLH mode and for that we will solve for the low frequency density variations and include in our considerations the convective derivative term $(\delta\mathbf{v}_i\cdot\nabla\delta\mathbf{v}_i)$, which leads to the ponderomotive force. We thus write down the following equations for ions:

$$\frac{\partial \delta\mathbf{v}_i^L}{\partial t} + \langle \delta\mathbf{v}_i\cdot\nabla\delta\mathbf{v}_i\rangle = \frac{e}{m_i}\left(\mathbf{E}^L + \frac{1}{c}\delta\mathbf{v}_i^L\times\mathbf{B}_o\right) - \frac{1}{m_i n_{io}}\nabla P_i^L, \tag{10}$$

where the angular brackets denote the averaging over a typical dust lower-hybrid wave period and wavelength, $\delta\mathbf{v}_i$ is the ion velocity for the DMLH waves. P_i^L is the ion pressure term and $\mathbf{E}^L$ is the electric field for the ultralow frequency field. From the ion continuity equation we get

$$\frac{\partial}{\partial t}\left(\frac{\delta n_i^L}{n_{io}}\right) + (\nabla\cdot\delta\mathbf{v}_i^L) = 0. \tag{11}$$

Dust dynamics are governed by the following equations of momentum and continuity:

$$\frac{\partial \delta\mathbf{v}_\mathbf{d}^L}{\partial t} = -\frac{Z_D e\mathbf{E}^L}{m_d} \tag{12}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\delta n_d^L}{n_{do}}\right) + (\nabla\cdot\delta\mathbf{v}_\mathbf{d}^L) = 0. \tag{13}$$

We have assumed that the ponderomotive force is not strong enough, due to the heavy mass of the dust grains to cause nonlinearity to appear in the dynamics of the dust grains. Using the quasineutrality condition $\delta n_i^L = Z_d \delta n_d^L$, and after performing some straightforward algebraic steps, we obtain the Zakharov-type equation,[37]

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_{ulh}^2 - u_o^2\frac{\partial^2}{\partial x^2}\right)\frac{\delta n_d^L}{n_{0d}} = \frac{Z_d}{m_d}\frac{\partial^2}{\partial x^2}\frac{1}{2}m_i\langle\,|\delta v_i|)^2\rangle, \tag{14}$$

where $u_o^2=(Z_d^2 n_{do}/n_{io})T_i/m_d$ and Ω_{ulh} are the dust acoustic velocity and dust-lower-hybrid frequency given by $[(Z_d n_{do}/n_{io})\Omega_{ci}\Omega_{cd}]^{1/2}$, respectively. The right-hand side of the above equation contains source term which appears due to the ponderomotive force term coming from ions on a fast times scale and contributes to excitation of DLH waves on the slow time scale.

Using the dispersion relation for dust lower-hybrid waves [Eq. (6)] and treating ω and k as operators given by[38,39]

$$\omega = \omega_o + i\frac{\partial}{\partial t}, \quad k = k_o - i\frac{\partial}{\partial x}, \tag{15}$$

we obtain the following evolution equation for ions on the slow time scale:

$$i\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial x}\right)\delta v_i + \frac{v_g}{2k_o}\frac{\partial^2}{\partial x^2}\delta v_i - \Delta\omega\,\delta v_i - \frac{\omega_\Gamma}{2}\left(\frac{Z_d\delta n_d^L}{n_{io}^o}\right)\delta v_i = 0, \tag{16}$$

where n_{io}^o and $v_g=k_o v_o^2/\omega_\Gamma$ represent the equilibrium density of ions on fast time scale and group velocity, respectively, and $\omega_\Gamma=[(n_{io}^o/n_{eo}^o)(\Omega_{ce}\Omega_{ci})]^{1/2}$. $\Delta\omega$ is the nonlinear frequency correction given by $(1/2\omega_o)[(\omega_\Gamma^2+k_o^2 v_o^2)-\omega_o^2]$. Equations (14) and (16) together constitute the NLS and Zakharov equations.

IV. MODULATIONAL INSTABILITY OF DMLH WAVES

In this section, we investigate the modulational instability of the dust-modified LH waves using Eqs. (14) and (16). To analyze the amplitude modulation of a DMLH wave, we use Madelung's representation[40] in the x direction only

$$\delta v_i \sim a(x,t)e^{i\zeta(x,t)}, \tag{17}$$

where the amplitude a and the phase ζ are real. Substituting Eq. (17) into Eq. (16), we obtain from the real and imaginary parts, respectively,

$$\frac{\partial\zeta}{\partial t} + \left(v_g\frac{\partial}{\partial x}\right)\zeta + \Delta\omega - \frac{v_g}{2k_o}\left[\frac{1}{a}\frac{\partial^2}{\partial x^2}a - \left(\frac{\partial}{\partial x}\zeta\right)^2\right] + \omega_\Gamma\left(\frac{Z_d\delta n_d^U}{2n_{io}^o}\right) = 0 \tag{18}$$

and

$$\frac{\partial}{\partial t}a^2 + \left(v_g\frac{\partial}{\partial x}\right)a^2 + \frac{v_g}{k_o}\frac{\partial}{\partial x}\left(a^2\frac{\partial}{\partial x}\zeta\right) = 0. \tag{19}$$

We now linearize the above equations with respect to the perturbations, in the amplitude a and the phase ζ as $a=a_0+\delta a$, $\zeta=\zeta_0+\delta\zeta$, and where a_0, ζ_0 denote the equilibrium values and δa, $\delta\zeta$ are small perturbations, and obtain

$$\frac{\partial\delta\zeta}{\partial t} + \left(v_g\frac{\partial}{\partial x}\right)\delta\zeta - \frac{v_g}{2a_ok_o}\frac{\partial^2}{\partial x^2}\delta a + \omega_\Gamma\left(\frac{Z_d\delta n_d^U}{2n_{io}^o}\right) = 0, \tag{20}$$

$$\frac{\partial}{\partial t}\delta a + a_o\left(v_g\frac{\partial}{\partial x}\right)\delta a + \frac{a_ov_g}{2k_o}\frac{\partial^2}{\partial x^2}\delta\zeta = 0. \tag{21}$$

Similarly, from the Zakharov equation [Eq. (16)] we get

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_{ulh}^2 - u_o^2\frac{\partial^2}{\partial x^2}\right)\frac{\delta n_{d1}}{n_{0d}} = \frac{Z_dm_ia_o}{m_d}\frac{\partial^2}{\partial x^2}\delta a. \tag{22}$$

Seeking a plane wave solution proportional to $\exp[i(qx-\Omega t)]$ (here q and Ω are the wave number and frequency of the modulation), we eventually obtain from Eqs. (20)–(22) the following dispersion relation for the modulation of a dust-modified lower-hybrid wave:

$$\left\{(\Omega - qv_g)^2 - \frac{v_o^4q^4}{4\omega_\Gamma^2}\right\}[\Omega^2 - \Omega_{lhd}^2 - q^2u_o^2] = \left(\frac{m_i}{m_d}\right)\left(\frac{Z_dn_{od}}{n_{io}}\right)\frac{v_o^2a_0^2q^4}{4}. \tag{23}$$

For simplicity, we discuss two limiting cases of this dispersion relation.

In the first case, we ignore the diffraction effects of the dust-lower-hybrid waves [second term in the first bracket on the left-hand side of Eq. (23)] and look for a solution where roots $\Omega=qv_g+\gamma$ and $\Omega=(\Omega_{lhd}^2+q^2u_o^2)^{1/2}+\gamma$ cross each other. We then get the following expression for the growth rate of oscillatory instabilities:

$$\mathrm{Im}\ \gamma = \frac{\sqrt{3}}{4}qv_o\left[\left(\frac{m_i}{m_d}\right)\frac{Z_dn_{od}}{n_{io}}\frac{qa_0^2}{v_o(\Omega_{lhd}^2+q^2u_o^2)^{1/2}}\right]^{1/3}. \tag{24}$$

In the second case, we assume the conditions $\Omega^2-q^2u_o^2 \ll \Omega_{lhd}^2$ and $(m_e/m_i)v_s^2 < a_o^2$ and as a result, obtain from Eq. (23) the imaginary part of Ω,

$$\mathrm{Im}\ \Omega = \frac{v_oa_0q^2}{2\Omega_{ci}}. \tag{25}$$

This expresses the growth rate of the oscillatory instabilities ($\mathrm{Re}\ \Omega=qv_g$) associated with the cusp solitons which we shall discuss in more detail in the next section.

V. SOLITON SOLUTIONS

Localized structures (solitons) which are usually formed from nonlinearly propagating waves are ubiquitous in plasmas.[20,41] Here, we shall use the standard approach to investigate solitons but will restrict ourselves to the consideration of stationary structures. We shall consider two types of soliton solutions.

In the first case, we assume $\partial^2/\partial t^2+\Omega_{ulh}^2 \ll u_o^2(\partial^2/\partial x^2)$ and from the Zakharov equations, i.e., Eq. (14) and we obtain the following expression for the perturbed dust density:

$$\delta n_d = -\left(\frac{n_{oi}m_i}{2T_i}\right)\langle\delta v_i^2\rangle. \tag{26}$$

Substituting this relation in Eq. (16) and shifting to a comoving frame of reference $\xi=x-v_gt$, such that the perturbations vanish at $\xi\to\pm\infty$, and introducing the notations $C=(1/2)(\omega_\Gamma/\Delta\omega)^{1/2}\delta v_i/v_{ti}$, and $Y=2/v_o(\omega_\Gamma\Delta\omega)^{1/2}\xi$, and then integrating once, we get

$$\frac{dC}{dY} = C\sqrt{1-C^2} \tag{27}$$

which has a solution of the form

$$Y = \log\left[\frac{C}{1+\sqrt{1-C^2}}\right]. \tag{28}$$

This is a standard bright soliton structure.

In the second case, we look for a stationary wave solution for which we assume that the phase velocity is almost the same as the dust sound speed of DLH waves ($\omega/k\approx u_o$). In this approximation we obtain from Eq. (14),

$$Z_d\delta n_d \approx n_{oi}\frac{\partial^2}{\partial x^2}\frac{\langle\delta v_i^2\rangle}{\Omega_{ci}^2}. \tag{29}$$

Comparing this with Eq. (26), we notice an interesting feature that the density perturbation is proportional to the second derivative of the amplitude of velocity which in turn leads to a soliton of a different kind. Substituting relation (29) in the second Zakharov equation [Eq. (17)], we obtain

$$\frac{\partial^2}{\partial\xi^2}F - F - \left(\frac{\partial^2}{\partial\xi^2}F^2\right)F = 0, \tag{30}$$

where $F=(m_i/m_e)^{1/2}\delta v_i/v_o$, and further by changing the variable from ξ to z, where $z=\xi/\sqrt{\alpha}$, and $\alpha=2\omega_\Gamma\Delta\omega/v_o^2$, and integrating Eq. (30) once, we obtain the equation

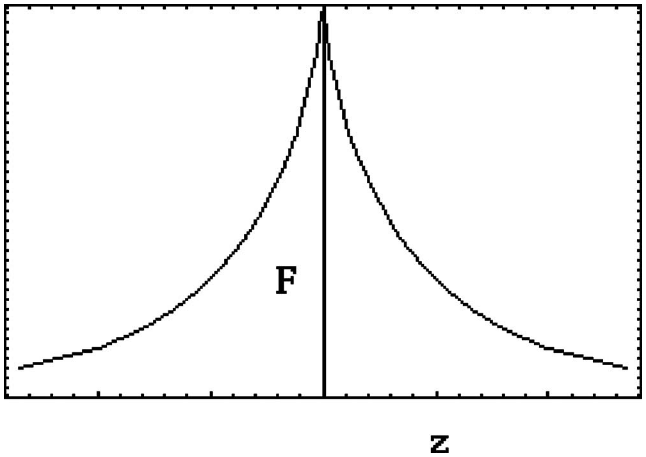

FIG. 1. A schematic illustration of cusp soliton solution (31) having general features that $\partial F/\partial z$ approaches infinity at its maxima.

$$\frac{dF}{dz}\sqrt{1-F^2}=F. \tag{31}$$

This equation shows $dF/dz \to \infty$ at maxima, and thus demonstrates the formation of a cusp soliton. Further integration of Eq. (31) leads to the following solution:

$$z=\sqrt{1-F^2}+\log|F|-\log|1+\sqrt{1-F^2}|. \tag{32}$$

The graph (Fig. 1) of this solution exhibits a discontinuous slope (cusp) at its crest, and thus justifies the term "cusp soliton," although all physical quantities remain continuous.

VI. RESULTS AND DISCUSSION

In the present work we have shown some new aspects of the lower-hybrid wave with spatial dispersion in a dusty plasma, which is a relatively higher frequency lower-hybrid (DMLH) wave and its subsequent decay into a lower frequency dust-modified lower hybrid wave and an ultralow frequency dust lower hybrid (DLH) wave. The linear dispersion relations have been obtained for these waves and via a three-wave interaction the DLH waves are shown to be excited. Zakharov equations for the DMLH wave and a Schrödinger equation for DLH waves are obtained with the nonlinear term coming from the DLH waves. The modulational instability of the DMLH wave is investigated. In two different approximations ordinary soliton and cusp soliton solutions have been obtained. Such investigations have not been undertaken earlier.

The results of our investigations may be used to interpret data from space, especially from near-Earth plasmas, where nonlinear structures are observed. Our study may also be extended to examine the effect of lower hybrid wave heating in tokamaks where it is known that dust is present not only in the edge region but also in the core of the tokamak plasma. We speculate that the excitation of DLH waves as proposed in our work may be used for diagnostic purposes in laboratory plasmas, especially in tokamaks.

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