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Effect of trapping in a degenerate plasma in the presence of a quantizing magnetic field

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Effect of trapping as a microscopic phenomenon in a degenerate plasma is investigated in the presence of a quantizing magnetic field. The plasma comprises degenerate electrons and non-degenerate ions. The presence of the quantizing magnetic field is discussed briefly and the effect of trapping is investigated by using the Fermi-Dirac distribution function. The linear dispersion relation for ion acoustic wave is derived in the presence of the quantizing magnetic field and its influence on the propagation characteristics of the linear ion acoustic wave is discussed. Subsequently, fully nonlinear equations for ion acoustic waves are used to obtain the Sagdeev potential and the investigation of solitary structures. The formation of solitary structures is studied both for fully and partially degenerate plasmas in the presence of a quantizing magnetic field. Both compressive and rarefactive solitons are obtained for different conditions of temperature and magnetic field. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4752416]

I. INTRODUCTION

Quantum or degenerate plasmas are of great interest due to their important applications in modern technology and astrophysics. Such plasmas have generated a lot of interest in the last decade owing to their importance in many areas of physics such as semiconductors, metals, microelectronics,[1] carbon nanotubes, quantum dots, and quantum wells.[2–4] Degenerate plasmas also play an important role in dense astrophysical objects like white dwarfs and neutron stars.[5]

A very substantial volume of literature has been produced looking at various aspects of degenerate or quantum plasmas. Special attention has been received by linear and nonlinear propagation characteristics of different electrostatic and electromagnetic modes. Most of the work on wave propagation properties is based on the quantum hydrodynamic model.[6] The role of quantum diffraction has been incorporated in this model and it becomes a valuable and sufficiently simple tool for the examination of linear, weakly nonlinear and fully nonlinear waves. Nonlinear ion acoustic modes have received special attention in many papers for, e.g., the effect of the Bohm potential (quantum diffraction) has been investigated in Ref. 7 and a nonlinear Schrodinger formalism has been developed in Ref. 8, for describing collective phenomena in dense quantum plasmas with degenerate electrons. Much of the literature produced in the last decade or so in the area of linear and nonlinear wave propagation in quantum plasmas has been effectively summarized and reviewed by Shukla and Eliasson.[9] As pointed out in Ref. 9 and other works, plasmas in general and quantum plasmas in particular need a differentiation between being weakly coupled or strongly coupled. Electrons in degenerate plasmas in general tend to be weakly coupled but ions which may often be treated classically can be strongly coupled which in turn needs the incorporation of viscous effects. In Ref. 9, the effect of ion viscosity has been included to account for wave propagation characteristics in strongly coupled plasmas and the subsequent evolution of shock like structures via the Burgers equation (see Ref. 9 and references therein) is considered. We would however like to point out here, since in this work, we will be considering the collisionless non viscous regime, a collisionless quantum plasma regime is relevant for phenomena appearing on the time scale of the order of femtosecond.[9] In the present work, we limit ourselves to considering only weakly coupled degenerate plasmas, where effects of ion viscosity are not considered because as pointed out in Ref. 10 ion viscosities can normally be neglected as long as the wave period is much larger than the time scale of the ion correlations and the damping rate due to the viscosities is much smaller than the work frequency of the wave. It has also been shown in Ref. 11 that there is a large range of validities for the hydrodynamic model in dense plasmas.

The effect of strong magnetic fields has not been much the focus of attention in degenerate plasmas. The presence of a strong ambient magnetic field qualitatively changes the properties of atoms, molecules, and condensed matter when the electron cyclotron energy $\hbar\omega_{ce}$ is larger than the typical Coulomb energy.[12] The usual perturbative treatment of magnetic effects like Zeeman splitting of atomic energy levels in a strong field regime does not apply in such a situation but instead, the Coulomb forces act as a perturbation to the magnetic forces. Owing to the extreme confinement of electrons in the transverse direction, the Coulomb force becomes much more effective in binding the electrons along the magnetic field direction.[13] As is well known, electron gas magnetization in a weak magnetic field has two independent parts; (i) paramagnetic, and (ii) the diamagnetic part. The intrinsic or spin magnetic moment of electrons gives rise to Pauli paramagnetism. The diamagnetic part is due to the fact that the orbital motion of electrons becomes quantized in a magnetic field.

This is also called Landau diamagnetism or Landau quantization.[14] The gas is degenerate if the temperature $T \ll \varepsilon_F$ (ε_F is the Fermi energy). If the Landau quantization of electron motion in a magnetic field is taken into account, then the field is called quantizing[15] and the condition $k_B T \ll \hbar\omega_{ce}$ must be fulfilled. However, the ground state and the Fermi energies remain the same because when summed, the pairs of energy levels of the Landau levels cancel out each other.

In a magnetic field, the spin energy of electrons (which is the additional energy) is $\pm\beta H$, where β is the Bohr magneton and equal to $|e|\hbar/2m_e c$ and the two signs are due to the two values of the spin components, i.e., $\pm 1/2$ along the magnetic field. The energy levels in a magnetic field of electron's orbital motion are[14]

$$\varepsilon_e^\ell = (2\ell + 1)\beta H + \frac{p_z^2}{2m_e},$$

where $\ell = 0, 1, 2, \ldots,$ are the Landau levels, and the momentum p_z is along the magnetic field and has a continuous range of values from $-\infty$ to $+\infty$. The total energy including the spin energy is[14]

$$\varepsilon_e^\ell = \frac{p_z^2}{2m_e} + 2\ell\beta H.$$

The effects of Landau quantization on the equations of state in degenerate plasmas in the presence of an ambient magnetic field have been discussed in detail by Eliezer *et al*.[16] for both the fully degenerate case and for a partially degenerate case when a small ($T < T_{Fe}$) but finite electron temperature is taken into account. The relationship of thermodynamic quantities such as number density, pressure, entropy, etc., on the Landau quantization level is also derived using the standard Fermi-Dirac integrals (see Eqs. (40) and (53) of Ref. 16). The significance of Landau quantization for better understanding of degenerate plasmas has been pointed out in Ref. 16. The electron density is modified by the presence of a finite electron temperature in the sense that the occupation number increases. This is a standard result.[14] The Landau quantization causes an analogous modification. We note here that in the absence of the quantizing magnetic field, the finite temperature T, and in the absence of the trapping potential φ the total number density becomes the same as that defined by the Fermi pressure p_F. These results cannot be derived from the inertialess equation of motion of the electrons and the proper treatment in the case of trapping can only be done kinetically by the use of Fermi integrals and Fermi Dirac distribution.

Much earlier, the pioneering work of Bernstein, Greene, and Kruskal,[17] showed that trapped particles have a prominent effect on the nonlinear dynamics of plasma, while trapping was considered by the wave itself. An alternative approach, developed a decade later, considered the effect of adiabatic trapping at the microscopic level was introduced by Gurevich,[18] and it was observed that the adiabatic trapping produced a 3/2 power nonlinearity instead of the usual quadratic one when trapping was absent. Experimental investigations[19] and computer simulations[20] confirmed the presence of trapping as a microscopic phenomenon. Maxwellian and non-Maxwellian distribution functions have been used to investigate the effect of trapping on the propagation characteristics of ion acoustic solitons.[21] In both the cases, it was seen that solitary dynamics changed considerably, in the latter case especially where spiky solitons were obtained. In classical plasmas, the trapping effect on the formation of vortices was considered, and the Hasegawa-Mima equation in modified form was derived and analyzed by considering shallow and deep potential wells, respectively.[22] The effect on the Sagdeev potential was also investigated. Luque *et al*.[23] carried out one of the first investigations in quantum plasmas, who took quantum corrected electron holes by perturbatively solving the Wigner-Poisson system.

Demeio[24] considered the trapping effects on Bernstein, Greene, and Kruskal equilibria and solved the Wigner-Poisson system using the perturbative technique in order to study the effect of trapping in quantum phase space. However, the statistical nature of trapping in Refs. 23 and 24 was not investigated as the Wigner-Poisson equation was used, which showed only the quantum diffraction effects.

Trapping in quantum plasma has been considered by us recently using the Gurevich approach[25] where the formation of one dimensional ion acoustic solitary structures both for fully degenerate plasma and for small temperature effects were investigated. This work was later extended to the case of fully relativistic degenerate plasma including both trapping and finite temperature effects.[26]

In the present work, we consider once again the propagation of one dimensional solitary ion acoustic waves in a quantum degenerate plasma taken into account the effect of trapped particles and finite temperature but this time in the presence of a quantizing magnetic field via Landau quantization. The Fermi-Dirac distribution function is as usual used to describe the massless electrons.

The layout of the present work is as follows: In Sec. II, we give a formulation of our basic equations and derive the linear dispersion relation for Ion acoustic waves in the presence of a quantizing magnetic field. In Sec. III, the Sagdeev potential is derived and investigated. In Sec. IV, we give an analysis of our results which are discussed graphically as well.

II. BASIC SET OF EQUATIONS

In order to derive the expressions for parallel propagating nonlinear ion acoustic waves in the presence of adiabatic trapping, we begin by considering the evaluation of the expression of number density of the electrons which are Landau quantized, degenerate, and trapped. We follow Landau and Lifshitz[14] to obtain the expression for the number densities of the trapped and free electrons, where the trapping of the electrons occurs in the potential of the ions. The motion of a particle in a plane perpendicular to the strong magnetic field is quantized.[27] The quantized electron energy levels ε_e^ℓ in the non-relativistic limit in the presence of a potential field φ are then given by[27]

$$\varepsilon_e^\ell = \ell\hbar\omega_{ce} + \frac{p_z^2}{2m_e} - e\varphi, \tag{1}$$

where $\omega_{ce} = eB_0/m_e c$ is the electron cyclotron frequency and $-e\varphi$ is the potential energy of the well in which the electrons are trapped and p_z is the parallel momentum associated with the electron. Here $\boldsymbol{B}_0 = \hat{z}B_0$ is the orientation of the external magnetic field in the Cartesian coordinate system. Electrons with energy $\varepsilon_e^\ell < 0$ and $\varepsilon_e^\ell > 0$ are trapped and free electrons, respectively. Trapping occurs when the condition $\varepsilon_e^\ell = 0$ is fulfilled.

The total occupation number for the Fermi-Dirac distribution after integration over the polar coordinates and change of variables from momentum p to energy ε is given by

$$n_e = \frac{p_{Fe}^2 \eta}{2\pi^2 \hbar^3} \sqrt{\frac{m_e}{2}} \sum_{\ell=0}^{\infty} \int_0^{\infty} \frac{\varepsilon^{-1/2}}{\exp\left\{\frac{\varepsilon - U}{\mathrm{T}}\right\} + 1} \mathrm{d}\varepsilon, \tag{2}$$

where $U = e\varphi + \mu - \ell\hbar\omega_{ce}$, here μ is the chemical potential. The summation above is over the Landau levels and we note here $\ell = 0$ refers to the case without a quantizing magnetic field. In the energy eigenvalue spectrum of a macroscopic system, there is an extremely high density of energy levels. The number of levels in a finite range of energy spectrum increases exponentially with N (number of particles) in the system, and the separation between the levels is proportional to 10^{-N}.[27] Therefore, we can conclude that it is reasonable to take a continuous energy spectrum instead of a discrete one. Thus, to obtain an expression of the density n_e after integration, we can separate the $\ell = 0$ case from the summation and replace the summation in Eq. (2) by integration ($\sum_1^{\ell_{max}} \to \int_1^{\ell_{max}} d\ell$), where $\ell_{max} = (1 + \frac{e\varphi}{\varepsilon_{Fe}})/\eta$ which is obtained from the condition that the integrand must remain a real quantity. We note here that as pointed in Ref. 28 anisotropy in a degenerate plasma may appear through the Fermi temperature T_{Fe} and is not due to the system (physical) temperature T where $T \ll T_{Fe}$. However, for propagation in one dimension along the magnetic field anisotropy in T_{Fe} (or the number density) will not play any role hence the distribution function considered by us is taken to be isotropic. Following the general treatment for Fermi integrals and the method elucidated in Refs. 25 and 26, we arrive at the following expression for the total number density

$$n_e = N_0 \left[\frac{3}{2}\eta(1+\Phi)^{\frac{1}{2}} + (1+\Phi-\eta)^{3/2} - \frac{\eta T^2}{2}(1+\Phi)^{-\frac{3}{2}} + T^2(1+\Phi-\eta)^{-1/2} \right]. \tag{3}$$

Here the effect of the quantizing magnetic field appears through $\eta = \hbar\omega_{ce}/\varepsilon_{Fe}$, and $\varepsilon_{Fe} = (\hbar^2/2m_e)(3\pi^2 N_0)^{2/3}$ is the electron Fermi energy, $N_0 = p_{Fe}^3/3\pi^2\hbar^3$ is the equilibrium number density for fully degenerate plasma and m_e is the mass of the electrons. For partially degenerate plasma the electron background number density is given by $n_{e0} = N_0\{(3-T^2)\eta/2 + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}\}$, which follows from Eq. (3) in the absence of the trapping potential. We further note that in the limit $\eta = 0$, we retrieve our previous results as Ref. 25. The Landau quantization parameter plays role similar to that of the small finite temperature T in modifying the electron occupation number density n_e given by Eq. (3). The chemical potential is not truly equal to the Fermi energy when $\mathrm{T} \neq 0$, however, in the case when $T/\varepsilon_{Fe} \ll 1$, it is reasonable to take $\mu = \varepsilon_{Fe}$. The potential φ and temperature T have been normalized in the following manner: $T = \pi T/2\sqrt{2}\varepsilon_{Fe}$ and $\Phi = e\varphi/\varepsilon_{Fe}$.

We now draw our attention to the ions; the ions are taken to be cold and non-degenerate due to their heavy mass as compared to the electrons. The ion equation of motion and the continuity equation are given by

$$\left[\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \vec{\nabla})\vec{v}_i\right] = -\frac{e}{m_i}\vec{\nabla}\varphi + \frac{e}{m_i}(\vec{v}_i \times \vec{B}) \tag{4}$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot (n_i \vec{v}_i) = 0. \tag{5}$$

Since we consider propagation along B_0, therefore, parallel propagating ion acoustic waves are not affected by the ambient magnetic field. For the case of propagation along the external magnetic field, the ion Lorentz force does not contribute. Finally, in order to have a closed set of equations, we include the Poisson's equation

$$\vec{\nabla} \cdot \vec{E} = 4\pi e(n_i - n_e). \tag{6}$$

After linearizing and applying a plane wave solution to the above set of equations, we obtain the following linear dispersion relation for parallel propagating ion acoustic wave in the presence of quantizing field with electrons having small but finite temperature

$$\frac{\omega}{k} = C_{sF}\left[\frac{2\left\{(3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}\right\}}{(1+T^2)\frac{3\eta}{2} + 3(1-\eta)^{1/2} - T^2(1-\eta)^{-3/2} + 2k^2\lambda_{TF}^2}\right]^{1/2}. \tag{7}$$

where C_{sF} is the Fermi ion sound velocity and λ_{TF} is the Thomas Fermi length and are given as

$$C_{sF} = \sqrt{\frac{\varepsilon_{Fe}}{m_i}} \quad \text{and} \quad \lambda_{TF} = \sqrt{\frac{\varepsilon_{Fe}}{4\pi e^2 N_0}},$$

respectively. Furthermore, the wave phase speed is much larger than the ion thermal speed. In order to find the nonlinear ion number density, we again use the ions equation of motion and continuity equation. As we consider the propagation of quantum ion acoustic wave in the z direction only and shift to a comoving frame of reference by taking $\xi = z - ut$, where u is the velocity of propagation of the perturbation. We integrate Eqs. (4) and (5) and apply the boundary conditions that all perturbations die out at infinity, i.e., when

$$\xi \to \infty, \quad \varphi, v_i \to 0, \quad \text{and} \quad n_i \to n_{0i}$$

where n_{0i} is the ions background number density. Taking the unperturbed electron and ion number densities to be equal, we get the following expression for ion number density

$$n_i = N_0\left[(3-T^2)\frac{\eta}{2} + (1-\eta)^{\frac{3}{2}} + T^2(1-\eta)^{-\frac{1}{2}}\right]\left(1-\frac{2\Phi}{M^2\alpha}\right)^{-1/2}. \tag{8}$$

Here the Mach number is defined as $M = \frac{u}{\omega/k}$, and α is a constant given by

$$\alpha = \frac{2\left\{(3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}\right\}}{(1+T^2)\frac{3\eta}{2} + 3(1-\eta)^{1/2} - T^2(1-\eta)^{-3/2}}.$$

III. SAGDEEV POTENTIAL

Now we derive an expression for the Sagdeev potential to investigate the presence of solitary waves. Putting the values of n_e and n_i from Eqs. (3) and (8) in the Poisson equation given by Eq. (6), we obtain

$$\frac{d^2\Phi}{d\xi^2} = \left[\left\{\begin{array}{c}\frac{3}{2}\eta(1+\Phi)^{\frac{1}{2}} + (1+\Phi-\eta)^{\frac{3}{2}} - \frac{\eta T^2}{2}(1+\Phi)^{-\frac{3}{2}} \\ +T^2(1+\Phi-\eta)^{-1/2}\end{array}\right\} - \left\{(3-T^2)\frac{\eta}{2} + (1-\eta)^{\frac{3}{2}} + T^2(1-\eta)^{-\frac{1}{2}}\right\}\left(1-\frac{2\Phi}{M^2\alpha}\right)^{-1/2}\right], \tag{9}$$

where ξ is normalized as $\xi = \xi/\lambda_{TF}$. Equation (9) can be expressed in the form of an energy integral as

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0. \tag{10}$$

By integrating Eq. (9) and making the use of boundary conditions

$$\xi \to \infty, \qquad \Phi = V(\Phi) = 0,$$

the final expression for the Sagdeev potential is

$$\begin{aligned} V(\Phi) = \Bigg[&(1+T^2)\eta + \frac{2}{5}(1-\eta)^{5/2} - \eta(1+\Phi)^{3/2} \\ &-\frac{2}{5}(1+\Phi-\eta)^{5/2} + M^2\alpha\beta - M^2\alpha\beta\left(1-\frac{2\Phi}{M^2\alpha}\right)^{1/2} \\ &-\eta T^2(1+\Phi)^{-1/2} - 2T^2(1+\Phi-\eta)^{1/2} \\ &+2T^2(1-\eta)^{1/2}\Bigg]. \end{aligned} \tag{11}$$

Here

$$\beta = (3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}.$$

Solitary waves are obtained when appropriate conditions are fulfilled[25,29] and compressive and rarefactive solitary waves are obtained when $V(\Phi) < 0$ for $0 < \Phi < \Phi_{max}$ for the compressive condition and $V(\Phi) < 0$, for $0 > \Phi > \Phi_{min}$ for the rarefactive condition.

In order to find the lower limit of the Mach number, Eq. (11) is Taylor expanded and the coefficients of quadratic terms in Φ are set equal to zero which yields

$$M \geq 1.$$

The upper limit of the Mach number is obtained from the ion term in Eq. (11) such that it does not produce an imaginary result. Thus, the upper limit is given by

$$M < \left\{\frac{(1+T^2)\frac{3\eta}{2} + 3(1-\eta)^{1/2} - T^2(1-\eta)^{-3/2}}{(3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}}\right\}^{1/2}.$$

Thus, the range of the Mach number is given by

$$1 \leq M < \left\{\frac{(1+T^2)\frac{3\eta}{2} + 3(1-\eta)^{1/2} - T^2(1-\eta)^{-3/2}}{(3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}}\right\}^{1/2}. \tag{12}$$

We take the different cases to discuss for fully and partially degenerate plasma, also with and without the presence of quantizing magnetic field. We discuss certain limiting cases given here as under

Case 1, when $\eta = 0$, i.e., in the absence of the magnetic field, the Sagdeev potential becomes

$$\begin{aligned} V(\Phi) = \Bigg[&\frac{2}{5} - \frac{2}{5}(1+\Phi)^{5/2} + M^2\alpha\beta - M^2\alpha\beta\left(1-\frac{2\Phi}{M^2\alpha}\right)^{1/2} \\ &+2T^2 - 2T^2(1+\Phi)^{1/2}\Bigg]. \end{aligned} \tag{13}$$

Here α and β reduce to

$$\alpha = \frac{2(1+T^2)}{3-T^2} \quad \text{and} \quad \beta = 1+T^2,$$

and the range of Mach number is given by

$$1 \leq M < \left\{\frac{3-T^2}{1+T^2}\right\}^{1/2}.$$

This is the same as our earlier result obtained in Ref. 25.

Case 2, when $T = 0$, i.e., for fully degenerate plasma, the Sagdeev potential is

$$V(\Phi) = \left[\eta + \frac{2}{5}(1-\eta)^{5/2} - \eta(1+\Phi)^{3/2} - \frac{2}{5}(1+\Phi-\eta)^{5/2} + M^2\alpha\beta - M^2\alpha\beta\left(1 - \frac{2\Phi}{M^2\alpha}\right)^{1/2}\right]. \quad (14)$$

Here α, β and range of the Mach number reduce to

$$\alpha = \frac{2\left\{(3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2}\right\}}{\frac{3\eta}{2} + 3(1-\eta)^{1/2}},$$
$$\beta = (3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2},$$
$$1 \le M < \left\{\frac{\eta + 2(1-\eta)^{1/2}}{\eta + \frac{2}{3}(1-\eta)^{3/2}}\right\}^{1/2}.$$

Case 3, when both $T = 0$, and $\eta = 0$, the Sagdeev potential is

$$V(\Phi) = \left[\frac{2}{5} - \frac{2}{5}(1+\Phi)^{5/2} + \frac{2}{3}M^2 - \frac{2}{3}M^2\left(1 - \frac{3\Phi}{M^2}\right)^{1/2}\right]. \quad (15)$$

This result is the same as in Ref. 25. For this range of the Mach number is given by

$$1 \le M < \sqrt{3}\ .$$

IV. RESULTS AND DISCUSSION

In this section, the theoretical results obtained in Secs. II and III have been presented graphically. The parameters that we have used in our graphs are the ones typically found for the white dwarfs.[30,31] The number density and the magnetic field have the values of the order of $10^{26}\,\mathrm{cm}^{-3}$ and 10^{10} G, respectively.[26,30] Using this number density, we have calculated the Fermi energy and the Fermi temperature given by $T_{Fe} = 9.14108 \times 10^6$ K and have taken the electron temperature $T \ll T_{Fe}$.[25]

We begin by considering the linear dispersion relation Eq. (7) and investigate the effect of the finite electron temperature and the effect of the quantizing magnetic field expressed through η. Fig. 1 shows dependence of the wave

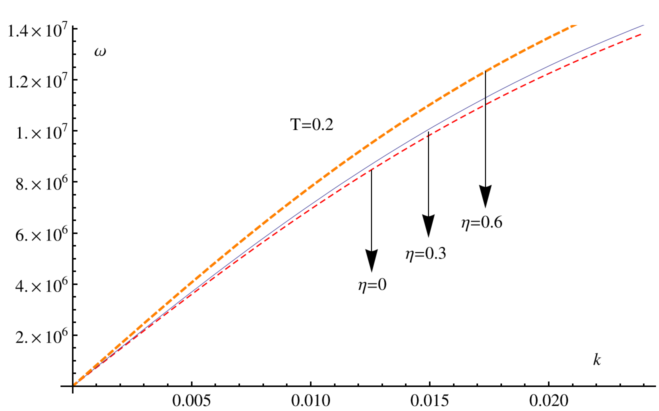

FIG. 1. Dispersion relation for $\boldsymbol{\omega}$ versus $\boldsymbol{k}$ for different values of η.

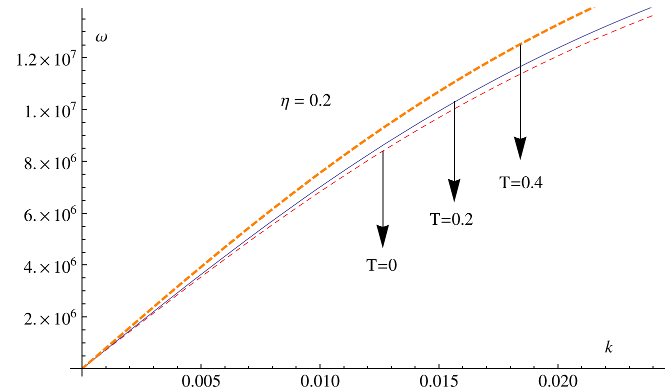

FIG. 2. Dispesrion relation for $\boldsymbol{\omega}$ versus $\boldsymbol{k}$ for different values of T.

frequency ω on the wave number k for different values of η keeping the temperature constant. It is seen that higher frequencies are observed with an increase in the magnetic field. Fig. 2 shows the wave frequency ω versus the wave number k plot for different values of temperature T keeping η fixed. It is observed that an increase in temperature enhances the frequency of the ion acoustic wave.

We now investigate the nonlinear properties of the ion acoustic wave via the Sagdeev potential and its dependence on the temperature T, the magnetic field through the parameter η, and the Mach number M. Fig. 3 depicts the plot between normalized potential Φ and Sagdeev potential V(Φ). We have used Eq. (11) to plot these graphs. These curves are plotted for fully degenerate plasma, i.e., $T = 0$ and Mach number is kept constant $M = 1.3$ but magnetic field is varied by changing η. From Fig. 3, we can see that with variation of magnetic field the depth and width of the Sagdeev potential increase, but the minimum of the Sagdeev potential decreases. Fig. 4 shows the corresponding soliton profiles obtained for the similar parameters used in Fig. 3. The amplitude of the soliton increases with the increasing magnetic field with the values of η ranging from 0.1 to 0.6 as shown in the Fig. 4. With these parameters we have only compressive solitons as shown in Figs. 3 and 4.

Again, using the Eq. (11), we have plotted the graphs between V(Φ) and Φ, by keeping the magnetic field effect fixed at $\eta = 0$, and also by keeping the Mach number fixed

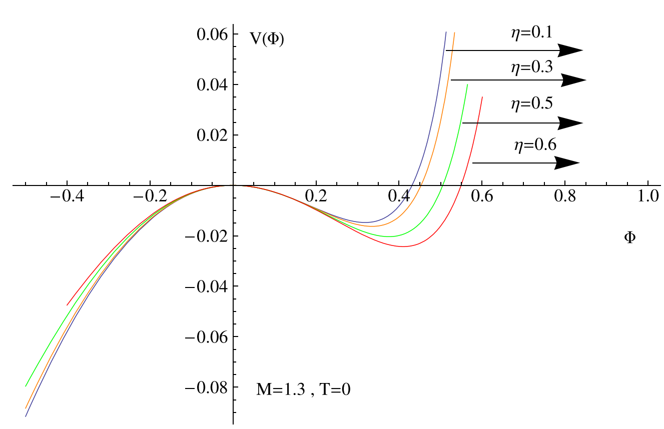

FIG. 3. Sagdeev potential V(Φ) versus Φ for different values of η when M = 1.3 and T = 0.

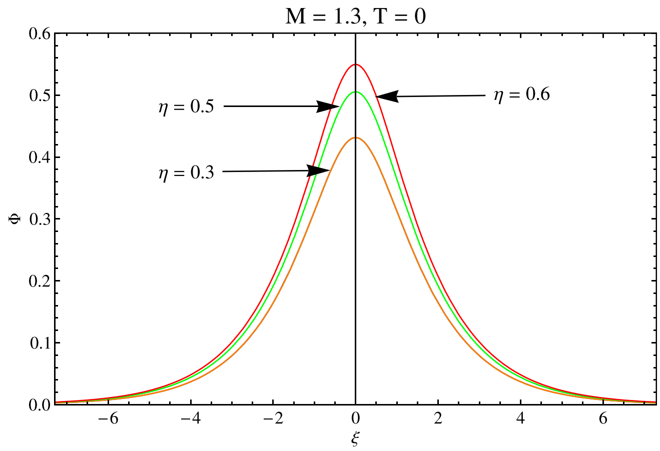

FIG. 4. Solitary wave amplitude Φ versus ξ corresponding to the Sagdeev potential V(Φ) shown in Fig. 3.

at $M = 1.3$. Here, the normalized electron temperature has been varied as shown in Fig. 5. In this figure, we observe that increasing the temperature enhances the width as well as the depth of Sagdeev potential. We have also two minima of V(Φ) for the temperature $T = 0.7$. The solitary structures of this Sagdeev potential are shown in Fig. 6. We have compressive solitons for certain range of electron temperature and above this range, i.e., for $T > 0.5$ rarefactive solitons are obtained as shown in Fig. 6. Such coupled solitary structures are observed in space plasmas.[31] The same effect has observed in Refs. 26 and 29. The amplitude of the soliton is found to increase with the increase in the temperature.

Fig. 7 shows the variation of V(Φ) and potential Φ for different values of Mach number. Here, the parameters like the temperature and the magnetic field are fixed as $T = 0.2$ and $\eta = 0.2$. It is observed that for greater value of Mach number, the potential Φ is larger and the Sagdeev potential is deeper. It is found that the width of the soliton decreases but amplitude increases with the increase of Mach number as shown in Fig. 8.

Fig. 9 is the graph of V(Φ) vs Φ for fixed values of Mach number and temperature, i.e., $M = 1.3$ and $T = 0.4$, but η is varied that includes the variation of magnetic field. We observe two minimum values of Sagdeev potential for each curve at large magnetic field. Also compressive and rarefactive solitary structures begin to form at higher magnetic fields ($B_0 \approx 3.5 \times 10^{10}$ G and 4×10^{10} G) for which $\eta = 0.5$ and $\eta = 0.6$, respectively. The related solitary structures are shown in Fig. 10. Similarly, Fig. 11 shows the graphs plotted between Sagdeev potential and potential Φ for varying electron normalized temperature T but Mach number M and η

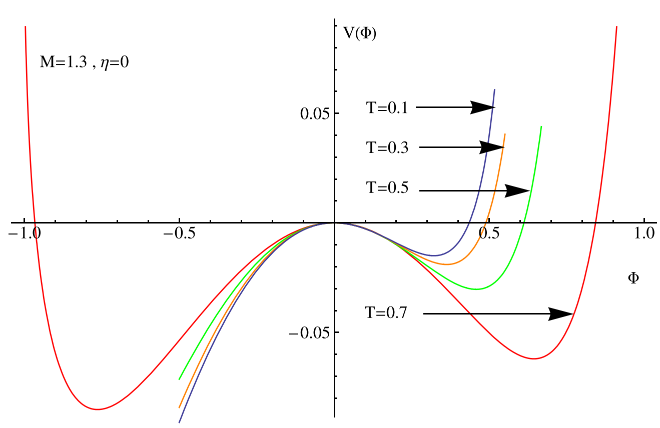

FIG. 5. Sagdeev potential V(Φ) versus Φ for different values of T when M = 1.3 and η = 0.

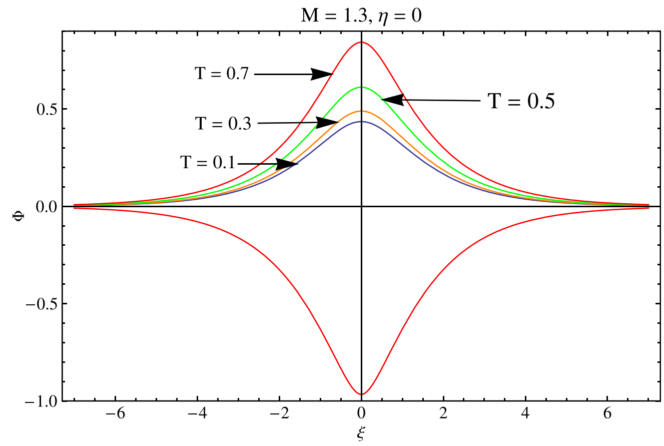

FIG. 6. Solitary wave amplitude Φ versus ξ corresponding to the Sagdeev potential V(Φ) shown in Fig. 5.

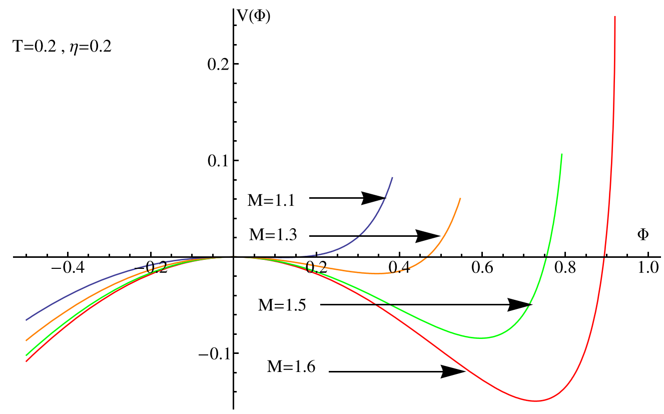

FIG. 7. Sagdeev potential V(Φ) versus Φ for different values of Mach number M when T = 0.2 and η = 0.2.

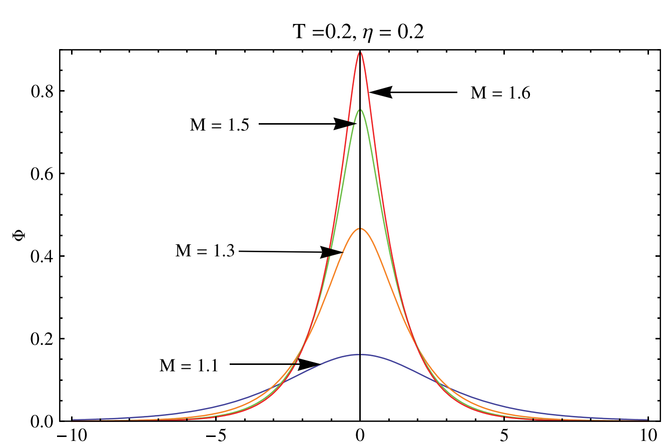

FIG. 8. Solitary wave amplitude Φ versus ξ corresponding to the Sagdeev potential V(Φ) shown in Fig. 7.

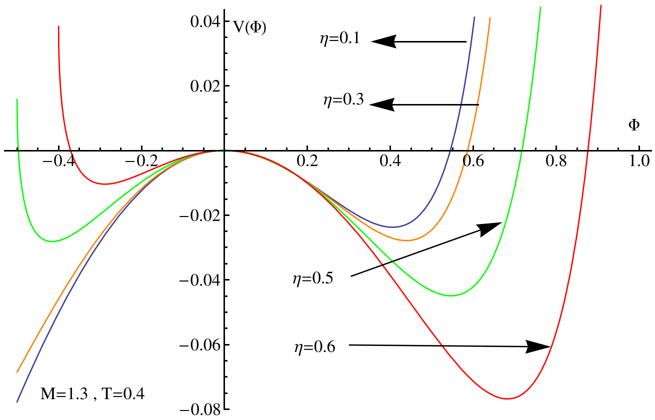

FIG. 9. Sagdeev potentials V(Φ) versus Φ for different values of η when M = 1.3 and T = 0.4.

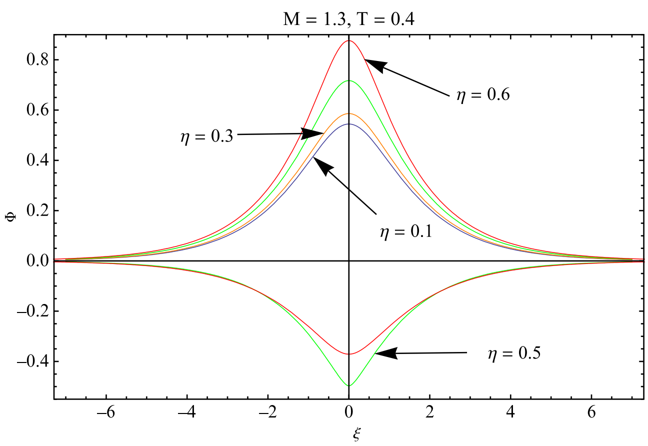

FIG. 10. Soliton wave amplitude Φ versus ξ corresponding to the Sagdeev potential V(Φ) shown in Fig. 9.

are fixed at the values $M = 1.3$ and $\eta = 0.2$. Due to increase in temperature, the Sagdeev potential becomes deeper and corresponding potential Φ becomes wider. But we have only one minima for each curve of V(Φ) as T is increased as compared to the previous result shown in Fig. 5, where we have two minimum at $T = 0.7$ and $\eta = 0$. The reason for this is that the values of T and η are so adjusted in Eq. (11) that as T is increased the Sagdeev potential V(Φ) has only negative values, i.e., V(Φ) < 0 as shown in Fig. 11. The corresponding solitons of these Sagdeev potentials of Fig. 11 are plotted in Fig. 12 between potential Φ and ξ. Here, we have only compressive solitons and only increase in amplitude of the solitons can be seen in Fig. 12.

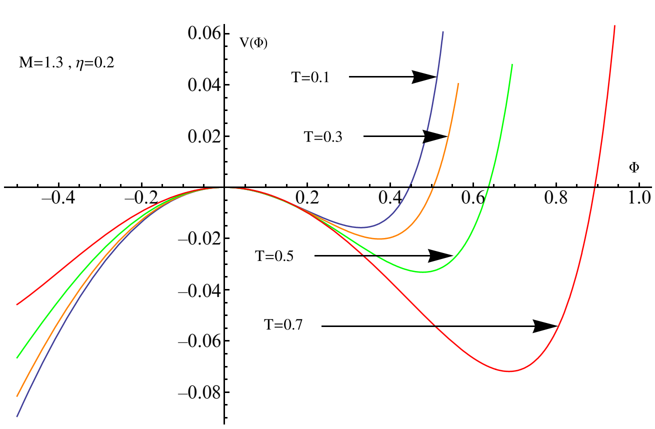

FIG. 11. Sagdeev potentials V(Φ) versus Φ for different values of temperature T when M = 1.3 and $\eta = 0.2$.

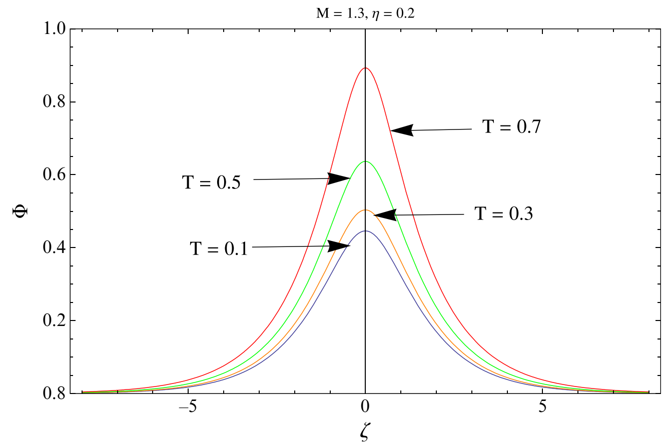

FIG. 12. Solitary wave amplitude Φ versus ξ corresponding to the Sagdeev potential V(Φ) shown in Fig. 10.

Fig. 13 shows the variation of soliton's amplitude by varying the Mach number. It is clear from the curves that the amplitude increases linearly with the increase of Mach number. Also the amplitude of soliton enhances with the increase in the magnetic field. It is also seen the amplitude of compressive and rarefactive solitons are different. Compressive solitons have higher amplitude by comparison with the rarefactive solitons. Such solitons are observed in the literature. Fig. 14 shows the amplitude of the soliton with the increasing Mach number curves for different values of temperature. Fig. 15 shows the dependence of Mach number M on η. It is seen that when we increase the value of η the corresponding Mach number decreases.

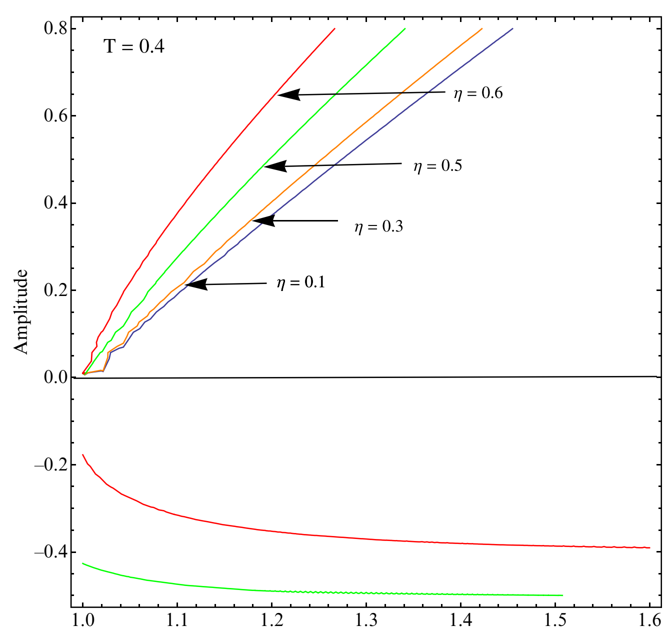

FIG. 13. Variation of compressive and rarefactive solitary waves amplitude Φ vs Mach number M.

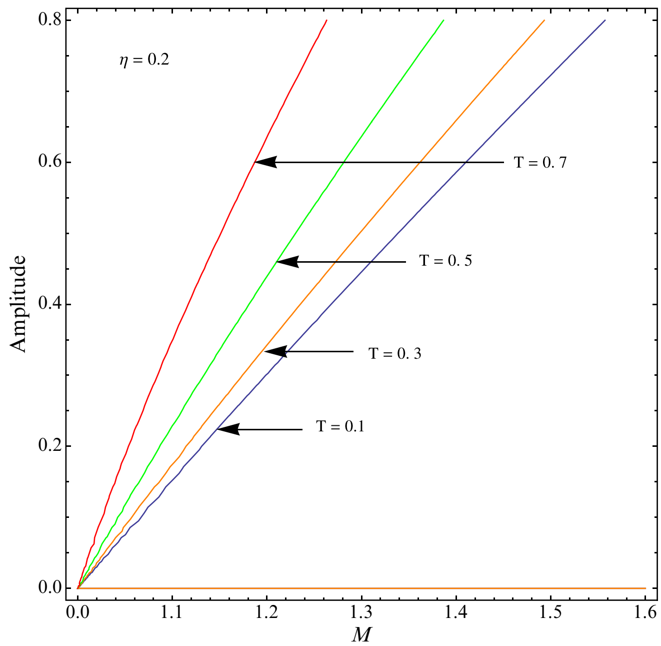

FIG. 14. Variation of solitary wave amplitude Φ vs Mach number M.

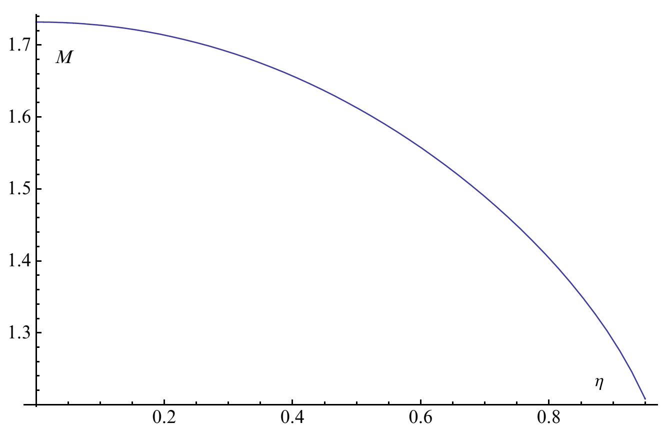

FIG. 15. Variation of Mach number M vs η.

V. CONCLUSION

In conclusion, we have investigated the effect of trapping as a microscopic phenomenon on the formation of solitary structures in degenerate and partially degenerate plasmas in the presence of a quantizing magnetic field. We have derived the modified linear dispersion relation for the ion acoustic wave. We have observed that in our case, the wave propagating along the magnetic field depends on the magnetic field as opposed to the classical case. We have investigated our theoretical results numerically for different parameter values like Mach number, magnetic field, and electron temperature. The results have been presented graphically showing the formation of solitary structures and their dependence on different plasma parameters. We have shown that only compressive solitons are formed for fully degenerate plasma in the presence of a magnetic field. For the case of partially degenerate plasma, we have both compressive and rarefactive solitons. These investigations may play an important role in the ultra strong laser plasma interactions as well as the description of complex phenomena that appear in the dense astrophysical objects like White dwarfs.

[1]A. Markowich, C. A. Ringhofer, and C. Schmeiser, *Semiconductor Equations* (Springer, Vienna, 1990).
[2]L. K. Ang, T. J. Kwan, and Y. Y. Lau, Phys. Rev. Lett. **91**, 208303 (2003).
[3]T. C. Killian, Nature (London) **441**, 297 (2006).
[4]Y. D. Jung, Phys. Plasmas **8**, 3842 (2001).
[5]M. Opher, L. O. Silva, D. E. Dauger, V. K. Decyk, and J. M. Dawson, Phys. Plasmas **8**, 2454 (2001).
[6]F. Haas, L. G. Garcia, J. Geodert, and G. Manfredi, Phys. Plasmas **10**, 3858 (2003).
[7]B. Eliasson and P. K. Shukla, J. Plasma Phys. **74**, 581 (2008).
[8]P. K. Shukla and B. Eliasson, Phys. Usp. **53**, 51–76 (2010)
[9]P. K. Shukla and B. Eliasson, Rev. Mod. Phys. **83**, 885 (2011).
[10]B. Eliasson and P. K. Shukla, Europhys. Lett. **97**, 15001 (2012).
[11]J. P. Mithen, J. Daligault, and G. Gregori, Phys. Rev. E. **83**, 015401 (2011).
[12]A. M. Abrahams and S. L. Shapiro, Astrophys. J. **374**, 652 (1991).
[13]D. Lai, Rev. Mod. Phys. **73**, 629–662 (2001)
[14]L. D. Landau and E. M. Lifshitz, *Statistical Physics I* (Pergamon, New York, 1980).
[15]A. Y. Potekhin, Astron. Astrophys. **306**, 999–1010 (1996).
[16]S. Eliezer, P. Norreys, J. T. Mendonca, and K. Lancaster, Phys. Plasmas **12**, 052115 (2005)
[17]I. B. Bernstein, J. M. Green, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).
[18]A. V. Gurevich, Sov. Phys. JETP **53**, 953 (1967).
[19]R. Z. Sagdeev, *Review of Plasma Physics* (Consultants Bureau, New York, 1996), Vol. 4.
[20]N. S. Erokhin, N. N. Zolnikova, and I. A. Mikhailovskaya, Fiz. Plazmy. **22**, 137 (1996).
[21]A. Mushtaq and H. A. Shah, Phys. Plasmas **13**, 012303 (2006).
[22]H. Siddiqui, H. A. Shah, and N. L. Tsintsadze, J. Fusion Energy **27**, 216 (2008).
[23]A. Luque, H. Schamel, and R. Fedele, Phys. Lett. A **324**, 185 (2004).
[24]L. Demeio, Transp. Theory State Phys. **36**, 137 (2007).
[25]H. A. Shah, M. N. S. Qureshi, and N. L. Tsintsadze, Phys. Plasmas **17**, 032312 (2010).
[26]H. A. Shah, W. Masood, M. N. S. Qureshi, and N. L. Tsintsadze, Phys. Plasmas **18**, 102306 (2011).
[27]L. N. Tsintsadze, “Quantization and excitation of longitudinal electrostatic waves in magnetized quantum plasmas,” AIP Conf. Proc. **1306**, 89 (2010).
[28]M. Akbari-Moghanjoughi, Phys. Plasmas **19**, 062106 (2012).
[29]A. A. Mamun, Phys. Rev. E **55**, 1852 (1997).
[30]D. Koester and G. Chanmugam, Rep. Prog. Phys. **53**, 837 (1990).
[31]S. S. Ghosh and G. S. Lakhina, Nonlinear Processes Geophys. **11**, 219 (2004).