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Adiabatic trapping in coupled kinetic Alfvén-acoustic waves

H. A. Shah,[1] W. Masood,[2,3,4] and Z. Ali[1]
[1]*Department of Physics, G.C. University, 54000 Lahore, Pakistan*
[2]*COMSATS, Institute of Information Technology, Park Road, Chak Shahzad, Islamabad 44000, Pakistan*
[3]*National Centre for Physics (NCP), Shahdara Valley Road, 44000 Islamabad, Pakistan*
[4]*Theoretical Plasma Physics Division, P. O. Nilore, Islamabad, Pakistan*

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In the present work, we have discussed the effects of adiabatic trapping of electrons on obliquely propagating Alfvén waves in a low β plasma. Using the two potential theory and employing the Sagdeev potential approach, we have investigated the existence of arbitrary amplitude coupled kinetic Alfvén-acoustic solitary waves in both the sub and super Alfvénic cases. The results obtained have been analyzed and presented graphically and can be applied to regions of space where the low β assumption holds true. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4794730]

I. INTRODUCTION

Alfvén waves have been very actively investigated and have found numerous applications in laboratory, space, astrophysical, and fusion plasmas.[1,2] Their role in heating, acceleration of particles, and transport of magnetic energy in various space plasmas is well established. In 1970, Stefant[2] pointed out that if the perpendicular wavelength of the wave is comparable to the ion gyro radius, then the ions do not follow the magnetic field lines but the lighter electrons due to their smaller mass and Larmor radius will follow the magnetic field lines. This subsequently produces a charge separation and there is a coupling of the Alfvén wave to the electrostatic longitudinal mode which leads us to the waves, known as kinetic Alfvén waves (KAWs), which have a dispersive character for oblique propagation.[1,2] Charge separation can also occur if electron inertia is taken into account and if the electron Larmor radius is comparable to the perpendicular wavelength, then such waves are called inertial Alfvén waves (IAWs). The linear and nonlinear propagation characteristics of both KAWs and IAWs have been investigated quite thoroughly over the past few decades. For instance, results from 1D Vlasov drift-kinetic plasma simulations have revealed how and where auroral electrons are accelerated along the Earth's geomagnetic field. The resulting electron distribution functions have been shown to compare favorably with *in situ* observations, demonstrating for the first time, a self-consistent link between Alfvén waves and electrons that form aurora.[3] The damping/excitation and propagation of Alfvén waves has also been suggested to be affected by the presence of dust and modify the behavior of these low frequency waves in dusty plasma.[4] Very recently, the first ion kinetic simulation of three-dimensional (3D) nonlinear physics of mode conversion from a fast-mode compressional wave to KAWs has been reported that occurs when a large amplitude compressional mode propagates across a plasma boundary into an increasing Alfvén velocity[5] and the study has been shown to be relevant for the Earth's magnetopause.

The novel idea of using the two potential theory for Alfvén waves in a low beta plasma was first introduced by Kadomtsev in 1965.[6] The main idea is that the low beta assumption allows one to describe the electric field by two potential fields that produce only shear perturbations in the magnetic field. Later, Hasegawa and coauthors[7,8] investigated the linear and nonlinear Alfvén waves in an electron-ion plasma by considering the finite Larmor radius effect.[6,7] Yu and Shukla[9] investigated the finite amplitude solitary Alfvén waves for small but finite β effects. The solitons were shown to have density humps with an upper limit on the amplitude. The work by Hasegawa and coauthors[7,8] and Yu and Shukla[9] was further extended to the case of electron-positron-ion plasma,[10] where the existence of solitons was shown for different values of β and positron concentration. Yinhua *et al.*[11] studied the KAWs in dusty plasma and reported the formation of both compressive and rarefactive nonlinear structures. It was also observed that the density humps were cusped and narrower by comparison with the dips.

In the present work, we investigate, for the first time, the effect of adiabatically trapped electrons on the propagation characteristics of arbitrary amplitude solitary kinetic Alfvén waves which is an extension of the work by Yu and Shukla[9] where the authors considered Boltzmannian response of the electrons. The adiabatic trapping of the particles was shown to be important while investigating the nonlinear characteristics of waves.[12] It was seen that adiabatic trapping produced 3/2 power nonlinearity instead of the usual quadratic one without trapping. Computer simulations[13] and experimental investigations[14] confirmed the existence of trapping as a microscopic phenomenon. The effect of trapping on the propagation characteristics of ion acoustic solitons was investigated using Maxwellian and non-Maxwellian distribution functions[15,16] and it was seen that solitary dynamics was considerably modified in both the cases. The effect of trapping on the vortex formation was also investigated and it was shown that the trapping produced stronger nonlinearities and the original Hasegawa Mima equation got considerably modified.[17] The effect of adiabatic trapping in degenerate quantum plasmas has also been investigated and it has been shown that both rarefactive and compressive solitons may be formed under different temperature conditions.[18,19] In a self-gravitating dusty quantum plasma, the trapping has also been found to play an effective role in the formation of solitary structure.[20]

The present work is organized in the following manner: In Sec. II, we present the governing set of equations for the problem under consideration and also present the linear dispersion relation for the coupled Kinetic Alfvén acoustic waves. In Sec. III, we present the nonlinear analysis whereas in Sec. IV, the results are discussed. Finally, in Sec. V, we recapitulate the main findings of this paper.

II. MATHEMATICAL FORMULATION

In this section, we follow the method outlined in Ref. 6 and introduce two potential fields. As pointed out by Kadomtsev and briefly in the introduction, the approach is valid only for low β plasmas. It follows that coupling between KAWs effectively takes place for a small but finite value of β such that $1 > \beta > m_e/m_i$. We assume that the variation exists in the xz plane, where z is the direction of the ambient magnetic field. The low β assumption allows us to use the two potential theories to solve the problem under consideration. In the two potential theories, we use the two potential fields φ and ψ to describe the electric field[7] given by

$$E_x = -\frac{\partial\varphi}{\partial x}, \qquad E_z = -\frac{\partial\psi}{\partial z}, \qquad E_y = 0.$$

Subsequently, potential fields will produce only shear perturbations in magnetic field,

$$B_z = B_0, \qquad B_x = 0.$$

The relevant field equations are the quasi neutrality condition, for ions and electron densities

$$n_e = n_i = n. \tag{1}$$

The electrons are assumed to follow the magnetic field lines since the phase velocity of the wave is much less than the electron thermal speed, v_{the} (equivalent to $v_{the} > V_A$).[2] Following,[12] we take into account the effect of adiabatic trapping in a slowly varying potential ψ. For the shallow well potential after evaluating the relevant integrals, the total number density n is given by

$$n = n_0\left(1 + \frac{\psi}{T_e} - \frac{4}{3\sqrt{\pi}}\left(\frac{\psi}{T_e}\right)^{\frac{3}{2}}\right). \tag{2}$$

We note here that the 1st term on the right hand side of Eq. (2) is the back ground number density and the 2nd term corresponds to the linear perturbation. The 3rd term is a nonlinear term of power 3/2 and it is this term that is related to the adiabatic trapping of electrons. Since we deal with a shallow potential well, the higher order nonlinear terms are neglected as their contribution is small compared to the $\psi^{3/2}$ order term.

We use the fluid equation of motion to describe the ion dynamics where the ions are considered cold ($T_i \ll T_e$). The x-component of the ion velocity may be written as[2,7–11]

$$v_{ix} = -\frac{m_i}{eB_0{}^2}\frac{\partial^2\varphi}{\partial x\partial t}. \tag{3}$$

The parallel equation of equation of ions is given by

$$\partial_t v_{iz} + v_{ix}\partial_x v_{iz} + v_{iz}\partial_z v_{iz} = -\mathrm{e}/\mathrm{m_i}\partial_z\psi. \tag{4}$$

The Ampère's law for j_z (Ref. 7–9) is

$$\partial_x^2\partial_z^2(\varphi - \psi) = \mu_0\partial_t\partial_z(j_z). \tag{5}$$

The ion continuity equation is given by

$$\partial_t n_i + \partial_x(n_i v_{ix}) + \partial_z(n_i v_{iz}) = 0, \tag{6}$$

where v_x, v_z, and j_z are perpendicular and parallel velocity components and the parallel current component, respectively. The current density j_z is given by[9,10]

$$\partial_z j_z = e\partial_t n_e + e\partial_z(n_i v_{iz}). \tag{7}$$

It is worth mentioning here that the electron continuity equation has been used to arrive at Eq. (7). Equations (1)–(7) are the governing equations of our model. Note that the governing equations have been derived by neglecting both the nonlinear terms and the ones that are of order higher than $\omega/\Omega_{ci} \ll 1$.[2,6–10]

A. Linear analysis

Assuming sinusoidal perturbation and using Eqs. (1)–(7), we obtain the following generalized linear dispersion relation for the coupled Kinetic Alfvén-acoustic waves:[2]

$$\left(1 - \frac{v_A^2 k_z^2}{\omega^2}\right)\left(1 - \frac{c_s^2 k_z^2}{\omega^2}\right) = \frac{v_A^2 k_z^2}{\omega^2}\lambda_s. \tag{8}$$

Here, $\lambda_s = k_x^2\rho_s^2 = k_x^2\frac{c_s^2}{\Omega_i^2}$ is the coupling parameter and

$$k_z = k\cos\theta, \qquad k_x = k\sin\theta.$$

The above relation shows the coupling of the Alfvén and the ion-acoustic waves with the coupling parameter λ_s. It should be noted here that the coupling between the two waves occur owing to the inclusion of the parallel motion of ions. If we set the ion-acoustic factor (2nd factor) on the left hand side of dispersion relation to unity we get the linear dispersion relation of KAW[2,6,8]

$$\omega^2 = v_A^2 k_z^2(1 + \lambda_s). \tag{9}$$

It should be noted here that the above wave reduces to the shear Alfvén wave for $\theta = 0$ whereas it disappears at $\theta = \pi/2$. It is therefore reasonable to conclude that the present investigation holds good for small angles of propagation.

By introducing normalized quantities in the following manner:

$$\beta = \frac{2c_s^2}{v_A^2} \qquad \lambda_s = \frac{c_s^2}{\Omega_i^2}k_x^2 = K_x^2 \qquad M_A = \frac{\omega}{k_z v_A},$$

where β is the ratio of thermal pressure to the magnetic pressure, M_A is the Alfvén Mach number which is the ratio of wave velocity to the Alfvén velocity, the linear dispersion relation for coupled KAW-acoustic wave in dimensionless form can be expressed as

$$M_A^2 = \frac{\left(\frac{\beta}{2}+1+K_x^2\right) \pm \left(\left(\frac{\beta}{2}+1+K_x^2\right)^2 - 2\beta\right)^{\frac{1}{2}}}{2}. \tag{10}$$

The above dispersion relation describes two modes. The positive (upper) sign represents the super Alfvénic, whereas the negative (lower) sign represents the sub Alfvénic mode.

III. FINITE AMPLITUDE SOLITARY STRUCTURES

In this section, we proceed to derive the Sagdeev potential and investigate the conditions necessary for the formation of solitary structures. In general, we follow the method and nomenclature developed in Refs. 9 and 10, but develop the Sagdeev potential when electrons are adiabatically trapped.

We now consider a wave solution which is obliquely propagating to the ambient magnetic field. Shifting to a co-moving frame of reference in normalized variables

$$\eta = K_x x + K_z z - Mt, \tag{11}$$

where normalizing has been done in the following manner:

$$n = \frac{n_e}{n_0}, \quad \psi = \frac{\psi}{T_e}, \quad M = \frac{v}{c_s}, \quad K = K\,\frac{c_s}{\Omega_i}, \quad t = \Omega_i\, t\,.$$

We can write the governing equations, i.e., Eqs. (3)–(7) as follows:

$$n = 1 + \psi + \alpha_1 \psi^{\frac{3}{2}}, \tag{12}$$

where $\alpha_1 = -\frac{4}{3\sqrt{\pi}}$

$$-M\frac{\partial v_z}{\partial \eta} + v_x K_x \frac{\partial v_z}{\partial \eta} + v_z K_z \frac{\partial v_z}{\partial \eta} = -K_z \frac{\partial \psi}{\partial \eta}, \tag{13}$$

$$v_{ix} = K_x M \frac{\partial^2 \varphi}{\partial \eta^2}, \tag{14}$$

$$2{K_x}^2{K_z}^2\partial_\eta^4(\varphi - \psi) = \beta(M^2{\partial_\eta}^2 n - MK_z{\partial_\eta}^2(nv_z)), \tag{15}$$

$$-M\partial_\eta n + K_x \partial_\eta(nv_x) + K_z\partial_\eta(nv_z) = 0. \tag{16}$$

By integrating Eqs. (13) and (14) and applying the boundary conditions,

$$\partial_\eta n = 0\,, \quad v_x,\, v_z,\, \phi,\, \psi \to 0 \;\text{ and }\; n = 1 \;\text{ at }\; \eta \to \infty.$$

We get

$$2{K_x}^2{K_z}^2\partial_\eta^2(\varphi - \psi) = \beta\left(M^2(n-1) - {K_z}^2 n(n-1)\right), \tag{17}$$

$$K_x v_x + K_z v_z = M\left(1 - \frac{1}{n}\right). \tag{18}$$

By putting the value of $K_x v_x$ from Eq. (16) in Eq. (11) and putting $n = 1 + \alpha\psi + \alpha_1\psi^{\frac{3}{2}}$ in the relation and integrating, we get

$$Mv_z = K_z\left(\psi + \frac{\alpha}{2}\psi^2 + \frac{2}{5}\alpha_1\psi^{\frac{5}{2}}\right). \tag{19}$$

Solving the above set of equations simultaneously, we get

$$\begin{aligned}
&-\frac{\beta {K_z}^2}{{M_A}^2}\left(\psi + \frac{1}{2}\psi^2 + \frac{2}{5}\alpha_1\psi^{\frac{5}{2}}\right) + 2\left(\frac{\psi + \alpha_1\psi^{\frac{3}{2}}}{1 + \psi + \alpha_1\psi^{\frac{3}{2}}}\right) \\
&\quad - 2{K_x}^2\partial_\eta(\partial_\eta\psi) = 2\frac{{M_A}^2}{{K_z}^2}\left(\psi + \alpha_1\psi^{\frac{3}{2}}\right) \\
&\quad - \beta\left(\psi + \frac{3}{2}\psi^2 + \frac{7}{5}\alpha_1\psi^{\frac{5}{2}}\right).
\end{aligned} \tag{20}$$

Here, we have used $M^2 = 2M_A^2/\beta$. By neglecting the higher order terms and keeping terms up to the order of $\psi^{\frac{3}{2}}$, we get

$$\begin{aligned}
&-\frac{\beta {K_z}^2}{{M_A}^2}\psi + 2\left(\psi + \alpha_1\psi^{\frac{3}{2}}\right) - 2\frac{{M_A}^2}{{K_z}^2}\left(\psi + \alpha_1\psi^{\frac{3}{2}}\right) + \beta\psi \\
&\quad = 2{K_x}^2\partial_\eta(\partial_\eta\psi), \\
&\left(-\frac{\beta {K_z}^2}{2{M_A}^2} + 1 - \frac{{M_A}^2}{{K_z}^2} + \frac{\beta}{2}\right)\partial_\eta\psi^2 + \left(\frac{4}{5}\alpha_1 - \frac{4}{5}\alpha_1\frac{{M_A}^2}{{K_z}^2}\right)\partial_\eta\psi^{\frac{5}{2}} \\
&\quad = {K_x}^2\partial_\eta(\partial_\eta\psi)^2.
\end{aligned} \tag{21}$$

After integrating, the above equation and using the boundary conditions, we get

$$\frac{1}{2}(\partial_\eta\psi)^2 + V(\psi) = 0, \tag{22}$$

where

$$V(\psi) = -\frac{1}{{K_x}^2}\left(A\,\psi^2 + B\,\psi^{\frac{5}{2}}\right). \tag{23}$$

Here

$$A = -\frac{\beta {K_z}^2}{4{M_A}^2} + \frac{1}{2} - \frac{{M_A}^2}{{K_z}^2} + \frac{\beta}{4},$$

$$B = \frac{2}{5}\alpha_1 - \frac{2}{5}\alpha_1\frac{{M_A}^2}{{K_z}^2}.$$

Equation (22) is in the form of energy integral and describes the motion of a pseudoparticle in a pseudopotential well $V(\psi)$ which is known as the Sagdeev potential.[14] Note that the coefficients of Sagdeev potential for the trapped electrons case is different from the one studied by Yu and Shukla[9] for the Boltzmannian electrons. It is pertinent to mention here that in order to form the solitary wave structure, the existence condition demands that A be positive which in turn gives the following condition on the parallel wave number:

$$K_z \geq \left[\frac{{M_A}^2}{\left(\frac{1}{2}+\frac{\beta}{4}\right) + \left(\left(\frac{1}{2}+\frac{\beta}{4}\right)^2 - \frac{\beta}{2}\right)^{\frac{1}{2}}}\right]^{\frac{1}{2}}. \tag{24}$$

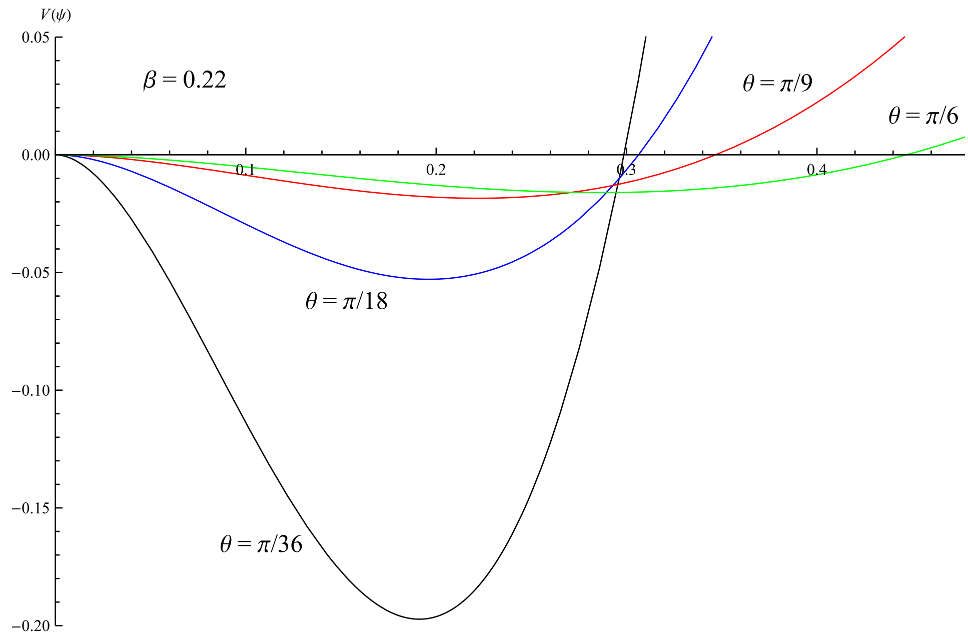

FIG. 1. Variation of Sagdeev potential $V(\psi)$ vs ψ for different values of angle of propagation. The value of plasma beta is assumed to be 0.2 in this case.

IV. DISCUSSION OF RESULTS

In this section, we numerically investigate the dependence of the Sagdeev potential $V(\psi)$ vs ψ for different plasma parameters. Fig. 1 shows the variation of Sagdeev potential $V(\psi)$ (Eq. (23)) vs potential ψ as a function of angle θ keeping the other plasma parameters fixed. It has been found that the depth of the Sagdeev potential decreases with the increasing angle of propagation θ but the amplitude of the soliton increases with the increase in θ. Fig. 2 shows the corresponding amplitude and width of the soliton vs the co-moving coordinate η and it is seen that the increasing angle of propagation enhances both the width and amplitude of the soliton. It is in the fitness of the situation to mention here that such solitons are formed only for the sub Alfvénic mode (i.e., lower sign of the dispersion relation given by Eq. (9)) as no solitary structure is formed for the super Alfvénic mode (i.e., upper sign of the dispersion relation given by Eq. (9)).

Fig. 3 shows the variation of Sagdeev potential $V(\psi)$ vs the potential ψ as a function of plasma β for a fixed value of propagation angle θ, i.e., $\theta = \pi/18$. It has been found that the depth of the Sagdeev potential enhances by increasing the plasma β and the amplitude of the soliton exhibits the similar behavior. Fig. 4 shows the corresponding amplitude and

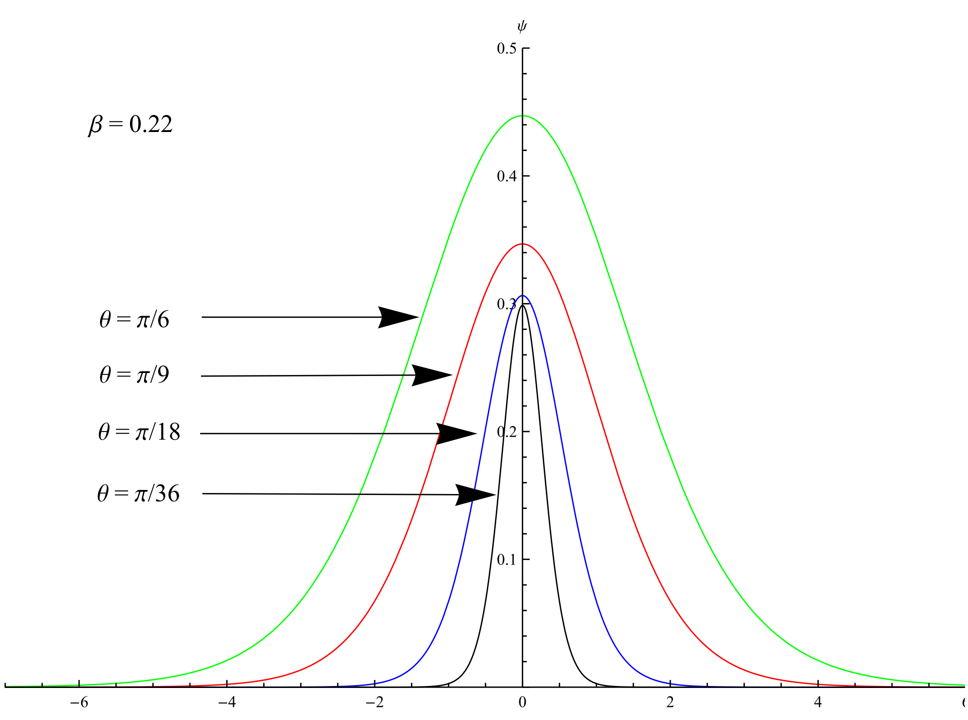

FIG. 2. Solitary wave amplitude ψ vs the co-moving coordinate η corresponding to the Sagdeev potential shown in Fig. 1.

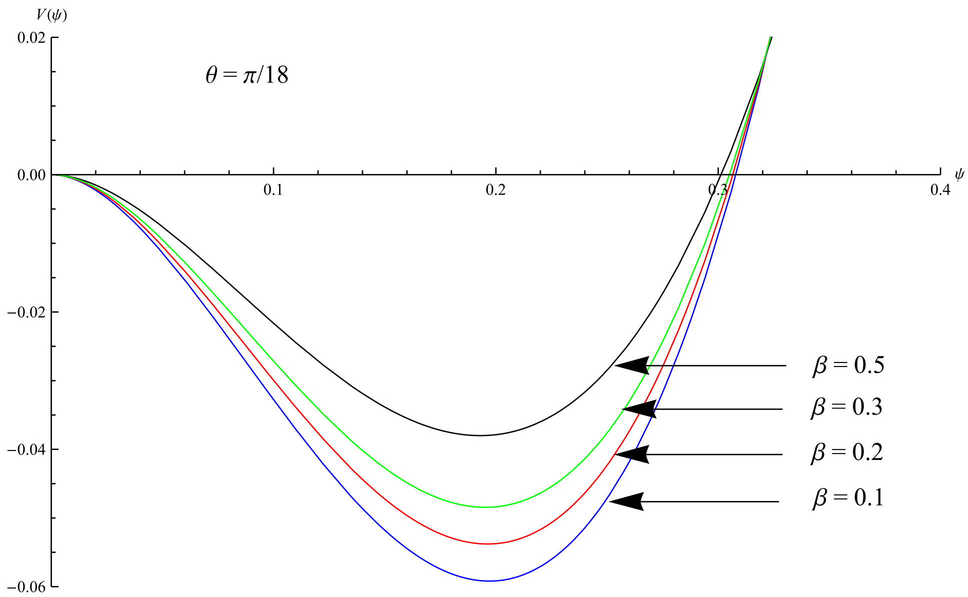

FIG. 3. Variation of Sagdeev potential $V(\psi)$ vs ψ for different values of plasma beta. The value of the angle of propagation is taken to be $\pi/18$ here.

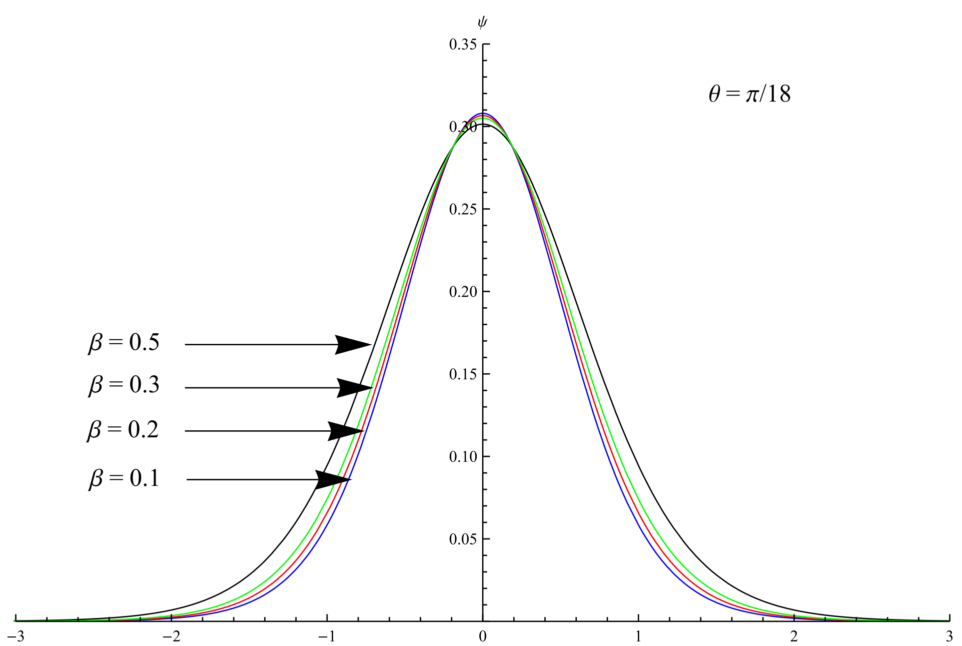

FIG. 4. Solitary wave amplitude ψ vs the co-moving coordinate η corresponding to the Sagdeev potential shown in Fig. 3.

width of the soliton vs the co-moving coordinate η and it is seen that the increasing plasma β enhances the amplitude but mitigates the width of the soliton.

V. CONCLUSION

In this paper, we have investigated the adiabatic trapping in coupled kinetic Alfvén-acoustic waves in low beta plasmas. Using the Sagdeev potential approach, we have studied the existence condition for the formation of the arbitrary amplitude solitary wave and have found that the solitons are formed for the sub Alfvénic mode, whereas no soliton is formed for the super Alfvénic mode. We have also explored the effects of the variation of the obliqueness angle θ and plasma β on the propagation characteristics of the nonlinear solitary structure. The present investigation may be beneficial in understanding the propagation of solitary structures in different space environments where trapped populations of electrons have been observed.

[1] A. Hasegawa and C. Uberoi, *The Alfven Wave* (Technical Information Centre, U.S. Department of Energy, Washington, DC, 1982).
[2] N. F. Cramer, *The Physics of Alfven Waves* (Wiley-VCH, Berlin, 2001).
[3] C. E. J. Watt and R. Rankin, Phys. Rev. Lett. **102**, 045002 (2009).
[4] N. Rubab, N. V. Erkaev, D. Langmayr, and H. K. Biernat, Phys. Plasmas **17**, 103704 (2010).
[5] Y. Lin, J. R. Johnson, and X. Wang, Phys. Rev. Lett. **109**, 125003 (2012).
[6] B. B. Kadomtsev, *Plasma Turbulence* (Academic, New York, 1965), p. 82.
[7] A. Hasegawa and L. Chen, Phys. Rev. Lett. **35**, 370 (1975).
[8] A. Hasegawa and K. Mima, Phys. Rev. Lett. **37**, 690 (1976).
[9] M. Y. Yu and P. K. Shukla, Phys. Fluids **21**, 1457 (1978).
[10] H. Kakati and K. S. Goswami, Phys. Plasmas **5**, 4229 (1998).
[11] C. Yinhua, L. Wei, and M. Y. Yu, Phys. Rev. E **61**, 809 (2000).
[12] L. D. Landau and E. M. Lifshitz, *Physical Kinetics* (Pergamon, New York, 1981).
[13] N. S. Erokhin, N. N. Zolnikova, and I. A. Mikhailovskaya, Fiz. Plazmy **22**, 137 (1996).

[14]R. Z. Sagdeev, *Reviews of Plasmas Physics* (Consultants Bureau, New York, 1996), Vol. 4.
[15]H. Abbasi, N. L. Tsintsadze, and D. D. Tshakaya, Phys. Plasmas **6**, 2373 (1999).
[16]A. Mushtaq and H. A. Shah, Phys. Plasmas **13**, 012303 (2006).
[17]H. Siddiqui, H. A. Shah, and N. L. Tsintsadze, J. Fusion Energy **27**, 216 (2008).
[18]H. A. Shah, M. N. S. Qureshi, and N. L. Tsintsadze, Phys. Plasmas **17**, 032312 (2010).
[19]H. A. Shah, W. Masood, M. N. S. Qureshi, and N. L. Tsintsadze, Phys. Plasmas **18**, 102306 (2011).
[20]M. Ayub, H. A. Shah, and M. N. S. Qureshi, Phys. Scr. **84**, 045505 (2011).