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ORIGINAL ARTICLE

Drift solitary structures in inhomogeneous degenerate quantum plasmas with trapped electrons

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Abstract In the present work, we have considered the nonlinear effects due to trapped electrons in an inhomogeneous degenerate quantum plasma. The formation of drift solitary structures has been investigated for both fully and partially degenerate plasmas. The Sagdeev potential approach has been employed to obtain arbitrary amplitude solitary structures. Interestingly, for a fixed value of density, not only compressive but rarefactive solitary structures have been obtained for a certain temperature range. Furthermore, it has been observed that the drift solitary structures exist only for the case when the drift velocity is smaller than the velocity of the nonlinear structure. The theoretical results obtained have been analyzed numerically for the parameters typically found in white dwarfs and the relevance of the results with regard to white dwarf asteroseismology is also pointed out.

Keywords Drift wave · Solitary structure · Trapped electron

1 Introduction

Degenerate plasmas are of special relevance in the investigation of dense astrophysical plasmas (Shukla and Ali 2004; Reach et al. 2004) such as those found in white dwarfs, neutron stars and giant planets. Degenerate plasmas have also relevance in the laboratory high intensity laser-produced plasma (Marklund and Shukla 2006) as well as in microelectronics (Markowich et al. 1990). In the last few years, a substantial volume of literature has been produced which has addressed different issues in degenerate plasmas. Most of the investigations have made use of the quantum hydrodynamic (QHD) model, which is based on the Schrodinger-Poisson formulation, and is a generalization of the classical fluid model of plasmas (Manfredi 2004). Although this model has the expected drawback that it fails to explain kinetic effects like Landau damping that are driven by resonant wave-particle interaction, nevertheless it has been extensively applied to investigate the linear and nonlinear phenomenon of different wave modes in degenerate quantum plasmas (Haas et al. 2003; Mamun and Shukla 2011; Ali et al. 2007; Mamun et al. 2011).

Many investigations have also been carried out to study the linear and nonlinear wave propagation in inhomogeneous degenerate quantum plasmas using the QHD model. Kendl and Shukla (2011) studied drift wave turbulence for a degenerate inhomogeneous magnetoplasma and investigated the growth rate of the collisional drift wave instability. It was observed that the quantum effects enhanced the growth rate of the collisional drift wave instability. Masood et al. (2009a, 2009b) studied nonlinear drift ion acoustic waves in inhomogeneous quantum magnetoplasma and showed the variation of shock strength with quantum diffraction term and positron concentration. Naeem et al. (2011) investigated low frequency wave propagation in inhomogeneous degenerate plasma and derived a modified dispersion relation for magnetic electron-drift vortex mode. Investigations on dust acoustic vortices involving quantum hydrodynamic model showed that the dust drift dissipative

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instability could grow if the parallel velocity shear is negative in an inhomogeneous quantum magnetoplasma (Masood et al. 2008).

Gurevich (1967) for the first time used Vlasov's equation along with Maxwell's equations to study adiabatic trapping of electrons at a microscopic level as a nonlinear phenomenon. In this paper, it was shown that $3/2$ order nonlinearity results due to adiabatic trapping instead of the usual quadratic nonlinearity. Nonlinear dynamics of ion cyclotron waves was investigated by Abbasi et al. (1999) and it was reported that the conditions for the existence of these waves change in the presence of the trapped particle distribution. Moreover, it was shown that the characteristic length of the modulation of the waves also gets modified by the trapping of particles. The effect of trapping in partially and fully degenerate plasmas was investigated by Shah et al. (2010) and a new type of nonlinearity was found that caused the formation of both compressive and rarefactive solitary structures under certain conditions. Recently, in degenerate quantum self-gravitating dusty plasmas, Ayub et al. (2011) used Sagdeev potential approach and investigated the solitary structures in the presence of trapped electrons and found that the features of the solitary wave structures are affected by the variation in Mach number and ion temperature. Shah et al. (2011) employed relativistic Fermi-Dirac distribution for trapped electrons and studied soliton formation for arbitrary amplitude perturbations in case of relativistic degenerate quantum plasmas and pointed out the relevance of their study in strong laser plasma interactions and in understanding astrophysical observations of dense objects such as white dwarfs.

In this paper, for the first time, we investigate the nonlinear drift waves by taking trapping of electrons in inhomogeneous degenerate quantum plasmas. In Sect. 2, we present the basic set of nonlinear equations of the system under consideration. Linear dispersion relation of the quantum drift wave in inhomogeneous quantum plasma is presented in Sect. 3. In Sect. 4, Sagdeev potential for arbitrary amplitude perturbations is derived. In Sect. 5, results are presented and discussed. Finally in Sect. 6, the main findings of the paper are recapitulated.

2 Governing equations of our model

We consider electrostatic wave propagation in inhomogeneous quantum plasma in the presence of a uniform magnetic field taken along the z-direction, i.e. $B_0 = B_0\hat{z}$. The plasma is supposed to be consisting of electrons and ions. The background number density is inhomogeneous and is assumed to be in the negative x-direction. We consider degenerate electrons which are adiabatically captured (Landau and Lifshitz 1981) in a potential well that vanishes at infinity. If potential energy of the well is $u = -e\varphi$, then the energy of the electrons is given by $\varepsilon = (p^2/2m) + u$, where p and m and are the momentum and mass of electrons respectively. The normalized occupation number for the Fermi-Dirac distribution after integration over spherical polar coordinates and change of variables from momentum p to energy ε is given by (Landau and Lifshitz 1980)

$$N = \frac{\sqrt{2}Vm^{3/2}}{\pi^2\hbar^3}\int_0^\infty \frac{\sqrt{\varepsilon}d\varepsilon}{e^{[\varepsilon-(\mu+e\varphi)]/T}+1} \tag{1}$$

where $\varphi(r,t)$ is the self-consistent potential field which traps the electrons and μ is the chemical potential. Electrons with $\varepsilon < 0$ and $\varepsilon > 0$ are the trapped and free electrons respectively in a single potential well which vanishes at infinity. Trapping occurs when the momentum $p = p_1 = (2m|u|)^{1/2}$, where the condition $\varepsilon = 0$ is fulfilled and separatrix motion occurs. Setting $U = e\varphi + \mu$ and using the trapping condition $\varepsilon - U = 0$, and making a change of variables $z = \varepsilon - U/T$, we obtain the following expression (following Landau and Lifshitz 1980).

$$\begin{aligned} n(r,t) = \frac{8\sqrt{2}\pi m^{3/2}T}{(2\pi\hbar)^3}\Bigg[&\int_0^{U/T}(U-Tz)^{1/2}\,dz \\ &+\int_0^\infty \frac{(U+Tz)^{1/2}-(U-Tz)^{1/2}}{e^z+1}\,dz\Bigg] \end{aligned} \tag{2}$$

The first integral in Eq. (2) is evaluated in a straightforward manner and represents the effect of trapped particles; however, the second integral is evaluated only approximately by taking the small temperature limit and expanding the functions in the numerator. We note here that the upper limit of the second integral is obtained as u/T and following Landau and Lifshitz (1980) in the small temperature limit $u/T \to \infty$. Thus upon integration, we obtain for the normalized number density $n_e(r,t) = n_e/n_{e0}$.

Following the procedure outlined in Shah et al. (2010), we arrive at the following expression of the total normalized number density for the free as well as the trapped particles

$$n_e(r,t) = (1+\Phi)^{\frac{3}{2}} + T^2(1+\Phi)^{-\frac{1}{2}} \tag{3}$$

where $n_e(r,t)$ is the normalized total density defined as $n_e(r,t) = n_e/n_{oe}(x)$, Φ is the normalized electrostatic potential, and T is the normalized electron temperature and normalized as $\Phi = e\phi/\epsilon_F(x)$ and $T(x) = \pi T/2\sqrt{3}\epsilon_F(x)$ and $\varepsilon_F(x) = P_F^2/2m = \hbar^2/2m(3\pi^2)^{2/3}n_0^{2/3}(x)$ is the Fermi energy, and p, m and e are the momentum, mass, and charge of electrons respectively. For fully degenerate plasma, the background number density is $n_{oe}(x)$ and if plasma is partially degenerate then the background number density will be $n_{oe}(x) = n_o(x)(1+T^2)$. The first term in Eq. (3) with $3/2$ order nonlinearity represents the effect of trapping and the second term is obtained by taking into account the small temperature effects for partially degenerate plasma.

In order to develop nonlinear equation for arbitrary amplitude perturbations, the normalized perturbed number density for trapped electrons is given as

$$\frac{\tilde{n_e}}{n_o(x)} = (1+\Phi)^{\frac{3}{2}} + T^2(1+\Phi)^{-\frac{1}{2}} - \left(1+T^2\right) \tag{4}$$

The Poisson's equation in normalized form is

$$\nabla^2(1+\Phi) = \frac{1}{n_o\lambda_{Fe}^2}(n_e - n_i) \tag{5}$$

where $\lambda_{Fe}(x) = \sqrt{\frac{\epsilon_F}{4\pi n_o e^2}}$ is the electron Fermi wavelength. Using Eq. (5), we obtain the following expression for the perturbed number density

$$\frac{\tilde{n_i}}{n_o(x)} = (1+\Phi)^{\frac{3}{2}} + T^2(1+\Phi)^{-\frac{1}{2}} - \lambda_{Fe}^2\nabla^2(1+\Phi) - \left(1+T^2\right) \tag{6}$$

The ions due to the largeness of their mass are treated classically and their equation of motion is given by

$$m_i n_i(\partial_t + v_i\cdot\nabla)v_i = en_i\left(E + \frac{1}{c}v_i \times B_o\right) \tag{7}$$

where $E = -\nabla\phi$ is the electrostatic field and n_i, m_i and e are the density, mass and charge of the ions respectively. Equation (7) is valid as long as the wave period is much larger than the time scale for the ion correlations as well as the work frequency is much larger than the damping rate caused by the ion fluid shear viscosity (Shukla and Eliasson 2011; Mithen et al. 2011; Eliasson and Shukla 2011). The perpendicular component of velocity from Eq. (7) can be written as (Masood et al. 2009a)

$$v_{i\perp} = \frac{c_s^2}{\Omega_{ci}}\left(\hat{z}\times\nabla(1+\Phi)\right) - \rho_i^2\partial_t\nabla_\perp(1+\Phi) \tag{8}$$

where $\Omega_{ci} = eB_o/cm_i$ is the ion cyclotron frequency, $c_s(x) = \sqrt{\epsilon_F/m_i}$ is the quantum ion acoustic speed, and $\rho_i(x) = c_s/\Omega_{ci}$ is the ion Larmor radius at electron temperature $T_F(x) = \hbar^2/2mk_B(3\pi^2)^{2/3}n_o^{2/3}(x)$ in the degenerate plasma. Here $(\perp)$ means perpendicular to B_o.

The ion continuity equation reads as

$$\partial_t\left(\frac{\tilde{n_i}}{n_o}\right) + \frac{1}{n_o}\nabla\cdot(n_i v_i) = 0 \tag{9}$$

Using Eq. (6) and Eq. (8) in Eq. (9), we get

$$\begin{aligned}&\partial_t(1+\Phi)^{\frac{3}{2}} + T^2\partial_t(1+\Phi)^{-\frac{1}{2}} - \lambda_{Fe}^2\partial_t\partial_y^2(1+\Phi)\\ &+\frac{3}{2}v_*\partial_y(1+\Phi) + \frac{3}{5}v_*\partial_y(1+\Phi)^{\frac{5}{2}}\\ &- v_*\partial_y(1+\Phi)^{\frac{3}{2}} - T^2v_*\partial_y(1+\Phi)^{\frac{1}{2}}\\ &- T^2v_*\partial_y(1+\Phi)^{-\frac{1}{2}} - \rho_i^2\partial_t\partial_y^2(1+\Phi) = 0\end{aligned} \tag{10}$$

where $v_* = 2c\epsilon_F\kappa_{ni}/3eB_o$ is the diamagnetic drift velocity in degenerate quantum plasma and $\kappa_{ni} = -1/n_{io}(\partial n_{io}/\partial x)$ is the inverse scale length of the density inhomogeneity.

3 Linear analysis

The dispersion relation of partially degenerate quantum drift waves can be found by linearizing Eq. (10) and assuming sinusoidal perturbation, we get

$$\omega = \frac{\frac{3}{2}\omega_*}{\frac{3}{2} - \frac{1}{2}T^2 + (\lambda_{Fe}^2 + \rho_i^2)k_y^2} \tag{11}$$

where $\omega_* = v_*k_y$ is the drift frequency, ω is the wave frequency, and k_y is the wave number. Here $3/2$ term appears due to the trapping effect and $T^2/2$ term due to small temperature effects. The term $\lambda_{Fe}^2k_y^2$ term appears because of charge separation, and $\rho_i^2k_y^2$ term arises when finite Larmor radius effect is taken into account. If we ignore the trapping and temperature effects then we are left with ordinary dispersion relation for drift waves. Here we see that the linear dispersion relation is modified due to trapping and finite but small temperature.

4 Nonlinear analysis

In order to find localized solution, we choose a coordinate ξ in a moving frame of reference such that $\xi = y - ut$, where "u" is an arbitrary velocity of the nonlinear structure moving with the frame. Equation (10) in the transformed frame can be written as

$$\begin{aligned}&d_\xi(1+\Phi) - Ad_\xi(1+\Phi)^{\frac{3}{2}} + \frac{2}{5}d_\xi(1+\Phi)^{\frac{5}{2}} + Bd_\xi^3(1+\Phi)\\ &- AT^2d_\xi(1+\Phi)^{-\frac{1}{2}} - \frac{2}{3}T^2d_\xi(1+\Phi)^{\frac{1}{2}} = 0\end{aligned} \tag{12}$$

where

$$A = \frac{(1+\frac{v_*}{u})}{3v_*/2u}, \qquad B = \frac{(\lambda_{Fe}^2 + \rho_i^2)}{3v_*/2u}$$

In order to find an expression of Sagdeev potential in the inhomogeneous quantum magnetoplasma, we integrate Eq. (11) and apply the boundary conditions that when $\xi \to \infty$, $\Phi \to 0$, we get

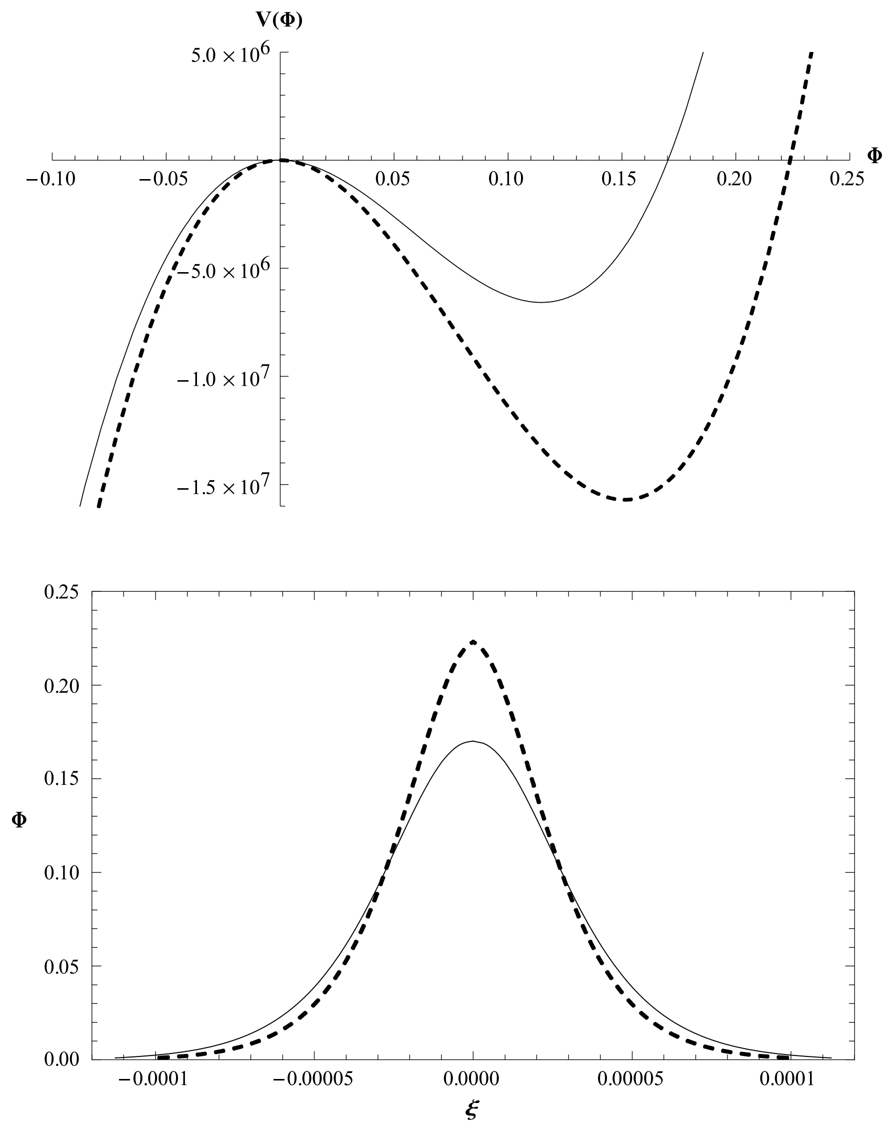

Fig. 1 Variation in Sagdeev potential $V(\Phi)$ with increasing magnetic field and the corresponding solitary structures. *Dotted lines* for $B_0 = 1.01 \times 10^9$ Gauss and *solid lines* for $B_0 = 1 \times 10^9$ Gauss. Other parameters are $n = 2.5 \times 10^{26}\ \mathrm{cm}^{-3}$ and $\kappa_n = -0.1\ \mathrm{cm}^{-1}$

$$d_\xi^2 \Phi = \frac{A}{B}(1+\Phi)^{\frac{3}{2}} - \frac{2}{5B}(1+\Phi)^{\frac{5}{2}} - \frac{1}{B}(1+\Phi) + \frac{AT^2}{B}(1+\Phi)^{-\frac{1}{2}} - \frac{2T^2}{3B}(1+\Phi)^{\frac{1}{2}} - \frac{1}{B}\left[A\left(1+T^2\right) - \frac{7}{5} + \frac{2T^2}{3}\right] \tag{13}$$

This equation enables us to investigate solitary structures in a co-moving frame of reference. In order to investigate solitary structure, we express Eq. (13) in the form of an energy integral in the following manner:

$$\frac{1}{2}(d_\xi \Phi)^2 + V(\Phi) = 0 \tag{14}$$

where $V(\Phi)$ is the Sagdeev Potential (Sagdeev 1996) which in our case is

$$V(\Phi) = \frac{1}{2B}(1+\Phi)^2 - \frac{2A}{5B}(1+\Phi)^{\frac{5}{2}} + \frac{4}{35B}(1+\Phi)^{\frac{7}{2}} - \frac{2AT^2}{B}(1+\Phi)^{\frac{1}{2}} - \frac{4T^2}{9B}(1+\Phi)^{\frac{3}{2}} + \frac{1}{B}\left[A\left(1+T^2\right) - \frac{7}{5} + \frac{2T^2}{3}\right](1+\Phi) + \frac{1}{B}\left(\frac{11}{14} - \frac{3}{5}A + AT^2 - \frac{2}{9}T^2\right) \tag{15}$$

Here the constant of integration is evaluated using boundary condition given earlier. Equation (15) is the expression of the Sagdeev potential for arbitrary amplitude solitons in a partially degenerate quantum magnetoplasma. It is evident from Eq. (15) that $V(\Phi) = \frac{dV(\Phi)}{d\Phi} = 0$, when $\Phi = 0$.

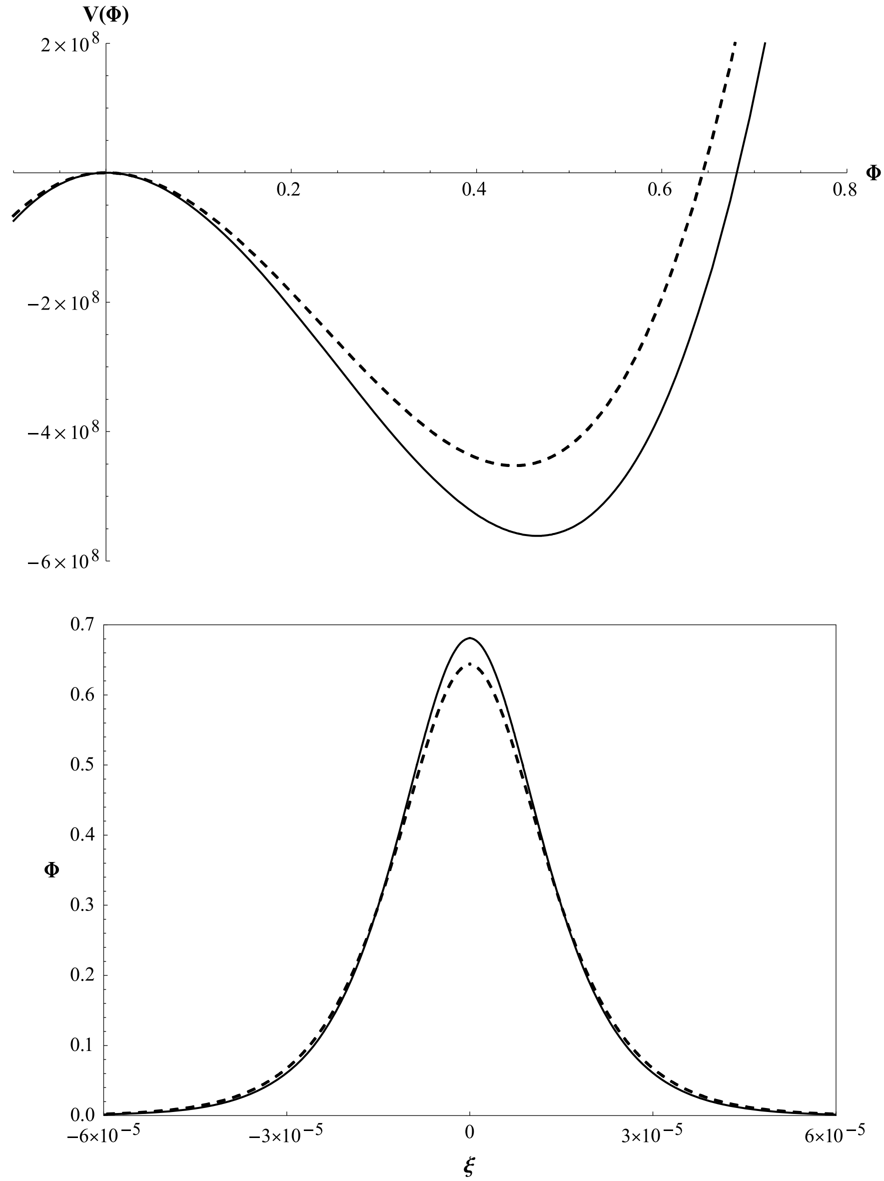

Fig. 2 Variation of Sagdeev potential $V(\Phi)$ for fully and partially degenerate quantum plasma and the corresponding solitary structures. *Dotted lines* are for $T = 0.2$ and *solid lines* are for $T = 0$. Other parameters are $B_0 = 1 \times 10^9$ Gauss, $n = 2.15 \times 10^{26}$ cm^{-3} and $\kappa_n = -0.1$ cm^{-1}

5 Results and discussion

In this section, we numerically investigate the dependence of Sagdeev potential of the quantum drift wave on the ambient magnetic field, temperature and ratio of drift to soliton velocity in co-moving frame. In highly dense plasmas like in dense astrophysical objects such as neutron stars and white dwarfs, the plasma densities are much higher and quantum effects can be ignored. It is pertinent to mention here that so far there are about two hundred observations of pulsating white dwarf stars. The pulsation period typically falls in the range from 2 to 35 minutes and can be attributed to non-radial gravity (g-mode) oscillation modes. The observations and theory of these pulsations is now well established, and the discipline of white-dwarf asteroseismology is used to study their rotation period, mass, equation of state, etc. (Winget and Kepler 2008; Fontaine and Brassard 2008). Moreover, the theory predicts the existence of acoustic modes (p-modes) where the ions provide the inertia and mainly the electron degeneracy pressure provides the restoring force. Typical oscillation periods of globally propagating p-modes are set by the time for the wave to travel across the star and lies in the range of a few seconds, two orders of magnitude shorter than g-mode oscillations. These modes were early predicted (Ostriker 1971), but are yet to be observed (Silvotti et al. 2011). However, the lack of observation does not imply that these modes are not excited. Recently, it has been suggested that large amplitude electrostatic structures could be excited in extreme events, such as supernova explosions at the outer

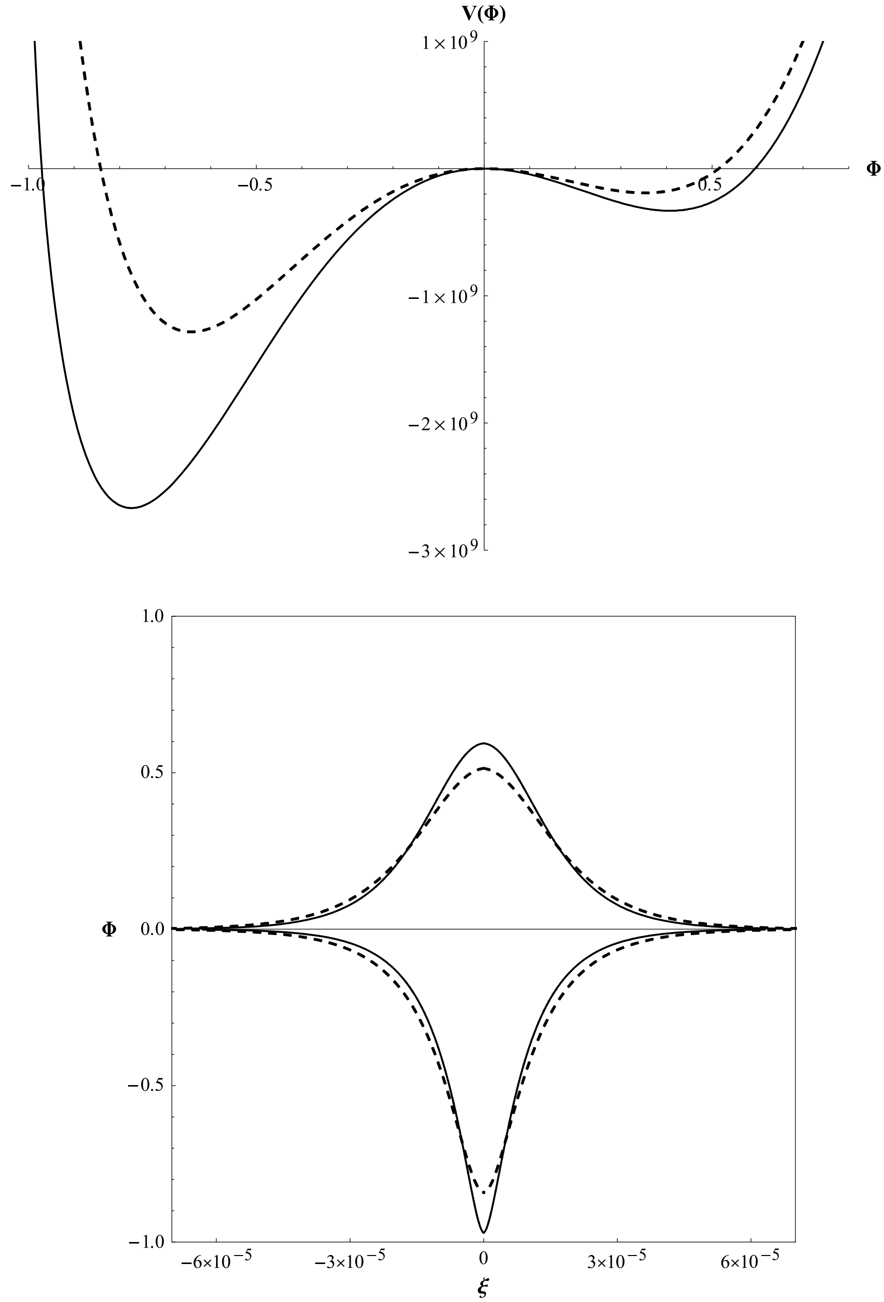

Fig. 3 Variation in Sagdeev potential $V(\Phi)$ with increasing temperature and the corresponding solitary structures. *Dotted lines* are for $T = 0.4$ and *solid lines* are for $T = 0.3$. Other parameters are $B_0 = 1 \times 10^9$ Gauss, $n = 2.15 \times 10^{26}$ cm^{-3} and $\kappa_n = -0.1$ cm^{-1}

shells of the star or during collisions of the white dwarf with other astrophysical bodies (Eliasson and Shukla 2012). We, therefore, choose here the parameters that are typically found in the white dwarfs, i.e. $n_0 \sim 10^{26}$–10^{28} cm^{-3} and $B_0 \sim 10^9$–10^{11} G (Koester and Chanmugam 1990; Masood et al. 2011; Shah et al. 2012). Graphical analysis of frequency of drift wave given by Eq. (10) is presented by plotting the wave number k_y and frequency ω. Figure 1 shows the effect of increasing magnetic field on the frequency of the drift wave. It is found that an increase in the magnetic field increases the frequency of drift wave. It is due to the fact that by increasing magnetic field the ion Larmor radius ρ_i in the denominator of dispersion relation decreases more rapidly than the drift velocity v_* in the numerator.

In Figs. 1–4, we plot the Sagdeev potential $V(\Phi)$ against the normalized electrostatic potential Φ. The depth of Sagdeev potential found to increase with the increasing magnetic field. This is due to the reason that an increase in magnetic field increases the coefficient of nonlinearity and decreases the coefficient of dispersion as shown Fig. 1.

In our investigation on partially degenerate quantum plasma, we find that (within the range of quantum regime

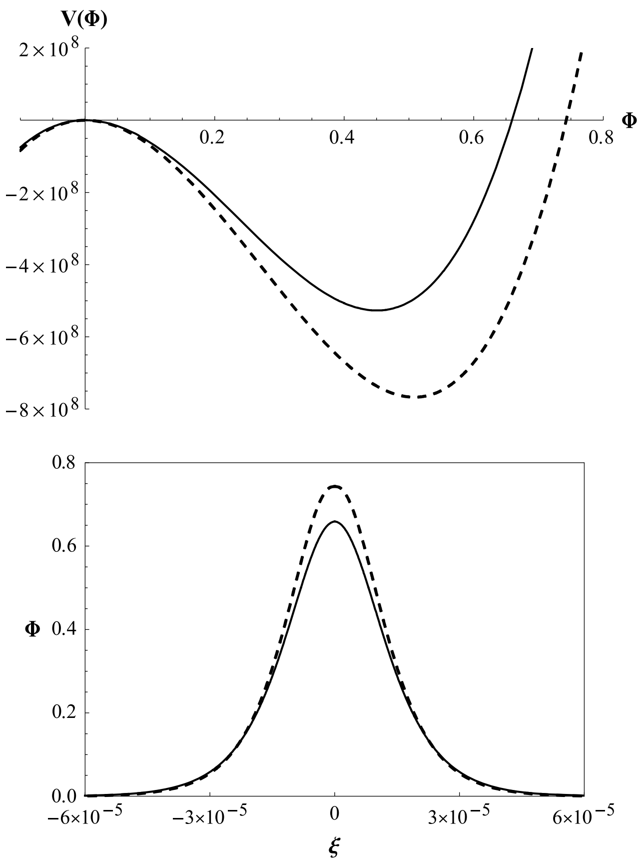

Fig. 4 Variation in Sagdeev potential $V(\Phi)$ with decreasing the ratio v_*/u and the corresponding solitary structures. *Dotted lines* are for $v_*/u = 0.86$ and *solid lines* are for $v_*/u = 0.88$. Other parameters are $B_0 = 1 \times 10^9$ Gauss, $n = 2 \times 10^{26}$ cm^{-3} and $\kappa_n = -0.1$ cm^{-1}

i.e. $T < 1$) by increasing the temperature T, for a fixed number density, the depth of Sagdeev potential decreases. This is due to the fact that the nonlinear terms appearing due to the small temperature effects becomes more significant. Interestingly in our results, for a fixed number density, we get a compressive solitary structure when $T < 0.3$ and therefore for fully degenerate quantum plasma we get only compressive solitary structure (Fig. 2). However we get a rarefactive solitary wave along with a compressive solitary wave (Witt and Lotko 1983; Mamun 1997) for $0.3 < T < 0.6$. For $T > 0.6$, no solitary structure is obtained. Note that a compressive solitary structure is obtained for $\Phi > 0$ and for negative values of Φ, we get a rarefactive one.

In Fig. 3, it is found that compressive and rarefactive solitary waves are not symmetrical. The depth of rarefactive part is greater in comparison to the depth of compressive part. Finally, Fig. 4 explores how the ratio of drift to soliton velocity in co-moving frame, i.e. v_*/u affects the solitary structure. It is found that in our case the solitary structure doesn't exist for $v_*/u > 1$ but in the limit when $v_*/u < 1$, the depth of Sagdeev potential increases with the increase in the velocity of the nonlinear structure in the co-moving frame.

6 Conclusion

To summarize, we have investigated the formation of drift solitary structures by using the trapped electrons in inhomogeneous degenerate quantum plasma. Such investigations, to the best of our knowledge, have been carried out for the first time, and the results of our work should help in understanding astrophysical observations of dense objects like white dwarfs. Linear and nonlinear propagation characteristics of drift solitary structures have been investigated by considering degenerate electrons and classical ions. In this regard, we have numerically investigated our theoretical results for different parameter values such as magnetic field, temperature and inhomogeneity scale length. The results have been presented graphically illustrating the formation of solitary wave structures and their dependence on the different parameters mentioned above. One of the important findings of the paper has been that, for a fixed density, the partially degenerate quantum plasma with trapped electrons admits both rarefactive and solitary wave structures for a range of temperature T. Finally, it has been observed that the solitary structure can only exist for the case when the diamagnetic drift velocity is smaller than the velocity of the nonlinear structure. The relevance of the present results with regard to the pulsating white dwarfs has also been pointed out.

References

Abbasi, H., Tsintsadze, N.L., Tskhakaya, D.D.: Phys. Plasmas **6**, 2373 (1999)
Ali, S., Moslem, W.M., Shukla, P.K., Kourakis, I.: Phys. Lett. A **366**, 606 (2007)
Ayub, M., Shah, H.A., Qureshi, M.N.S.: Phys. Scr. **84**, 045505 (2011)
Eliasson, B., Shukla, P.K.: Europhys. Lett. **97**, 15001 (2011)
Eliasson, B., Shukla, P.K.: Europhys. Lett. **97**, 15001 (2012)
Fontaine, G., Brassard, P.: Publ. Astron. Soc. Pac. **120**, 1043 (2008)
Gurevich, A.V.: Sov. Phys. JETP **53**, 953 (1967)
Haas, F., Garcia, L.G., Goedert, J., Manfredi, G.: Phys. Plasmas **10**, 3858 (2003)
Kendl, A., Shukla, P.K.: Phys. Lett. A **375**, 3138 (2011)
Koester, D., Chanmugam, G.: Rep. Prog. Phys. **53**, 837 (1990)
Landau, L.D., Lifshitz, E.M.: Statistical Physics, Part I. Butterworth-Heinemann, Oxford (1980)
Landau, L.D., Lifshitz, E.M.: Physical Kinetics. Pergamon Press, New York (1981)
Mamun, A.A.: Phys. Rev. E **55**, 1852 (1997)
Mamun, A.A., Shukla, P.K.: Europhys. Lett. **94**, 65002 (2011)
Mamun, A.A., Shukla, P.K., Mendis, D.A.: J. Plasma Phys. **78**, 143 (2011)
Manfredi, G.: Fields Inst. Commun. **36**, 162 (2004)
Marklund, M., Shukla, P.K.: Rev. Mod. Phys. **78**, 491 (2006)
Markowich, A., Ringhofer, C., Schmeiser, C.: Semiconductor Equations. Springer, Vienna (1990)
Masood, W., Mirza, A.M., Nargis, S.: Phys. Plasmas **15**, 103703 (2008)
Masood, W., Karim, S., Shah, H.A., Siddiq, M.: Phys. Plasmas **16**, 112302 (2009a)
Masood, W., Karim, S., Shah, H.A., Siddiq, M.: Phys. Plasmas **16**, 042108 (2009b)

Masood, W., Eliasson, B., Shukla, P.K.: Phys. Rev. E **83**, 105401 (2011)
Mithen, J.P., Daligault, J., Gregori, G.: Phys. Rev. E **83**, 105401 (2011)
Naeem, I., Mirza, A.M., Masood, W., Farid, T.: J. Plasma Phys. **77**, 375 (2011)
Ostriker, J.P.: Annu. Rev. Astron. Astrophys. **9**, 535 (1971)
Reach, W.T., Kuchner, M.J., Hippel, T.V., Burrow, A., Mullally, F., Kilic, M., Winget, D.E.: Astrophys. J **624**, L161 (2004)
Sagdeev, R.Z.: Review of Plasma Physics, vol. 4. Consultants Bureau, New York (1996)
Shah, H.A., Qureshi, M.N.S., Tsintsadze, N.L.: Phys. Plasmas **17**, 032312 (2010)
Shah, H.A., Masood, W., Qureshi, M.N.S., Tsintsadze, N.L.: Phys. Plasmas **18**, 102306 (2011)
Shah, H.A., Iqbal, M.J., Tsintsadze, N.L., Masood, W., Qureshi, M.N.S.: Phys. Plasmas **19**, 092304 (2012)
Shukla, P.K., Ali, S.: Phys. Plasmas **11**, 113401 (2004)
Shukla, P.K., Eliasson, B.: Rev. Mod. Phys. **83**, 885 (2011)
Silvotti, R., Fontaine, G., Pavlov, M., et al.: Astron. Astrophys. **525**, A64 (2011)
Winget, D.E., Kepler, S.O.: Annu. Rev. Astron. Astrophys. **46**, 157 (2008)
Witt, E., Lotko, W.: Phys. Fluids **26**, 2176 (1983)