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# Alfven solitary waves in nonrelativistic, relativistic, and ultra-relativistic degenerate quantum plasma

M. A. Rehman,[1] H. A. Shah,[2] W. Masood,[3,4] and M. N. S. Qureshi[1]
[1]Department of Physics, GC University, Kachery Road, Lahore 54000, Pakistan
[2]Department of Physics, Forman Christian College, Ferozepur Road, Lahore 54600, Pakistan
[3]COMSATS, Institute of Information Technology, Park Road, Chak Shehzad, Islamabad 44000, Pakistan
[4]National Centre for Physics (NCP) Shahdra Valley Road, Islamabad, Pakistan

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Nonlinear circularly polarized Alfvén waves are studied in magnetized nonrelativistic, relativistic, and ultrarelativistic degenerate Fermi plasmas. Using the quantum hydrodynamic model, Zakharov equations are derived and the Sagdeev potential approach is used to investigate the properties of the electromagnetic solitary structures. It is seen that the amplitude increases with the increase of electron density in the relativistic and ultrarelativistic cases but decreases in the nonrelativistic case. Both right and left handed waves are considered, and it is seen that supersonic, subsonic, and super- and sub-Alfvénic solitary structures are obtained for different polarizations and under different relativistic regimes. © *2015 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4932072]

## I. INTRODUCTION

The nonlinear dynamics of plasma waves in isotropic as well as magnetoactive plasmas has been fairly well studied in the past forty years or so.[1–13] More recently, a large body of literature devoted to collective behavior in degenerate or quantum plasmas has been produced[14] (for a detailed review, see Ref. 14 and the references therein). The applications of the quantum systems include microelectronics,[14] plasmas produced by laser matter interactions, and very dense astrophysical objects[15–17] (e.g., white dwarfs, neutron stars, etc.), and thin metal films.[18–25] In degenerate electron plasma, quantum effects play a significant role[26–29] since with an increase in electron number density the electron Fermi energy cannot be ignored in comparison to the electron rest mass energy, and the electron speed on the Fermi surface can become relativistic for highly dense plasmas. It is well known that the equation of state for the degenerate electrons[30] for the nonrelativistic degenerate electrons has the form $P \propto {n_e}^{5/3}$, which changes to $P \propto {n_e}^{4/3}$ for a strongly relativistic case.[31] The relativistic effects are prominent in super-dense astrophysical objects (white dwarfs and magnetars), with electron number density $10^{32}\mathrm{m}^{-3}$ and beyond.[32–34] Since the electron degeneracy pressure depends on the number density of electrons and not on the temperature,[35] is what supports a white dwarf against gravitational collapse.[36] It is thought that the most common constituents of the interior of white dwarf stars are fully ionized helium, carbon, and oxygen.[37,38] It was predicted by Blacket[39] and Ginzburg[40] that strong magnetic field (of the order of 1 MG) is present in white dwarfs stars.

The quantum hydrodynamic (QHD) model in plasma is the modified form of classical fluid model to investigate the quantum effects.[41] In this model, the equations of state are modified by transport of momentum and energy of charged particles with quantum statistical pressure through Fermi pressure and quantum tunneling effect through Bohm potential.[42] Electromagnetic solitons in degenerate relativistic electron-positron plasma were studied,[43] and the existence of soliton solution in non-relativistic as well as in relativistic degenerate plasma was shown by Marklund and Brodin.[44] The aforementioned authors studied spin solitons in electron-positron magnetized plasma and obtained the modified Korteweg-de Vries (KdV) equation for Alfvén solitary structures with spin effects. Ion-acoustic solitons in a fully relativistic plasma were studied,[44] and it was found that the features of ion-acoustic solitons are considerably modified in a relativistically degenerate plasma.

In the present work, we will use the full set of nonlinear QHD equations for a relativistically degenerate electron ion plasma to investigate circularly polarized Alfvén waves via the Zakharov equations. The organization of paper is as follows. In Section II, mathematical formulation is given and the linear dispersion relation for Alfvén waves is derived. In Section III, the derivation of nonlinear relativistic electromagnetic Sagdeev potential is performed, which is necessary for the discussion of soliton solution for relativistic degenerate circularly polarized Alfvén waves. In Section IV, the main results of the paper are discussed, and the summary of the work is given in Section V.

## II. MATHEMATICAL FORMULATION

As stated in the Introduction, we consider a two component electron-ion magnetoplasma. The background magnetic field is taken in the z direction $\boldsymbol{B}_0 = B_0\hat{z}$, and propagation is considered only in the parallel direction. The electrons are considered inertialess as we look at low frequency Alfvén waves only. The electrons are taken to be degenerate and fully relativistic whereas the ions are taken be cold and are treated classically owing to their large mass. The equations of motion and continuity for both components of charged particles along with Maxwell's equations are used to construct an effective one fluid model to study the Alfvén waves in the plasma. The fundamental set of equations is as follows.

 

The electrons are considered to be inertialess, which is valid approximation in the case of low frequency, and, therefore, no subsequent relativistic mass corrections are needed here. Thus, the equation of motion for electron is

$$0 = -n_e e\boldsymbol{E} - en_e(\boldsymbol{v_e} \times \boldsymbol{B}) - \nabla p_{eR}. \tag{1}$$

Here, $n_e$ is the electron number density, $e$ is the magnitude of the electron charge, $\boldsymbol{v_e}$ is electron fluid velocity, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field vectors, and $p_{eR}$ is the relativistic electron degenerate pressure, which will be introduced later.

The momentum equation for inertial cold ions is

$$m_i n_i \frac{d\boldsymbol{v_i}}{dt} = Zen_i\boldsymbol{E} + Zen_i(\boldsymbol{v}_i \times \boldsymbol{B}), \tag{2}$$

where $m_i$ is the mass of ions, $n_i$ is the number density of ions, $Z$ is the charge number of ion, and $\boldsymbol{v}_i$ is the ions fluid velocity.

The ion continuity equation is

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{v}_i) = 0. \tag{3}$$

By ignoring the displacement current (as we consider low frequency waves only), the Maxwell's equations are

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{4}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}_P. \tag{5}$$

Here, $\mu_0$ is the magnetic permeability. The current density is given by

$$\boldsymbol{j}_P = e(Zn_i\boldsymbol{v}_i - n_e\boldsymbol{v}_e). \tag{6}$$

By following a standard procedure,[45] we obtain the normalized magnetic induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v}_i \times \boldsymbol{B}) - \frac{v^2{}_A}{\Omega}\nabla \times ((\nabla \times \boldsymbol{B}) \times \boldsymbol{B}). \tag{7}$$

And the one fluid momentum equation is given by

$$\frac{d\boldsymbol{v_i}}{dt} = v^2{}_A(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \frac{\nabla p_{eR}}{m_i n_i}. \tag{8}$$

Here, $v_A = \sqrt{\frac{B_0{}^2}{\mu_{0 n_{i0} m_i}}}$ is the Alfvén velocity and $\Omega = \frac{ZeB_0}{m_i}$ is the ion gyro-frequency and $P_{eR}$ is the electron relativistic pressure, the expression for which will be given later.

Now for circularly polarized waves, we take $B_\pm = B_x \pm iB_y$ and $v_\pm = v_x \pm iv_y$. The upper and lower signs are used for right and left circularly polarized forward propagating Alfvén waves. Also, expressing the convective fluid derivative as $\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v_i}.\nabla$, the perpendicular and parallel components of Eq. (8) are

$$\left(\frac{\partial}{\partial t} + v_{iz}\frac{\partial}{\partial z}\right)\boldsymbol{v}_{\pm i} = v^2{}_A\frac{\partial}{\partial z}\boldsymbol{B}_\pm, \tag{9}$$

$$\frac{\partial v_{iz}}{\partial t} + \frac{1}{2}\frac{\partial}{\partial z}|v_{iz}|^2 = -\frac{1}{2}v^2{}_A\frac{\partial}{\partial z}|B|^2 - \frac{\frac{\partial}{\partial z}P_{eR}}{m_i n_i}. \tag{10}$$

Eq. (7) can be written as

$$\frac{\partial \boldsymbol{B}_\pm}{\partial t} = \mp i\frac{v^2{}_A}{\Omega}\frac{\partial^2}{\partial z^2}\boldsymbol{B}_\pm + \frac{\partial}{\partial z}v_{\pm i} - \frac{\partial}{\partial z}(v_{iz}\boldsymbol{B}_\pm). \tag{11}$$

Operating Eq. (11) by $\left(\frac{\partial}{\partial t} + v_{iz}\frac{\partial}{\partial z}\right)$ and using Eq. (9) we obtain

$$\begin{aligned}\frac{\partial^2 \boldsymbol{B}_\pm}{\partial t^2} &+ v_{iz}\frac{\partial^2 \boldsymbol{B}_\pm}{\partial z \partial t}\\ &= \mp i\frac{v^2{}_A}{\Omega}\left(\frac{\partial^3 \boldsymbol{B}_\pm}{\partial z^2 \partial t} + v_{iz}\frac{\partial^3}{\partial z^3}\boldsymbol{B}_\pm\right) + v^2{}_A\frac{\partial^2}{\partial z^2}\boldsymbol{B}_\pm\\ &\quad + \frac{\partial^2}{\partial z \partial t}(v_{iz}\boldsymbol{B}_\pm) - v_{iz}\frac{\partial^2}{\partial z^2}(v_{iz}\boldsymbol{B}_\pm).\end{aligned} \tag{12}$$

The quantities $\boldsymbol{v}_{iz}$ and $n_i$ are related via the continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i\boldsymbol{v}_{iz}) = 0. \tag{13}$$

Equations (10), (12), and (13) form a complete set of nonlinear equations which relate $\boldsymbol{B}_\pm$, $n_i$, and the parallel ion fluid velocity $\boldsymbol{v}_{iz}$.

We now briefly consider the linear dispersion relation for circularly polarized Alfvén waves by linearizing Eq. (12). We note that $n_i$ and $\boldsymbol{v}_{iz}$ are taken to be constant within the linear approximation and so we obtain

$$\left[\frac{\partial^2}{\partial t^2} - v^2{}_A\frac{\partial^2}{\partial z^2} \pm i\frac{v^2{}_A}{\Omega}\frac{\partial^3}{\partial z^2 \partial t}\right]\boldsymbol{B}_\pm = 0\,. \tag{14}$$

By using a plane wave solution, we obtain from Eq. (14)

$$\omega^2 \mp \frac{\omega\omega^2{}_A}{\Omega} - \omega^2{}_A = 0\,. \tag{15}$$

Here, $\omega_A = k_A v_A$, and $k_A$, $\omega$ are the wave number and frequency. As $\frac{\omega_A}{\Omega} \ll 1$ for low frequency waves, Eq. (15) yields

$$\omega_\pm = \omega_A\left(1 \mp \frac{\omega_A}{2\Omega}\right). \tag{16}$$

This is the linear dispersion relation for finite amplitude Alfvén waves. We note here that this is the same expression as obtained[45] for classical plasma since quantum effects will appear through the relativistic pressure, which plays an important role when nonlinear effects are accounted for in the ensuing section.

## III. NONLINEAR EVOLUTION EQUATION

In this section, we will derive the nonlinear evolution equation and look for the possibility of formation of solitary structures. We will derive the modified set of Zakharov equations by using the full set of nonlinear equations given by (10), (12), and (13).

We begin by incorporating a slowly varying amplitude for fluctuating quantities in the following manner:

$$\boldsymbol{B}_{\pm} = b(z,t)\exp\left[i(k_A z - \omega_{\pm} t)\right], \tag{17}$$

using the assumption $\frac{\omega_A}{\Omega} \ll 1$ and considering the scaling $\frac{\partial b}{\partial t} \simeq \frac{b}{\tau}$, $\frac{\partial b}{\partial z} \simeq \frac{b}{v_A \tau}$, $\delta \boldsymbol{v}_{iz} \simeq v_A b^2$, and $\delta n_i \simeq b^2$.[45] Here, $\delta n_i$ and $\delta \boldsymbol{v}_{iz}$ are perturbed ion number density and parallel perturbed ion velocity.

We now use the full expression in Eq. (10) for the relativistic pressure given by[30]

$$P_{eR} = \frac{\pi m^4{}_e c^5}{h^3}\left[\gamma_e\left(2\gamma^2{}_e - 3\right)\left(\gamma^2{}_e + 1\right)^{\frac{1}{2}} + 3\sinh^{-1}(\gamma_e)\right]. \tag{18}$$

By Taylor expansion Eq. (18) around $n_e = n_{e0}$ in the expression for pressure and retaining the terms up to $\delta n_e$, Eq. (18) takes the form

$$P_{eR} \simeq P_{e0} + \left(\frac{\partial P_{eR}}{\partial n_e}\right)_{n_e = n_{e0}}, \quad n_e = P_{e0} + \frac{2}{3\gamma_{e0}}\epsilon_{Fe}\delta n_e \tag{19}$$

and $n_e = Z n_i$

Here, $\gamma_{e0} = \sqrt{(1+\gamma^2{}_e)}$ is the relativistic gamma factor of an electron with $\gamma_e = \frac{p_{Fe}}{m_e c}$ and $p_{Fe} = \left(\frac{3h^3 n_e}{8\pi}\right)^{\frac{1}{3}}$ is the momentum of the electrons on the Fermi surface.

Using expression (17) in Eq. (12) and combining the equation of continuity and the parallel equation of motion (i.e., Eqs. (10) and (13)) along with Eqs. (18) and (19), we obtain

$$i\frac{\partial b}{\partial t} + i v_g \frac{\partial b}{\partial z} + \frac{\omega_A}{2} b \delta n_i - k_A \delta v_{iz} b \mp \frac{v^2{}_A}{2\Omega}\frac{\partial^2 b}{\partial z^2} = 0\,, \tag{20}$$

$$\left[\frac{\partial^2}{\partial t^2} - \frac{2}{3}c^2{}_{sr}\frac{\partial^2}{\partial z^2}\right]\delta n_i = \frac{v^2{}_A}{2}\frac{\partial^2 |b|^2}{\partial z^2}, \tag{21}$$

and

$$\frac{\partial \delta n_i}{\partial t} + \frac{\partial \delta \boldsymbol{v}_{iz}}{\partial z} = 0, \tag{22}$$

where $v_g = v_A\left(1 \mp \frac{\omega_A}{\Omega}\right)$ is the group velocity of Alfvén waves. The above set of expressions constitute Zakharov's equations[1] and it is further noted that these equations are fully relativistic and all relativistic effects are incorporated in $c_{sr} = \sqrt{\frac{2Z\varepsilon_{Fe}}{3m_i\gamma_{e0}}}$ and $\varepsilon_{Fe} = \frac{p^2{}_{Fe}}{2m_e}$.

In order to solve Eqs. (20) and (21) to obtain the Sagdeev potential, we shift to a comoving frame of reference through the following transformation:

$$\xi = z - vt, \tag{23}$$

where $v$ is the arbitrary velocity of propagation of the nonlinear structure. Now using Eq. (23) in Eq. (21), we get

$$\delta n_i = \frac{v^2{}_A}{2(v^2 - c^2{}_{sr})}|b|^2. \tag{24}$$

Eq. (22) gives us

$$\delta v_{iz} = v\delta n_i. \tag{25}$$

After substituting the values of $\delta v_{iz}$ and $\delta n_i$, Eq. (20) can be written as

$$-i(v - v_g)\frac{\partial b}{\partial \xi} \mp \frac{v^2{}_A}{2\Omega}\frac{\partial^2 b}{\partial \xi^2} + \frac{k_A v_A}{2(v^2 - c^2{}_{sr})}\left(\frac{v_A}{2} - v\right)b|b|^2 = 0\,. \tag{26}$$

Further by expressing the complex perturbed magnetic field as

$$b = A(\xi)e^{i\phi(\xi)}, \tag{27}$$

where $A$ and $\varphi$ are real quantities. Eq. (26) can be separated into real and imaginary parts and we obtain

$$(v - v_g)A\frac{\partial \phi}{\partial \xi} \mp \frac{v^2{}_A}{2\Omega}\left(\frac{\partial^2 A}{\partial \xi^2} - A\left(\frac{\partial \phi}{\partial \xi}\right)^2\right) + \frac{k_A v_A}{2(v^2 - c^2{}_{sr})}\left(\frac{v_A}{2} - v\right)A^3 = 0, \tag{28}$$

$$-(v - v_g)\frac{\partial A}{\partial \xi} \mp \frac{v^2{}_A}{2\Omega}\left(2\frac{\partial A}{\partial \xi}\frac{\partial \phi}{\partial \xi} + A\frac{\partial^2 \phi}{\partial \xi^2}\right) = 0. \tag{29}$$

Eq. (29) can be integrated[46] to yield

$$\phi = \pm\frac{\Omega}{v^2{}_A}(v - v_g)\xi\,. \tag{30}$$

Further by inserting Eq. (30) in Eq. (28), we can write Eq. (28) as

$$\frac{d^2 A}{d\xi^2} + \frac{d}{dA}V(A) = 0\,, \tag{31}$$

where

$$V(A) = \mp\frac{k_A\Omega\left(\frac{v_A}{2} - v\right)}{4(v^2 - c^2{}_{sr})}A^4 - \frac{3\Omega^2(v - v_g)^2}{v^4{}_A}A^2 \tag{32}$$

is the Sagdeev or pseudopotential.

This potential can be manipulated in the usual manner. For example, the roots of $V(A) = 0$ gives $A = 0$ and

$$A = \pm\sqrt{\mp\frac{12\Omega(v - v_g)^2(v^2 - c^2{}_{sr})}{k_A v^4{}_A\left(\frac{v_A}{2} - v\right)}}. \tag{33}$$

The upper and lower signs in the square root denote the right and left-handed circularly polarized Alfvén waves. We can further analyze where the real minima and maxima occurs for the right and left hand polarized waves.

## IV. DISCUSSION OF THE RESULTS

In this section, we analyze main results of Section III, i.e., the general expression obtained for the Sagdeev potential Eq. (32). The conditions for the formation of solitary structures through Sagdeev potential are well established,[47] and by using them, we obtain conditions for the formation of solitary structures for different polarizations and relativistic regimes.

As we know that the amplitude $A$ of the Sagdeev potential $V(A)$, given by Eq. (32), must always be real; therefore, for amplitude to remain real for the right-handed circularly polarized Alfvén waves, the following conditions must be fulfilled simultaneously in the different velocity regimes:

(1) $v^2 < c^2{}_{sr}$ (subacoustic) and $\frac{v_A}{2} > v$ (sub-Alfvénic)
(2) $v^2 > c^2{}_{sr}$ (super acoustic) and $\frac{v_A}{2} < v$ (super-Alfvénic)

Thus, for the formation of solitary waves for the right handed circularly polarized waves, the wave must either be subacoustic and sub-Alfvénic or super acoustic and super-Alfvénic.

Similarly for the real value of $A$, the left-handed circularly polarized Alfvén wave must fulfill the following relations:

(3) $v^2 < c^2{}_{sr}$ (subacoustic) and $\frac{v_A}{2} < v$ (super-Alfvénic)
(4) $v^2 > c^2{}_{sr}$ (super acoustic) and $\frac{v_A}{2} > v$ (sub-Alfvénic)

Thus, a different set of conditions for solitary wave propagation is obtained in this case by comparison with the right hand circularly polarized waves. It is observed that solitary structures are formed when the Alfvén wave is subsonic and super-Alfvénic or when the wave is super acoustic and sub-Alfvénic.

### A. Limiting cases

There exists a strong magnetic field up to $10^9 - 10^{14}$ G in white dwarfs and neutron stars.[48–51] In dense astrophysical plasmas like atmosphere of neutron stars, magnetars, and the interior and outer shell of massive white dwarfs, the variation in the electron plasma density ranges from $10^{32} - 10^{38}/\mathrm{m}^3$, and so, this wide range of variation can be discussed in the context of nonrelativistic degenerate, relativistic degenerate, and ultra-relativistic degenerate Fermi plasmas $\gamma_e \ll 1$ or $\gamma_{e0} = 1, \gamma e > 1$ and $\gamma_e \gg 1$.

We will discuss these cases for heavy carbon ions, which exist in the dense astrophysical plasmas. The parameters chosen here are $B_0 = 10^7\,\mathrm{T}$, $m_e = 9.1 \times 10^{-31}$ kg, $Z = 6$, $m_i = 2 \times 10^{-26}$kg (carbon) $n_e = 10^{33}\mathrm{m}^{-3}$ (nonrelativistic) $n_e = 10^{34}\mathrm{m}^{-3}$ (relativistic) $n_e = 10^{38}\mathrm{m}^{-3}$ (ultrarelativistic), which matches the neutron stars and interior of white dwarfs.

#### 1. Nonrelativistic case

The Sagdeev potential represented by Eq. (32) for right handed circularly polarized wave and for nonrelativistic degenerate plasma is plotted in Figure 1. It is observed that as the value of $\beta = \frac{c^2{}_{sr}}{v^2{}_A}$ (i.e., electron number density) increases, the amplitude of the Sadgeev potential and solitary structure decreases, i.e., the solitary structure becomes less super acoustic and more super-Alfvénic. The amplitude $A$ versus space coordinate $\xi$ is plotted in Figure 2. For left handed circularly polarized waves, solitons are also obtained, and the Sagdeev potential can be plotted. These plots show a similar trend to the one obtained in Fig. 1 for the right hand circularly polarized waves.

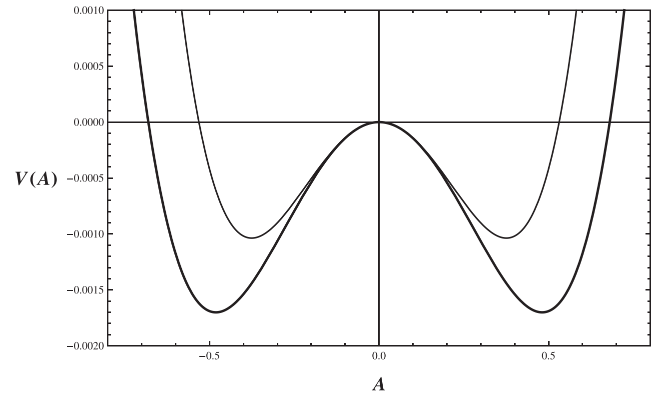


FIG. 1. Variation of Sagdeev potential V(A) (normalized by ion gyro radius $\rho_i (= \frac{v_A}{\Omega})$) versus amplitude A for the right circularly polarized wave for $v_g = 0.5\, v_A$ and $\omega_A = 0.5\,\Omega$, $v = 0.57\, v_A$, $\beta = 0.04$ (bold line), and $\beta = 0.1$ (normal line).

#### 2. Relativistic case

For right handed circularly polarized waves, the relativistic regime in a degenerate plasma is obtained with increasing $\beta$ (i.e., the electron number density in relativistic limit) and the Sagdeev potential is plotted in Figure 3. It is observed that the amplitude of the Sagdeev potential and solitary structure enhances with the increase in number density making the solitary structure less super acoustic and more super-Alfvénic. The amplitude $A$ versus space coordinate $\xi$ for relativistic case is shown in Figure 4. Similar trends are observed for left handed circularly polarized wave in this regime.

#### 3. Ultrarelativistic case

It is observed that in the ultrarelativistic regime, the solitons exist only for left handed circularly polarized wave. For the right handed circularly polarized, no solitary structures are obtained. The Sagdeev potential for ultrarelativistic degenerate plasma is plotted in Figure 5 with increasing $\beta$ (i.e., the electron number density in ultrarelativistic limit). It is observed that as the wave becomes more subrelativistic acoustic, the Sagdeev potential becomes deeper and have larger amplitude, which is a nonlinear phenomenon. The

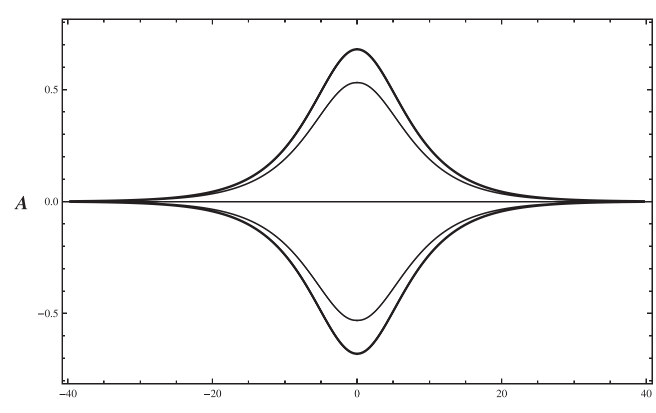


FIG. 2. Variation of soliton amplitude A versus coordinate $\xi$ for right circularly polarized wave with same numerical values as in Figure 1.

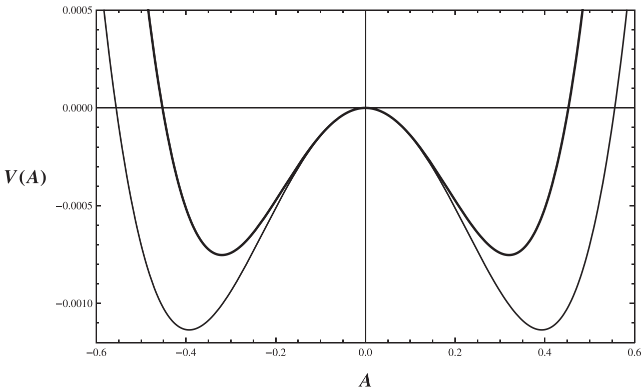


FIG. 3. Variation of Sagdeev potential V(A) (normalized by ion gyro radius $\rho_i (= \frac{v_A}{\Omega})$) versus amplitude A for the right circularly polarized wave for $v_g = 0.5\, v_A$, $\omega_A = 0.5\,\Omega$ and $v = 0.57 v_A$, $\beta = 0.14$ (bold line), and $\beta = 0.20$ (normal line).

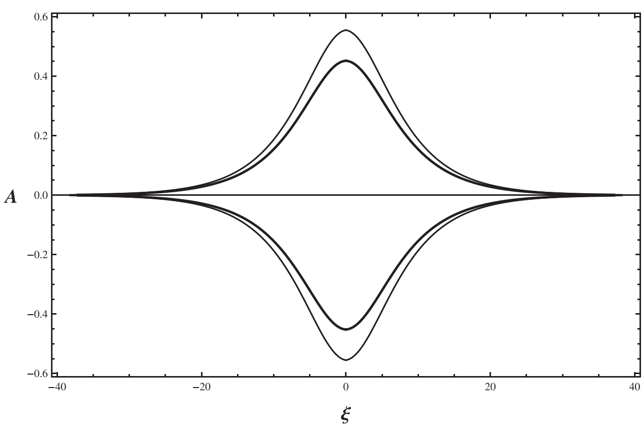


FIG. 4. Variation of soliton amplitude A versus coordinate $\xi$ for right circularly polarized wave with same numerical values as in Figure 3.

amplitude $A$ versus space coordinate $\xi$ ultrarelativistic case is shown in Figure 6.

The rapid development of laser technology since the invention of chirped pulse amplification[52] has sprung forth unprecedented intensities on target to be realized. Lasers are now routinely focused to an irradiance on target of $I\lambda^2 = 10^{21}$ Wcm$^{-2}$ $\mu$m$^2$ (where $I$ is the intensity and $\lambda$ is the wavelength of the laser radiation).[53,54] A number of new lasers promise many more order of magnitude increases in intensity in the near[55,56] and medium-term future.[57] This field has engendered a lot of interest and attention owing to the fact that it that has allowed the investigation of many novel relativistic plasma physics issues, ranging from compact particle accelerators[58–60] to high energy density laboratory astrophysics[61–63] and fast ignition inertial fusion.[64]

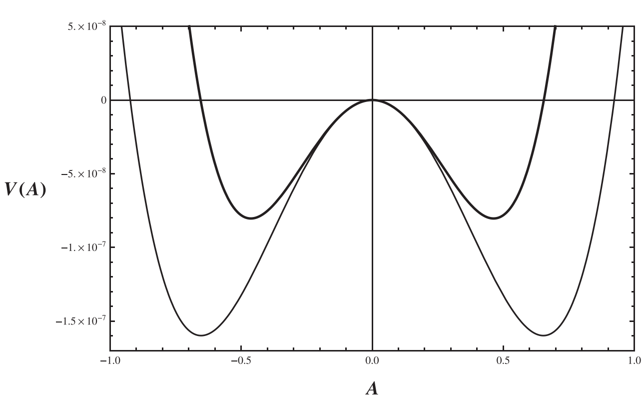


FIG. 5. Variation of Sagdeev potential V(A) (normalized by ion gyro radius $\rho_i (= \frac{v_A}{\Omega})$) versus amplitude A for the left circularly polarized wave for $v_g = 1.5995\, v_A$, $\omega_A = 0.5995\,\Omega$ and $v = 1.6 v_A$, $\beta = 94231.5$ (bold line), and $\beta = 187390.0$ (normal line).

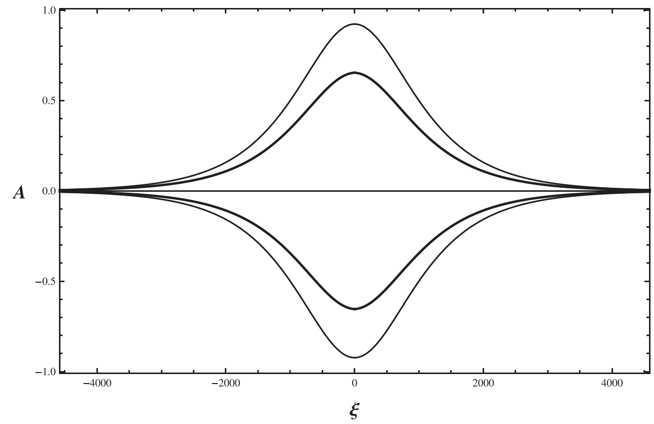


FIG. 6. Variation of soliton amplitude A versus coordinate $\xi$ for the left circularly polarized wave with same numerical values as in Figure 5.

It is in the fitness of the situation to mention here that the *in situ* observations of waves in dense plasmas in extreme environments are very difficult. However, the rapid development of laser technology as mentioned above would hopefully make it possible for us to compare the theory with experiments. Nevertheless, it is imperative that we develop the theory of dense plasmas owing to its applications to laser-solid and compressed plasmas in the laboratory in addition to the astrophysical applications.

## V. SUMMARY AND CONCLUSIONS

In this paper, we have considered the propagation of nonlinear Alfvén waves in a magnetized quantum plasma with relativistically degenerate electrons. We have assumed parallel propagation $(\frac{\partial}{\partial x} = 0 = \frac{\partial}{\partial y})$ and background magnetic field in the z-direction ($\boldsymbol{B}_0 = B_0 \hat{z}$). The set of quantum hydrodynamic equations in the presence of relativistic degenerate electron Fermi pressure has been derived from the Chandrasekhar's equation of state and subsequently used in this paper. Nonlinear coupling of Alfvén and acoustic waves in the presence of relativistic and ultra-relativistic degenerate Fermi pressure has also been investigated. In this regard, Zakharov equations for relativistic degenerate Fermi plasma have been derived. The soliton solutions for non-relativistic, relativistic, and ultrarelativistic degenerate Fermi plasma have also been discussed. It has been found that the amplitude of the soliton mitigates with the increase in number density for nonrelativistic regime whereas the amplitude has been observed to enhance in relativistic and ultrarelativistic regimes.

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