ORIGINAL ARTICLE

Finite amplitude solitary structures of coupled kinetic Alfven-acoustic waves in dense plasmas

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Received: 5 August 2014 / Accepted: 23 October 2014 / Published online: 4 December 2014 © Springer Science+Business Media Dordrecht 2014

Abstract In this paper, we have investigated the nonlinear propagating coupled Kinetic Alfven-acoustic waves in a low beta degenerate quantum plasma in the presence of trapped Fermi electrons using the quantum hydrodynamic (QHD) model. By using the two potential theory and the Sagdeev potential approach, we have investigated the formation of solitary structures for coupled kinetic Alfven-acoustic waves in the presence of quantum mechanically trapped electrons. We have shown that there are regions of propagation and non-propagation for such solitary structures. We have also highlighted the differences between the classical and quantum mechanically trapped electrons. Interestingly, it has been found that the nature of the nonlinearity for the quantum mechanically trapped electrons is different from its classical counterpart. The results presented here may have applications in white dwarf asteroseismology as well as next generation laserplasma experiments where low beta plasma condition is met.

Keywords Kinetic Alfven waves · Quantum trapping · QHD model

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1 Introduction

The low-frequency Alfven wave dispersion relation gets modified when the perpendicular wavelength becomes comparable with the thermal ion Larmor radius. In this case the wave is called the kinetic Alfven wave (KAW). When the ion parallel motion is taken into account in a classical plasma, the Alfven wave couples to the ion acoustic wave via the parameter $\lambda_s = k_x^2 c_s^2 / \Omega_i^2$, where Ω_i is the ion cyclotron frequency and c_s is the ion sound speed respectively producing coupled kinetic Alfven-acoustic waves (CKAAWs). Kinetic Alfven waves (KAWs) are believed to play an important role in plasma heating, particle acceleration, and anomalous transport (Stasiewicz et al. [2000](#page-7-0); Wu [2010](#page-7-1)).

Kadomtsev [\(1965](#page-7-2)) introduced the idea of using two potential theory for Alfven waves in a low *β* (ratio of thermal pressure to the magnetic pressure) plasma. Using the twopotential method, linear and nonlinear KAW in an electronion plasma by including the finite Larmor radius effect was investigated in the references of Hasegawa and Chen [\(1975](#page-7-3)), Hasegawa and Mima [\(1976\)](#page-7-4). Yu and Shukla [\(1978](#page-7-5)) studied finite amplitude solitary KAWs for small but finite *β* effects. Solitons with density humps were observed with an upper limit on the amplitude. This work was further extended by Kakati and Goswami [\(1998](#page-7-6)) to the case of electron-positronion plasma. Dust kinetic Alfven waves were investigated by Yinhua et al. [\(2000](#page-7-7)) and it was observed that the density humps are cusped and narrower than the dips.

The study of numerous collective interactions in dense plasmas are relevant in the context of intense laser-solid density plasma experiments (Azechi et al. [1991](#page-7-8), [2006;](#page-7-9) Hu and Keitel [1999;](#page-7-10) Marklund and Shukla [2006](#page-7-11); Malkin et al. [2007](#page-7-12); Lee et al. [2009](#page-7-13); Norreys et al. [2009](#page-7-14)) the cores of giant planets and the crusts of old stars (Guillot [1999;](#page-7-15) Fortney

et al. [2009](#page-7-16)), superdense astrophysical objects (Craighead [2000;](#page-7-17) Shapiro and Teukolsky [2004;](#page-7-18) Chabrier et al. [2002\)](#page-7-19) (e.g., interiors of white dwarfs and magnetospheres of neutron stars and magnetars); micro and nano-scale objects (e.g., quantum diodes Lau et al. [1991](#page-7-20); Ang et al. [2006](#page-7-21); Shukla and Eliasson [2008\)](#page-7-22) quantum dots and nanowires (Shpatakovskaya [2006](#page-7-23)), nanophotonics (Barnes et al. [2003](#page-7-24); Chang et al. [2006](#page-7-25)), plasmonics (Marklund et al. [2008](#page-7-26)), ultra-small electronic devices (Markovich et al. [1990;](#page-7-27) Abrahams et al. [2001](#page-7-28); Magnus and Schoenmaker [2002](#page-7-29)), metallic nanostructures (Crouseilles et al. [2008\)](#page-7-30), microplasmas (Becker et al. [2006\)](#page-7-31) and quantum X-ray free-electron lasers (Serbeto et al. [2008](#page-7-32); Piovella et al. [2008](#page-7-33)). Furthermore, a Fermi-degenerate dense plasma may also arise when a pellet of hydrogen is compressed to many times the solid density in the fast ignition scenario for inertial confinement fusion (Piovella et al. [2008](#page-7-33); Lindl [1995](#page-7-34); Tabak et al. [2005](#page-7-35)).

The rapid development of laser technology since the invention of chirped pulse amplification (Strickland and Mourou [1985\)](#page-7-36) has given rise to unprecedented intensities on target to be realized. Lasers are now routinely focused to an irradiance on target of $I\lambda^2 = 10^{21}$ W cm⁻² µm² (Perry et al. [1999](#page-7-37); Danson et al. [2004](#page-7-38)) (where *I* is the intensity and λ is the wavelength of the laser radiation). A number of new lasers promise many more order of magnitude increases in intensity in the near (Hooker et al. [2006;](#page-7-39) Yanovsky et al. [2008\)](#page-7-40) and medium-term future (Gerstner [2007](#page-7-41)). This field has engendered a lot of interest and attention owing to the fact that it that has allowed the investigation of many novel relativistic plasma physics issues, ranging from compact particle accelerators (Tajima and Dawson [1979](#page-7-42); Pukhov and Meyer-ter Vehn [2002;](#page-7-43) Mangles et al. [2004](#page-7-44); Geddes et al. [2004](#page-7-45); Faure et al. [2004](#page-7-46)) to high energy density laboratory astrophysics (Ryutov et al. [1999](#page-7-47), [2001;](#page-7-48) Ryutov and Remington [2003\)](#page-7-49) and fast ignition inertial fusion. (Tabak et al. [1994;](#page-7-50) Kodama et al. [2001](#page-7-51)).

The study of radiative blast waves in atomic cluster media using intense laser pulses is reported (Smith [2008\)](#page-7-52). Atomic clusters have been shown to be very efficient absorbers of intense laser radiation. They can be used to create high energy density plasmas that drive strong shocks (*>*Mach 50) and radiative blast waves. Careful application of these equations and similarities allow experiments to be scaled to astrophysical phenomena that have spatial and temporal scales that are greater by as much as 15–20 orders of magnitude. In this way, the radiative blast waves in the laboratory have been scaled those experienced in supernova remnants and the physics governing their dynamics investigated under controlled conditions. It is in the fitness of the situation to mention here that the *in-situ* observations of waves in dense plasmas in extreme environments are very difficult. However the rapid development of laser technology as mentioned above would hopefully make it possible for us to compare the theory with experiments. Nevertheless it is imperative that we develop the theory of dense plasmas owing to its applications to laser-solid and compressed plasmas in the laboratory, in addition to the astrophysical applications.

The most frequently employed approaches to describe the statistical and hydrodynamic behavior of charged species at quantum scales in dense plasmas is the Wigner-Poisson and the Schrödinger-Poisson models. These two approaches are the quantum equivalent of kinetic and fluid treatments of classical plasmas. The two approaches have been vividly explained in a review article by Manfredi ([2005\)](#page-7-53). The quantum hydrodynamic (QHD) model is based on the Schrödinger-Poisson formulation. It has been extensively applied to study the linear and nonlinear propagation of several waves in the quantum plasma (Haas et al. [2003;](#page-7-54) Haas [2005](#page-7-55); Marklund et al. [2005;](#page-7-56) Masood and Mushtaq [2008\)](#page-7-57).

Gurevich ([1967\)](#page-7-58), Landau and Lifshitz ([1981\)](#page-7-59) showed that adiabatic trapping modifies the behavior of nonlinear ion acoustic waves and a 3*/*2 power nonlinearity appears instead of the usual quadratic nonlinearity. The existence of trapping as a microscopic phenomenon has been confirmed by computer simulations (Erokhin et al. [1996](#page-7-60); Sagdeev [1996\)](#page-7-61) and experimental investigations (Hansen et al. [1996\)](#page-7-62). Maxwellian and non-Maxwellian distribution functions were used to investigate the trapping effect on the propagation characteristics of ion acoustic solitons and it was seen that solitary dynamics was modified in both cases (Abbasi et al. [1999;](#page-7-63) Mushtaq and Shah [2006\)](#page-7-64). In classical plasmas, the effect of trapping on the vortex formation was investigated and the modified Hasegawa-Mima equation was derived (Siddiqui et al. [2008\)](#page-7-65). The effect of trapping as a microscopic phenomenon in a degenerate plasma has also been investigated in the presence of quantizing magnetic field (Shah et al. [2012\)](#page-7-66). In a self-gravitating dusty quantum plasma, adiabatic trapping has also been found to play an effective role in the formation of solitary structures (Ayub et al. [2011](#page-7-67)). Using two potential theory, the effect of adiabatic trapping on obliquely propagating coupled Kinetic Alfven-acoustic waves in a low *β* plasma was investigated (Shah et al. [2013](#page-7-68)) for the first time.

In the present paper, we investigate coupled kinetic Alfven-acoustic solitary structures in a low *β* degenerate quantum plasma by including the effect of adiabatic trapping of electrons. The primary difference is that electrons in this case are governed by the Fermi-Dirac distribution and therefore the frame work of obtaining the expression for number density as well as the nature of nonlinearity differs quite significantly here. The layout of present work is as follows: In Sect. [2](#page-2-0), we give the formulation of basic equations and derive the linear dispersion relation of the coupled kinetic Alfven-acoustic in a degenerate quantum plasma. In Sect. [3,](#page-2-1) the nonlinear Sagdeev potential is derived and investigated. In Sect. [4,](#page-3-0) the results are discussed and finally in Sect. [5](#page-5-0), the findings of the current investigation are recapitulated.

2 Mathematical formulation

At the outset, we would like to state that the electrons are considered degenerate and follow the Fermi-Dirac distribution function; however the ions due to their heavy mass are assumed to behave in a classical manner.

In this section, we follow the method illustrated in Cramer ([2001\)](#page-7-69). The limit $m_e/m_i < \beta < 1$ allows us to neglect the electron mass and leads to the investigation of coupled kinetic Alfven-acoustic waves in the low frequency limit. The variation exists in the $x-z$ plane, where z is the direction of ambient magnetic field. The low−*β* assumption allows us to use the two potential fields φ and ψ to describe the electric field in the x and z direction in the following manner $E_x = -\partial \varphi / \partial x$, $E_z = -\partial \psi / \partial z$, $E_y = 0$. As a consequence of the two-potential theory, only shear perturbations in magnetic field are present which is mathematically expressed as $B_z = B_0$, $B_x = 0$. The quasi neutrality condition for ions and electrons densities leads to $n_e = n_i = n$.

For low frequency perturbations the phase velocity of the wave is much less than the electron Fermi speed and electrons are assumed to follow the magnetic field lines. By following the method of integration used by Shah et al. [\(2010](#page-7-70)), the expression for total number density *n* including trapped electrons is obtained for a Fermi-Dirac distribution for a fully degenerate plasma and is given by

$$
n = n_0(1 + \Psi)^{3/2} \tag{1}
$$

 $\Psi = e\psi/\varepsilon_{Fe}$ is the normalized electrostatic potential, where *εFe* is the Fermi energy given by $\varepsilon_{Fe} = \hbar^2 (3\pi^2 n_0)^{3/2}/2m_e$. It is worth mentioning here that the implicit assumption in the derivation of Eq. (1) (1) is that the electron Fermi energy is so high that the electron Landau quantization effects can be ignored.

Although the equations for ions have been given before in an earlier paper, we reproduce them here for the sake of completeness. Following the procedure out lined in references (Hasegawa and Mima [1976;](#page-7-4) Kakati and Goswami [1998;](#page-7-6) Shah et al. [2013\)](#page-7-68), the *x* component of ion velocity is given by

$$
v_{ix} = -\frac{m_i}{eB_0^2} \frac{\partial^2 \varphi}{\partial x \partial t}
$$
 (2)

The parallel equation of motion for ions is

$$
\partial_t v_{iz} + v_{ix} \partial_x v_{iz} + v_{iz} \partial_z v_{iz} = -\frac{e}{m_i} \frac{\partial \psi}{\partial z}
$$
 (3)

From Ampere's law (Hasegawa and Mima [1976](#page-7-4)), we have

$$
\mu_0 \partial_t j_z = \partial_z \partial_x^2 (\varphi - \psi) \tag{4}
$$

The ion continuity equation is

$$
\partial_t n_i + \partial_x (n_i v_{ix}) + \partial_z (n_i v_{iz}) = 0 \tag{5}
$$

and

$$
\partial_z j_z = e \partial_t n_e + e \partial_z (n_i v_{iz}) \tag{6}
$$

The equation above is derived by using electron continuity equation. The algebraic manipulation of Eqs. (1) (1) – (6) (6) yield the following linear dispersion relation

$$
\left(1 - \frac{v_A^2 k_z^2}{\omega^2}\right) \left(\frac{3}{2} - \frac{c_{sF}^2 k_z^2}{\omega^2}\right) = \frac{v_A^2 k_z^2}{\omega^2} \lambda_{sF}
$$
(7)

where $\lambda_{sF} = k_x^2 c_{sF}^2 / \Omega_i^2$ $\sqrt{\varepsilon_F/m_i}$ is the ion sound speed at the Fermi energy ε_{Fe} , *v_A* is the Alfven velocity and $Ω_i$ is the usual ion cyclotron frequency. The wave numbers in the *x* and *z* directions are expressed through the obliqueness angle with respect to the magnetic field and given by $k_z = k \cos \theta$, $k_x = k \sin \theta$. It is pertinent to mention here that the linear dispersion relation for coupled kinetic Alfven-acoustic wave given by Eq. ([7\)](#page-2-4) differs from its classical counterpart (Shah et al. [2013](#page-7-68)) quite appreciably. Note that the second bracket of left hand side contains not only the ion sound velocity defined at Fermi energy but also a 3*/*2 factor which is the consequence of the effect of adiabatic trapping on the linear dispersion relation. In a classical plasma, the linear dispersion relation remains unaffected by adiabatic trapping as it manifests itself only in the nonlinear regime.

Equation [\(7](#page-2-4)) shows the coupling of Alfven wave with the ion-acoustic wave through the coupling parameter λ_{s} . If we set the 2nd factor on the left hand side to unity we get the linear dispersion relation of KAW (Hasegawa and Mima [1976\)](#page-7-4).

$$
\omega^2 = v_A^2 k_z^2 (1 + \lambda_{sF}) \tag{8}
$$

3 Sagdeev potential

In this section, we derive the Sagdeev potential to investigate the formation of solitary structures and to this end we shift to a co-moving frame of reference in normalized variables

$$
\eta = K_x x + K_z z - Mt \tag{9}
$$

The normalized variables are given by

$$
n = n_{e,i}/n_0, \qquad \Phi = e\varphi/T_e, \qquad M = v/c_s,
$$

$$
K = Kc_s/\Omega_i, \qquad t = \Omega_i t,
$$

where n, Φ, M, K, t are the normalized density, potential, effective Mach number, wave number, and time respectively.

Equations (2) (2) – (6) (6) are recast in dimensionless form and are given below

$$
-M\frac{\partial v_{iz}}{\partial \eta} + v_x K_x \frac{\partial v_{iz}}{\partial \eta} + v_z K_z \frac{\partial v_{iz}}{\partial \eta} = -K_z \frac{\partial \Psi}{\partial \eta}
$$
 (10)

$$
v_{ix} = K_x M \frac{\partial^2 \Phi}{\partial \eta^2}
$$
 (11)

$$
2K_x^2 K_z^2 \partial_\eta^4 (\Phi - \Psi) = \beta \left(M^2 \partial_\eta^2 n - M K_z \partial_\eta^2 (n v_{iz}) \right) \tag{12}
$$

$$
-M\partial_{\eta}n + K_{x}\partial_{\eta}(nv_{ix}) + K_{z}\partial_{\eta}(nv_{iz}) = 0
$$
\n(13)

Integrating Eqs. (12) (12) and (13) (13) and applying the boundary conditions v_{iz} , v_{ix} , φ , $\psi \rightarrow 0$, $n_0 = 1$ as $\eta \rightarrow \infty$, we obtain

$$
2K_x^2 K_z^2 \partial_{\eta}^2 (\Phi - \Psi) = \beta \big(M^2 (n - 1) - M K_z (n v_{iz}) \big) \tag{14}
$$

$$
K_x v_{ix} + K_z v_{iz} = M\left(1 - \frac{1}{n}\right) \tag{15}
$$

Using Eqs. (1) (1) , (10) (10) , and (15) (15) , we get the following expression for the parallel ion velocity *viz*

$$
v_{iz} = \frac{K_z}{M(1+T^2)} \left[(1+\Psi)^{3/2} \right] \Psi \tag{16}
$$

The algebraic manipulation of Eqs. (11) (11) , (15) , and (16) (16) yield the following expression

$$
2K_x^2 \frac{\partial^2 \Psi}{\partial \eta^2} = 2\left(\frac{(1+\Psi)^{3/2}-1}{(1+\Psi)^{3/2}}\right) - \frac{\beta K_z^2}{M_A^2} (\Psi(1+\Psi)^{3/2})
$$

$$
-2\frac{M_A^2}{K_z^2}((1+\Psi)^{3/2}-1) + \beta(\Psi(1+\Psi)^3)
$$
(17)

In the above equation, we have used $M^2 = 2M_A^2/\beta$, where *MA* is the ratio of wave velocity to the Alfven velocity and is generally referred to as the Alfvenic Mach number. Equation [\(17](#page-3-6)) can be expressed in the form of an energy integral through the Sagdeev or pseudo-potential in the following manner

$$
\frac{1}{2}\left(\frac{d\Psi}{d\xi}\right)^2 + V(\Psi) = 0\tag{18}
$$

where $V(\Psi)$ is given by

$$
V(\Psi) = -\frac{1}{K_x^2} \left[\left(\Psi + \frac{2}{\sqrt{1 + \Psi}} \right) -\frac{\beta K_z^2}{2M_A^2} \left\{ \frac{2}{35} (1 + \Psi)^{\frac{5}{2}} (-2 + 5\Psi) \right\} -\frac{M_A^2}{K_z^2} \left\{ \frac{2}{5} (1 + \Psi)^{\frac{5}{2}} - \Psi \right\} + \frac{\beta}{2} \left\{ \frac{\Psi^2}{2} + \Psi^3 + \frac{3\Psi^4}{4} + \frac{\Psi^5}{5} \right\} + \frac{2}{5} \frac{M_A^2}{K_z^2} - \frac{2}{35} \frac{\beta K_z^2}{M_A^2} - 2 \right]
$$
(19)

4 Results and discussions

In this section, we present our analysis in order to obtain the solitary structures. The Mach number is found to obey the following conditions:

$$
\sqrt{\frac{\beta}{3}}K_z < M_A < M_1 \tag{20a}
$$

$$
M_2 < M_A < K_z \tag{20b}
$$

where

$$
M_1 = \frac{1}{2} \sqrt{\left(\frac{a-b}{(1+\Psi)(1-\sqrt{1+\Psi}+\Psi(2+\Psi))}\right)}
$$

\n
$$
M_2 = \frac{1}{2} \sqrt{\left(\frac{a+b}{(1+\Psi)(1-\sqrt{1+\Psi}+\Psi(2+\Psi))}\right)}
$$

\n
$$
a = K_z^2 \left(2(-1+\sqrt{1+\Psi})+\Psi\sqrt{1+\Psi}(2+\beta(1+\Psi)^4)\right)
$$

\n
$$
b = \left[K_z^4 \left(8-8\sqrt{1+\Psi}+\Psi(\beta^2\Psi(1+\Psi)^9\right) -4\beta(1+\Psi)^4(1-\sqrt{1+\Psi}+\Psi(2+\Psi))\right)
$$

\n
$$
+4\left(3-2\sqrt{1+\Psi}+\Psi(3+\Psi)\right)\right)\right]^{\frac{1}{2}}
$$

The above conditions are obtained by taking second derivative of Eq. [\(19](#page-3-7)) and setting the coefficients of quadratic term equal to zero. Here M_1 and M_2 are the lower and higher values of Mach number and between these values no solitary structure is formed. Solitary structures are observed to formed only below M_1 and above M_2 .

We now present the graphical analysis of our model by using the values of plasma parameters that are typically found in the vicinity of white dwarf stars (Koester and Chanmugam [1990](#page-7-71); Ghosh and Lakhina [2004\)](#page-7-72). It has been suggested that electrostatic structures could be excited in extreme events, such as supernova explosions at the outer shells of the star (Eliasson and Shukla [2012](#page-7-73)). It was also remarked that electromagnetic waves should also be studied in future. Motivated by this suggestion, we have investigated the coupling of electromagnetic kinetic Alfven wave with the acoustic wave in the presence of quantum mechanically trapped electrons. It is worth mentioning here that for the values of number density and magnetic field used here, the value of plasma beta $(\beta = 2nk_BT_F\mu_0/B_0^2)$ turns out to be less than one and hence the use of two potential theory is justified. Figure [1](#page-4-0) shows the plot of Sagdeev potential for different values of number density by keeping the values of magnetic field and Mach number fixed. We see that by increasing the number density the maximum value of potential increases but depth of the potential decreases. The corresponding solitary structures are shown in Fig. [2](#page-4-1), in which the amplitude as well as the width of the soliton increases with the increase in the number density.

Fig. 1 Sagdeev potential $V(\Psi)$ versus *Ψ* for different values of n_0 when $B_0 = 0.7 \times 10^7$ T and

 $M_A = 0.38$

Fig. 2 Solitary structures corresponding to the Sagdeev potential $V(\Psi)$ shown in Fig. [1](#page-4-0)

In Fig. [3](#page-5-1), the variation of Sagdeev potential is explored for different values of magnetic field by keeping the other plasma parameters fixed. It is observed that depth of the potential increases but the maximum value of potential decreases by increasing the magnetic field. The corresponding solitary structures are plotted in Fig. [4](#page-5-2). It is found that the amplitude as well as the width of the soliton decrease with the increase of magnetic field.

Figure [5](#page-6-0) exhibits the change in Sagdeev potential by varying the Mach number for fixed values of magnetic field strength and number density. It is observed that the depth of the potential increases till the upper limit on Mach number (M_1) given in condition [\(20a](#page-3-8)) is reached. With a further increase in the Mach number, Sagdeev potential does not intersect the potential axis and consequently there will be no corresponding solitary structure. However, when the Mach number further increases and reaches a certain value $(M₂)$ satisfying the condition ([20b\)](#page-3-9), the Sagdeev potential again intersects the potential axis giving rise to the formation of the solitary structures again.

Figure [6](#page-6-1) is plotted for the maximum amplitude of the soliton vs the Mach number for different values of angle of propagation $\theta = 45^\circ$ (thick black line), $\theta = 60^\circ$ (thick dashed line), $\theta = 75^{\circ}$ (black line). It is clear from the Fig. [6](#page-6-1) that for a particular angle, the maximum amplitude of the soliton first increases with the increase in Mach number and then decreases for further increase in the Mach number. Note that the gaps in maximum amplitude values for each angle indicate the absence of solitary structure formation as they correspond to those values of Mach number that do not satisfy the conditions given by Eq. $(20a)$, $(20b)$ $(20b)$. It can also be seen that with the increase in the obliqueness the gap and

Fig. 3 Sagdeev potential $V(\Psi)$ versus *Ψ* for different values of *B*₀ when $n_0 = 3 \times 10^{33}$ m⁻³ and $M_A = 0.38$

range of Mach number decreases. The solitons corresponding to the Fig. [5](#page-6-0) are plotted in Fig. [7](#page-6-2). Note that the amplitude as well as the width of the soliton enhances with the increase in the Mach number as long as condition (given by Eq. $(20a)$ is satisfied, however, both amplitude and width of the soliton show a decrease with the increase in Mach number when the condition (given by Eq. ([20b\)](#page-3-9)) is satisfied.

5 Summary

In conclusion, we have investigated the adiabatic trapping of electrons in a low beta quantum plasma for a coupled kinetic Alfven-acoustic wave. It has been observed that the framework for obtaining the expression of number density of quantum mechanically trapped electrons is quite different from its classical counterpart. Most importantly, the nature of the nonlinearity is found to be different for quantum mechanically trapped electrons by comparison with its classically trapped counterparts. By using the Sagdeev potential approach, we have studied the finite amplitude nonlinear structures and also mentioned the conditions which determine the existence regimes of the solitary structures. We have also explored the variation of the structure of the solitary waves by using different plasma parameters of interest such as obliqueness, magnetic field strength and number density. The present work may be beneficial in enhancing our understanding of the solitary structures in astrophysical environments with special reference to the pulsating white dwarfs and also in laboratory experiments on chirped laser plasma interactions where many astrophysical phenomena can be mimicked on the laboratory scale.

Fig. 5 Sagdeev potential $V(\Psi)$ versus *Ψ* for different values of M_A when $B_0 = 0.7 \times 10^7$ T and $n_0 = 3 \times 10^{33}$ m⁻³

 -5

 $\boldsymbol{0}$

 ζ

Fig. 6 Maximum value of potential versus Mach number for different values of angle

Fig. 7 Solitary structures corresponding to the Sagdeev potential $V(\Psi)$ shown in Fig. [5](#page-6-0)

5

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Acknowledgement This research was supported by the Higher Education Commission (HEC) grant No. 20-2595/NRPU/R&D/HEC/13.

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