Nonlinear coupling of kinetic Alfven waves with acoustic waves in a self-gravitating dusty plasma with adiabatic trapping

A. Sabeen, W. Masood, M. N. S. Qureshi, and H. A. Shah

Citation: Physics of Plasmas **24**, 073704 (2017); doi: 10.1063/1.4990700
View online: http://dx.doi.org/10.1063/1.4990700
View Table of Contents: http://aip.scitation.org/toc/php/24/7
Published by the American Institute of Physics

PHYSICS OF PLASMAS 24, 073704 (2017)

Nonlinear coupling of kinetic Alfven waves with acoustic waves in a self-gravitating dusty plasma with adiabatic trapping

A. Sabeen,[1,2] W. Masood,[3,4] M. N. S. Qureshi,[1,a)] and H. A. Shah[1]
[1]Department of Physics, GC University, Lahore 54000, Pakistan
[2]Department of Physics, University of Lahore, Gujrat Campus, Gujrat, Pakistan
[3]COMSATS Institute of Information Technology, Park Road, Chak Shahzad, Islamabad 44000, Pakistan
[4]National Center for Physics (NCP), Islamabad 44000, Pakistan

(Received 27 January 2017; accepted 16 June 2017; published online 5 July 2017)

In this paper, linear and nonlinear coupling of kinetic Alfven and acoustic waves has been studied in a dusty plasma in the presence of trapping and self-gravitation effects. In this regard, we have derived the linear dispersion relations for positively and negatively coupled dust kinetic Alfven-acoustic waves. Stability analysis of the coupled dust kinetic Alfven-acoustic wave has also been presented. The formation of solitary structures has been investigated following the Sagdeev potential approach by using the two-potential theory. Numerical results show that the solitary structures can be obtained only for sub-Alfvenic regimes in the scenario of space plasmas. *Published by AIP Publishing.*
[http://dx.doi.org/10.1063/1.4990700]

I. INTRODUCTION

The importance of nonlinear phenomena in various branches of plasma physics has led to a great deal of research to understand them.[1–13] The presence of dust grains in an electron-ion plasma introduces new normal modes in the system.[14] In dusty plasmas, the relatively tenuous electrons and ions are assumed to be Boltzmannian while the inertia is provided by the dust particles. The existence of dust-acoustic waves (DAWs) in laboratory plasma has been confirmed by experiments.[15–18] Many studies have discussed the properties of dusty plasmas.[19–23]

Dusty plasmas are found in a variety of physical situations of interest like planetary rings,[24] asteroid zones,[25] and Earth's atmosphere.[26] Dust particles in the planetary and cometary environments have been observed by *in situ* spacecraft observations.[27–31] The presence of dust grains in many space/astrophysical regions may also play an important role in the dynamics of Alfven waves such as in the breaking down of the ideal MHD fluid approximation.[32] It could also affect the way magnetic flux evanesces in the interstellar clouds.[33] The dust grains can be important in the dynamical evolution of Alfven waves in dusty plasma since most of the time the inertial effects of dust are considered.[34]

Dust has also been observed in fusion devices and plasma processing in the laboratory plasmas.[35–38] A theoretical frame work to understand the propagation mechanism of dust-acoustic waves (DAWs) was given for the first time by Rao *et al.*[39] Shukla and Silin[40] investigated the onset of dust-ion-acoustic waves (DIAWs) in dusty plasma. Dust acoustic waves in cometary magnetospheres have been investigated using a kinetic model.[41] Later, experiments confirmed the presence of these new modes.[42,43] The propagation of parallel and perpendicular fluid modes has also been explored in dusty plasmas.[44] It has been found that the spatio-temporal regimes for the propagation of these modes alter significantly in the presence of massive dust particles. The presence of the gargantuan dust mass makes it imperative to incorporate the gravitational effects. Numerous studies[45–47] that investigate the wave propagation characteristics in self-gravitating dusty plasmas indicate a constant tug of war between the electrostatic repulsion among the charged dust grains and the gravitational self-attraction.

We have also analyzed the positive dust which is found in the Jupiter's magnetosphere.[48,49] Alfven waves have found several applications in space, laboratory, and astrophysical plasmas.[50,51] The propagation characteristics of Alfven waves have been observed to modify when the heavy dust component gets added to the ordinary electron-ion plasma.[52] It has been found[51] that the different behaviors of ions and electrons give rise to charge separation which leads to coupling of the Alfven wave with the electrostatic longitudinal mode.

Pioneer work on the two potential theory for Alfven waves in a low β plasma was initiated by Kadomtsev.[53] Since Alfven waves are essentially electromagnetic waves, therefore, using electrostatic potentials was innovative at that time and this led Hasegawa and coauthors[54,55] to investigate the linear and nonlinear Alfven waves in an electron-ion plasma by using this novel technique with the inclusion of the finite Larmor radius effect. Yu and Shukla[56] deliberated upon the nonlinear propagation of Alfven waves for finite β effects and found that the system under consideration allowed the formation of density humps. Nonlinear Alfven solitary structures were also investigated in electron-positron-ion plasmas[57] and it was reported that an exact solitary solution exists for small but finite β. Density dips and cusp solitons have been found out in a similar kind of analysis for a low-$\beta_d \left(= \frac{z_d \mu_0 n_{d0} k_B T_i}{B_0^2}\right)$ collisionless dusty plasma.[58]

Gurevich was the first one to deliberate upon trapping as microscopic phenomena.[59] The behavior of nonlinear ion-acoustic waves was observed to be altered by the adiabatic trapping owing to the change in the nature of nonlinearity.[60]

a)Author to whom correspondence should be addressed: nouman_sarwar@yahoo.com

The existence of trapping has also been validated by experimental investigations[63] and computer simulations.[61,62] It has been shown that the use of thermal as well as non-thermal trapped distributions modifies the propagation characteristics of ion acoustic solitons.[64,65] Trapping has also been considered in the study of the nonlinear drift vortex for which the modified Hasegawa–Mima equation was obtained.[66] The adiabatic trapping plays an effective role in self-gravitating dusty quantum plasmas.[67] The effect of adiabatic trapping using the two potential theory in a low β plasma has been investigated for obliquely propagating Alfven waves.[68]

In this paper, we consider the coupling of kinetic Alfven and nonlinear density excitations in a low β dusty plasma with the inclusion of trapping and self-gravitation effects. The paper is organized as follows: the basic set of governing equations to study the linear and nonlinear coupling of dust kinetic Alfven waves with dust acoustic waves with adiabatic trapping in a self-gravitating dusty plasma is presented in Sec. II. The dispersion characteristics for both negative and positively charged dust species are derived in Sec. III. In Sec. IV, stability analysis is given with the inclusion of the Jeans term. In Sec. V, the nonlinear Sagdeev potential is derived both for negative and positive dust particles. The results are presented and discussed in Sec. VI. Finally, the main findings of the paper are recapitulated in Sec. VII.

II. MODEL EQUATIONS

In this section, we consider a multi-component plasma comprising of electrons, ions, and negatively/positively charged dust grains and present the model equations which will first be used to derive the linear dispersion relation for coupled dust kinetic Alfven-acoustic waves in a self-gravitating dusty plasma by considering adiabatic trapping of electrons.[51,69] We neglect the mass of electrons and ions in the limit $m_i/m_d < \beta_d < 1$ which leads to the investigation of coupled dust kinetic Alfven-acoustic waves in the low frequency limit. Further in the low-β assumption, we can consider two potential fields ϕ and ψ to describe the electric field $E_x = -\frac{\partial\phi}{\partial x}$, $E_z = -\frac{\partial\psi}{\partial z}$, $E_y = 0$ in the x- and z-directions, respectively, where z is the direction of the ambient magnetic field. It is noted that only the shear perturbations in the magnetic field are present. Mathematically, $B_z = B_0$, $B_x = 0$ are a consequence of the two-potential theory.

The equations of motion and continuity for dust grains, respectively, are

$$m_d n_d\left(\frac{\partial}{\partial t} + \mathbf{v}_d\cdot\nabla\right)\mathbf{v}_d = Q_d n_d(\mathbf{E} + \mathbf{v}_d\times\mathbf{B}) - m_d n_d\nabla\varphi, \tag{1}$$

$$\frac{\partial n_d}{\partial t} + \nabla\cdot(n_d\mathbf{v}_d) = 0\,. \tag{2}$$

Here, m_d, n_d, Q_d, and ϕ are the mass, number density, charge number, and gravitational potential of dust, respectively. The dust charge Q_d is defined as $Q_d = \pm z_d e$ with upper signs and lower signs for the positive and negative dust charges, respectively. Since the dust is massive in comparison to ions and electrons, therefore, we consider the Jeans effect in the linear study of coupled dust kinetic Alfven-acoustic waves in a dusty plasma. The inclusion of Jeans effect in the nonlinear study is beyond the scope of this work.

The electrostatic and gravitational Poisson's equations are

$$\nabla^2\psi = \frac{e}{\varepsilon_0}(z_d n_d + n_e - n_i), \tag{3}$$

$$\nabla^2\varphi = \frac{1}{\varepsilon_0}G m_d n_d, \tag{4}$$

where n_i and n_e are the number densities of ions and electrons, respectively, ψ is the electrostatic potential, and G is the gravitational constant. Equations (1)–(4) are the governing equations of our model.

Since we are interested in the effect of adiabatic trapping of the charged particles by an electrostatic potential, we consider two cases separately here, i.e., first when the dust is negatively charged and ions are trapped and second when the dust is positively charged and electrons are trapped. As mentioned in the Introduction, we follow the method outlined in Landau and Lifshitz[60] where the trapped particle distribution is divided into free and trapped parts and the number densities are calculated by taking the relevant distributions into account.

Thus, we consider both the cases: (i) when dust is negatively charged, ions are adiabatically trapped, and electrons are considered Boltzmannian and (ii) when dust is positively charged, electrons are adiabatically trapped, and ions are assumed to be Boltzmannian. Following Ref. 68, we obtain the following expressions for the densities of the trapped ions and Boltzmannian electrons:

$$n_e = n_{e0}e^{\frac{e\psi}{T_e}}, \tag{5}$$

$$n_i = n_{i0}\left(1 - \frac{e\psi}{T_i} + \alpha\left(\frac{e\psi}{T_i}\right)^{\frac{3}{2}}\right), \tag{6}$$

where $\alpha = \frac{4}{3\sqrt{\pi}}$. On the other hand when the dust is positively charged, we consider the adiabatic trapping of electrons and assume that ions follow the Boltzmann distribution given by

$$n_i = n_{i0}e^{-\frac{e\psi}{T_i}}. \tag{7}$$

The trapped density of electrons is given by[60]

$$n_e = n_{e0}\left(1 + \frac{e\psi}{T_e} + \alpha\left(\frac{e\psi}{T_e}\right)^{\frac{3}{2}}\right). \tag{8}$$

The current density is given by[57]

$$\partial_z j_z = e\partial_t n_e - e\partial_t n_i - e\partial_z(z_d n_d v_{dz}). \tag{9}$$

III. LINEAR DISPERSION RELATION

To derive the linear dispersion relation, we ignore the inertia of electrons and ions in comparison to the dust mass for coupled dust kinetic Alfven-acoustic waves and obtain the following dispersion relation from Eqs. (1)–(6):

$$(\sigma_{id}+\gamma\sigma_{ed})\left[1-\frac{k_z^2c_{sd}^2}{\omega^2}\right]\left[1-\frac{k_z^2v_{Ad}^2}{\omega^2}\right] \mp \frac{k_z^4\, v_{Ad}^2\omega_{jd}^2\beta_d z_d}{\omega^4}\left[1-\frac{k_x^2\,\omega^2}{k_z^2\,\Omega_d^2}-\frac{\omega^2}{k_z^2\,v_{Ad}^2}\right] \times\left[\frac{\frac{\omega^2}{k_z^2}\frac{\sigma_{ed}}{\beta_d\, z_d\, v_{Ad}^2}(1+\gamma)\mp\frac{k_x^2v_{Ad}^2}{\Omega_d^2}\pm\frac{k_z^2v_{Ad}^2}{\omega^2}-1+\frac{\omega^2}{k_z^2}\frac{\gamma}{c_{sd}^2}}{k_x^2\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)+k_z^2\left(1+\frac{\omega_{jd}^2}{\omega^2}\right)\mp\frac{\omega_{jd}^2}{v_{Ad}^2}}\right] = \lambda_s\frac{k_z^2\,v_{Ad}^2}{\omega^2}. \tag{10}$$

In Eq. (10) and thereafter (specified otherwise), the upper sign corresponds to negative dust while the lower sign corresponds to positive dust. Equation (10) represents the dispersion relation for coupled dust (positive as well as negative) kinetic Alfven-acoustic waves in the self-gravitating dusty plasma. Here, $c_{sd}=\sqrt{\frac{z_dk_BT_i}{m_d}}$ is the dust acoustic speed, $v_{Ad}=\sqrt{\frac{B_0^2}{\mu_0 n_{d0} m_d}}$ is the dust Alfven speed, $\omega_{jd}=\sqrt{\frac{1}{\varepsilon_0}Gm_d n_{d0}}$ is the Jeans frequency, whereas $\sigma_{ed}=\frac{n_{e0}}{n_{d0}}$ and $\sigma_{id}=\frac{n_{i0}}{n_{d0}}$ are the ratios of unperturbed electron and ion densities to the unperturbed dust density, $\gamma=\frac{T_i}{T_e}$ is the ion to electron temperature ratio, and $\lambda_s=\frac{z_dk_x^2c_{sd}^2}{\Omega_d^2}$ is the coupling parameter here and $\Omega_d=\frac{q_dB_0}{m_d}$. Here, we note that in the linear dispersion relation, the trapping effect cannot be seen since trapping is a nonlinear phenomenon.

If we neglect the Jeans term, the dispersion relation for the coupled dust Kinetic Alfven-acoustic wave [(given by Eq. (10)] can be written in dimensionless form as

$$\begin{aligned} D_+^2 &= \frac{(\beta_d+\sigma_{id}+\gamma\,\sigma_{ed}+z_dK_x^2)+\sqrt{(\beta_d+\sigma_{id}+\gamma\,\sigma_{ed}+z_dK_x^2)^2-4\,\beta_d(\sigma_{id}+\gamma\,\sigma_{ed})^2}}{2(\sigma_{id}+\gamma\,\sigma_{ed})},\\ D_-^2 &= \frac{(\beta_d+\sigma_{id}+\gamma\,\sigma_{ed}+z_dK_x^2)-\sqrt{(\beta_d+\sigma_{id}+\gamma\,\sigma_{ed}+z_dK_x^2)^2-4\,\beta_d(\sigma_{id}+\gamma\,\sigma_{ed})^2}}{2(\sigma_{id}+\gamma\,\sigma_{ed})}, \end{aligned} \tag{11}$$

where $\beta_d=\frac{z_d\mu_0 n_{d0}k_BT_i}{B_0^2}=c_{sd}^2/v_{Ad}^2$, $\lambda_s=z_d\frac{c_{sd}^2}{\Omega_d^2}k_x^2=z_dK_x^2$, $D_{+,-}=\omega/k_zv_{Ad}$. In the absence of the Jeans term, i.e., $\omega_{jd}=0$ and by putting $m_d\rightarrow m_i$, $n_{d0}\rightarrow n_{i0}$, we obtain the same expression as obtained for the kinetic Alfven wave,[51] i.e.,

$$\omega^2=k_z^2\,v_A^2(1+\lambda_s). \tag{12}$$

IV. STABILITY ANALYSIS WITH JEANS TERM

In this section, stability analysis is performed from the dispersion relation [Eq. (10)], which can be written as

$$\omega^6+a1\,\omega^4+a2\,\omega^2+a3=0, \tag{13}$$

where

$$\begin{aligned} a1 = {} & \Bigg\{\pm\lambda_s\,k_z^2\,\omega_{jd}^2-\lambda_s\,k_z^4\,v_{Ad}^2-\lambda_s\,k_x^2\,k_z^2\,v_{Ad}^2\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)-\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)(1+\beta_d)C\,v_{Ad}^2\,k_x^2\,k_z^2+\omega_{jd}^2\,k_z^2C-(1+\beta_d)\,k_z^4\,v_{Ad}^2C\\ & \mp k_z^2\,\omega_{jd}^2A\mp z_dc_{sd}^2\,k_z^2\,\omega_{jd}^2B-B\,\frac{z_dc_{sd}^2v_{Ad}^2\,k_z^2\,k_x^2\omega_{jd}^2}{\Omega_d^2}\pm k_z^2\,\omega_{jd}^2C(1+\beta_d)\Bigg\}\Bigg/\left\{\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)C\,k_x^2\mp C\frac{\omega_{jd}^2}{v_{Ad}^2}+AB\,\omega_{jd}^2+C\,k_z^2\right\},\\ a2 = {} & \Bigg\{C\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)k_x^2c_{sd}^2v_{Ad}^2\,k_z^4-\lambda_sv_{Ad}^2\omega_{jd}^2\,k_z^4-(1+\beta_d)C\,\omega_{jd}^2\,k_z^4v_{Ad}^2+C\,v_{Ad}^2c_{sd}^2\,k_z^6\mp c_{sd}^2\omega_{jd}^2\,k_z^4\,C\pm z_dc_{sd}^2\omega_{jd}^2\,k_z^4\\ & +\frac{z_dc_{sd}^2v_{Ad}^2\omega_{jd}^2\,k_x^2\,k_z^4}{\Omega_d^2}+Bz_dc_{sd}^2v_{Ad}^2\omega_{jd}^2\,k_z^4\Bigg\}\Bigg/\left\{\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)C\,k_x^2\mp C\frac{\omega_{jd}^2}{v_{Ad}^2}+AB\,\omega_{jd}^2+C\,k_z^2\right\},\\ a3 = {} & \left\{v_{Ad}^2c_{sd}^2\omega_{jd}^2\,k_z^6C-z_dc_{sd}^2v_{Ad}^2\omega_{jd}^2\,k_z^6\right\}\Bigg/\left\{\left(1-\frac{\omega_{jd}^2}{\Omega_d^2}\right)C\,k_x^2\mp C\frac{\omega_{jd}^2}{v_{Ad}^2}+AB\,\omega_{jd}^2+C\,k_z^2\right\},\\ & A=\sigma_{ed}(1+\gamma)+\gamma\,z_d,\quad B=\frac{1}{v_{Ad}^2}+\frac{k_x^2}{\Omega_d^2},\quad C=(\sigma_{id}+\gamma\,\sigma_{ed}), \end{aligned}$$

are the coefficients. If $(a1\,a2-a3)>0$, then the system is Jeans stable.[70] Conversely, the system will become Jeans unstable if $(a1\,a2-a3)<0$.

V. SAGDEEV POTENTIAL

In this section, the Sagdeev potential is derived without the Jeans term for a wave propagating obliquely to the ambient magnetic field by writing the set of Eqs. (1)–(8) in the nonlinear form. For this, we move to a co-moving frame which in a normalized form is given by[57]

$$\eta = K_x x + K_z z - M_a t. \tag{14}$$

We also use the following normalized variables:

$$n_{e,i,d} = \frac{n_{e,i,d}}{n_0}, \quad \Psi = \frac{e\psi}{T_i}, \quad \Phi = \frac{e\phi}{T_i}, \quad \varphi = \frac{\varphi}{c_s^2},$$
$$M_a = \frac{v}{v_{Ad}}, \quad K = k\frac{c_s}{\Omega_d}, \quad t = \Omega_d t,$$

where K, Ψ, n, t, M_a are the wave number, potential, normalized density, normalized time, and Mach number, respectively.

From Ampere's law,[55] we can write

$$K_x^2 K_z^2 \partial_\eta^4(\Phi - \Psi) = \frac{\beta_d}{z_d}\left(-\mathrm{M}_a^2\partial_\eta^2(\mathrm{z_d n_d}) + M_a K_z \partial_\eta^2(\mathrm{z_d n_d v_{dz}})\right). \tag{15}$$

By taking the parallel component of the equation of motion, we can have

$$-M_a\frac{\partial \mathrm{v_{dz}}}{\partial\eta} + \mathrm{v_x}K_x\frac{\partial \mathrm{v_{dz}}}{\partial\eta} + \mathrm{v_z}K_z\frac{\partial \mathrm{v_{dz}}}{\partial\eta} = K_z\frac{\partial\Psi}{\partial\eta} - K_z\frac{\partial\varphi}{\partial\eta}. \tag{16}$$

Integrating Eqs. (15) and (16) and using the boundary conditions such that $(\mathrm{v_{dx}}, \mathrm{v_{dz}}, \Phi, \Psi) \to 0$ as $\eta \to \infty$ and dropping the gravitational potential term, we obtain

$$\begin{aligned} &-\frac{\beta_d K_z^2}{z_d M_{Ad}^2}\left[(\sigma_{id} - \sigma_{ed})^2\Psi\right] + \frac{1}{z_d}\left[(\sigma_{id} + \gamma\,\sigma_{ed})\Psi\right. \\ &\quad \left. + \alpha\,\sigma_{ed}\,\gamma^{3/2}\,\Psi^{3/2}\right] - \frac{M_{Ad}^2}{z_d K_z^2}\left[(\sigma_{id} - \sigma_{ed})(\sigma_{id} + \gamma\sigma_{ed})\Psi\right. \\ &\quad \left. + \alpha\,\sigma_{ed}\gamma^{3/2}(\sigma_{id} - \sigma_{ed})\Psi^{3/2}\right] + \frac{\beta_d}{z_d^2}\left[(\sigma_{id} - \sigma_{ed})^3\Psi\right] \\ &= (\sigma_{id} - \sigma_{ed})K_x^2\partial_\eta(\partial_\eta\Psi). \end{aligned} \tag{17}$$

Here, $M_{Ad}^2 = M_a^2\beta_d$. After integrating and using the boundary conditions described earlier, we get the energy integral equation in the following manner for positive as well as negative dust charges.

$$\frac{1}{2}\left(\frac{d\Psi}{d\eta}\right)^2 + V(\Psi) = 0, \tag{18}$$

where $V(\Psi)$ is the Sagdeev potential and is given by

$$\begin{aligned} V(\Psi) = -\frac{1}{K_x^2}&\left[\pm\frac{\beta_d}{2}\left(1 \mp \frac{K_z^2}{M_{Ad}^2}\right)^2\Psi^2 + \frac{1}{2z_d}(\sigma_{id} + \gamma\,\sigma_{ed})\right. \\ &\left.\times\left\{1 - \frac{M_{Ad}^2}{K_z^2}\right\}\Psi^2 + \frac{2}{5z_d}\alpha\,\sigma_{ed}\,\gamma^{\frac{3}{2}}\Psi^{\frac{5}{2}}\left\{1 - \frac{M_{Ad}^2}{K_z^2}\right\}\right]. \end{aligned} \tag{19}$$

In order to have the soliton solution, the Sagdeev potential (19) must satisfy the conditions;[72] (i) $V(\Psi) = \frac{dV(\Psi)}{d\Psi} = 0$ and $\frac{d^2V(\Psi)}{d\Psi^2} < 0$ at $\Psi = 0$, (ii) $\frac{dV(\Psi)}{d\Psi} > 0$ at $\Psi = \Psi_{max}$ and (iii) $V(\Psi) = 0$ when $0 < \Psi < \Psi_{max}$. The first condition implies that the fixed point is unstable at the origin and gives a lower bound on the Mach number. The same can also be obtained by Taylor expanding the Sagdeev potential (19) in Ψ and putting the coefficient of the second order term equal to zero. The upper limit on Mach number can be found out from the second condition. Thus, in order to obtain the soliton solution, the Mach number M_{Ad} must satisfy the following inequality in terms of different constants, such as α, γ, etc.:

$$\frac{k_z z_d\sqrt{\gamma\,\beta_d}}{\sqrt{2\gamma\sigma_{ed} + 2\sigma_{id}}} < M_{Ad} < \frac{k_z z_d\sqrt{5\gamma\beta_d}}{\sqrt{10\gamma\sigma_{ed} + 8\alpha\gamma\sigma_{ed} + 10\sigma_{id}}}. \tag{20}$$

The above range of Mach number defines the existence regimes of the formation of solitary structures in dusty plasmas. It can readily be seen that the inclusion of dust component modifies the upper and lower limits of the existence regime of the formation of electrostatic solitary structures under investigation here.

VI. RESULTS AND DISCUSSION

In this section, we first investigate the linear dispersion relation for the coupled dust kinetic Alfven-acoustic waves [i.e., Eq. (10)] for the case of negative dust when the negative dust number density and magnetic field values are 10^9 m^{-3} and 10^{-4}T, respectively.[71] In Fig. 1, real frequency is plotted against the normalized wave number for different values of magnetic field. We note that first the wave frequency increases directly with the wave number and its slope increases with the increase in the magnetic field strength and then the wave frequency becomes constant when $\frac{k_z c_{sd}}{\Omega_d} > 0.01$.

In Fig. 2, real frequency is plotted against the wave number by varying the values of negative dust number density. We can see that the real frequency increases with the wave number but remains higher for the smaller values of number density. Figure 3 is plotted for the growth rate

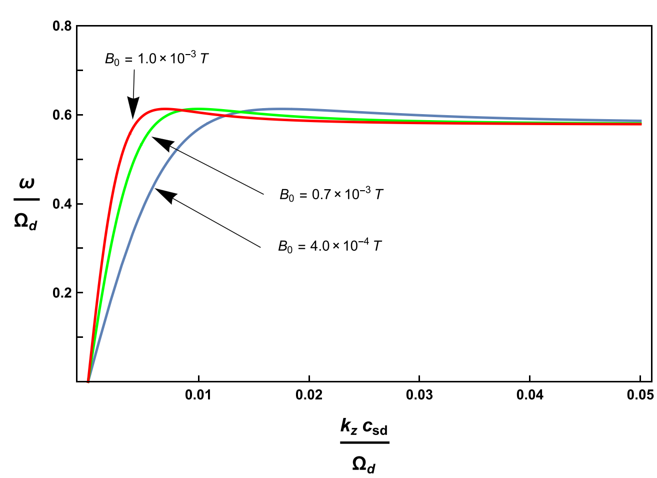

FIG. 1. Dispersion relation for normalized frequency vs. normalized wave number for different values of magnetic field for negatively charged dust.

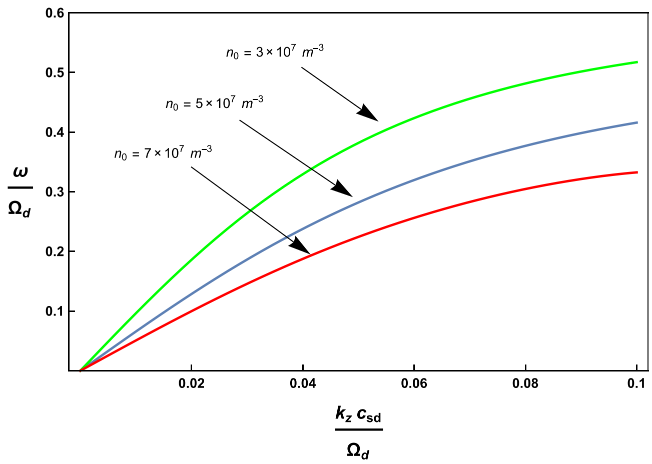

FIG. 2. Dispersion relation for normalized frequency vs. normalized wave number for different values of number density for negatively charged dust.

against the normalized wave number for different values of negative dust number density. We can see that increase in the number density enhances the growth rate and widens the range of wave number over which the wave grows.

We investigate the Sagdeev potential [given by Eq. (19)] and corresponding solitary solutions numerically for different plasma parameters. Figure 4 depicts the profiles of Sagdeev potential for different values of Mach number when $B_0 = 0.2\times10^{-4}$ T and $n_{d0} = 10^7$ m^{-3}. We find that when the Mach number increases, the depth as well as the maximum value of the potential experiences an enhancement. In Fig. 5, soliton structures are plotted corresponding to the Sagdeev potential in Fig. 4 for the same values of Mach number. We can see that as the Mach number increases, the amplitude of the soliton increases.

We now investigate the linear dispersion relation for the coupled dust Kinetic Alfven-acoustic waves [given by Eq. (10)] for the case of positive dust when the positive dust number density and magnetic field values are 10^5 m^{-3} and 10^{-4} T, respectively.[48] In Fig. 6, real frequency is plotted against the wave number for different values of magnetic field. We can see that the wave frequency first increases with the wave number and then becomes constant when the wave number

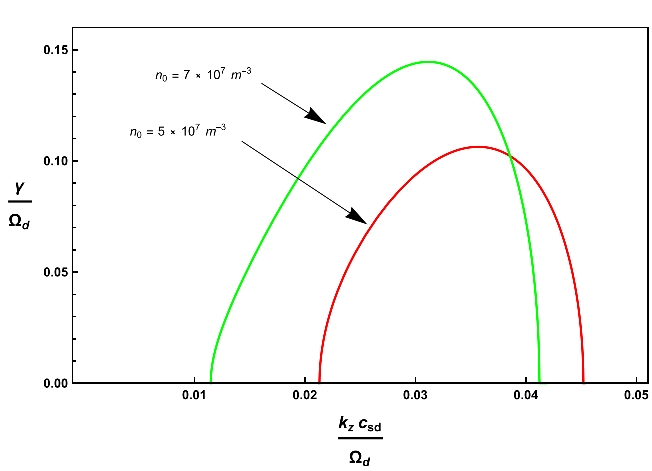

FIG. 3. Growth rate vs. normalized wave number for different values of number density for negatively charged dust.

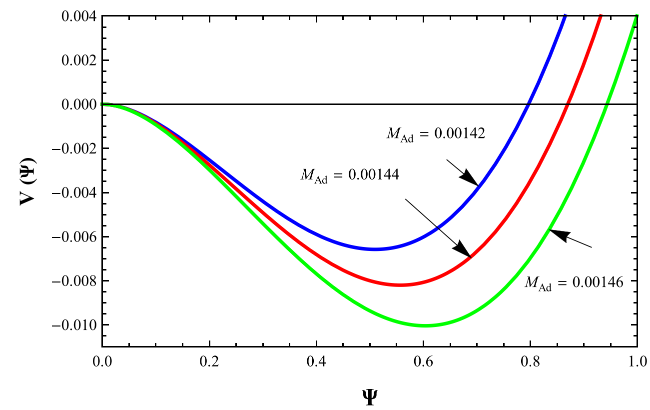

FIG. 4. Sagdeev potential $V(\Psi)$ versus Ψ for different values of M_{Ad} when $B_0 = 0.2\times10^{-4}$ T and $n_{d0} = 10^7$ m^{-3}.

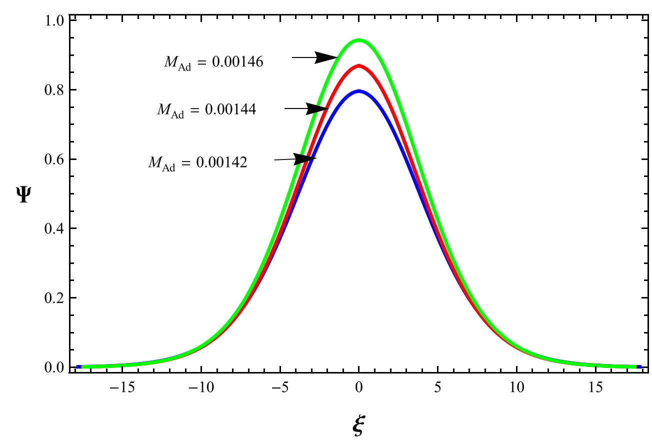

FIG. 5. Solitary structures corresponding to the Sagdeev potential $V(\Psi)$ shown in Fig. 3.

becomes $\frac{k_z c_{sd}}{\Omega_d} > 0.1$; however, the frequency remains higher for a larger value of magnetic field. In Fig. 7, the real frequency is plotted against the wave number by varying the values of positive dust number density. It can be seen that the real frequency increases with the wave number but remains higher for smaller values of number density.

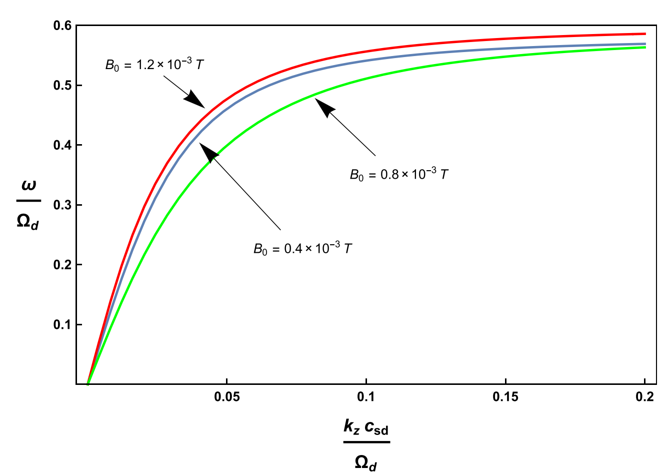

FIG. 6. Dispersion relation for normalized frequency vs. normalized wave number for different values of magnetic field for positively charged dust.

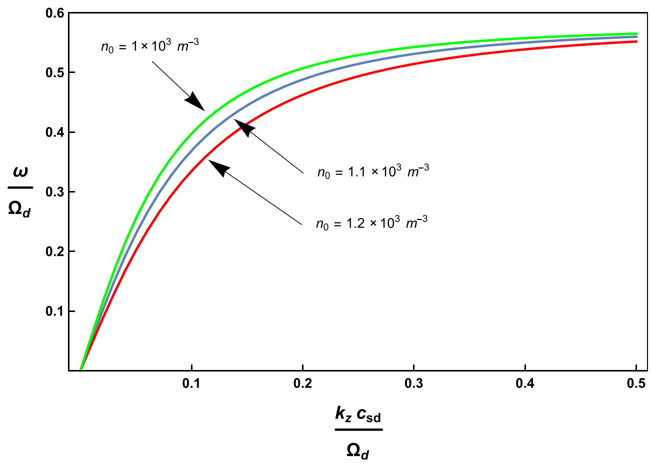

FIG. 7. Dispersion relation for normalized frequency vs. normalized wave number for different values of number density for positively charged dust.

Figure 8 depicts the profiles of Sagdeev potential for different values of Mach number when $B_0 = 0.2 \times 10^{-4}$ T, $\theta = \pi/3$, and $n_{d0} = 10^3\,\mathrm{m}^{-3}$. It is found that as the Mach number increases, the depth as well as the maximum value of the potential increases. In Fig. 9, soliton structures are plotted corresponding to the Sagdeev potentials in Fig. 8 for the same values of Mach number. We find that the increase in Mach number enhances the amplitude of the soliton.

Figure 10 shows the profiles of Sagdeev potential for different values of angle of propagation θ for $B_0 = 0.2 \times 10^{-4}$ T, $M_{Ad} = 0.0000032$, and $n_{d0} = 10^3\,\mathrm{m}^{-3}$. It is found that as the angle of propagation increases, the depth as well as the maximum value of the potential also increases. In Fig. 11, soliton structures are plotted corresponding to the Sagdeev potentials in Fig. 10 for the same values of angle of propagation. It can be seen that an increase in angle enhances the amplitude and width of the soliton.

Finally, it is imperative that we highlight the important physical difference between dusty plasmas and ordinary e-i plasmas. The gigantic dust mass by comparison with ions and electrons introduces new spatio-temporal scales. Note that the frequencies being considered here are very slow compared with ordinary e-i plasmas and the dust acoustic Larmor radius is very large compared with the ion acoustic Larmor radius. This fact has an important ramification,

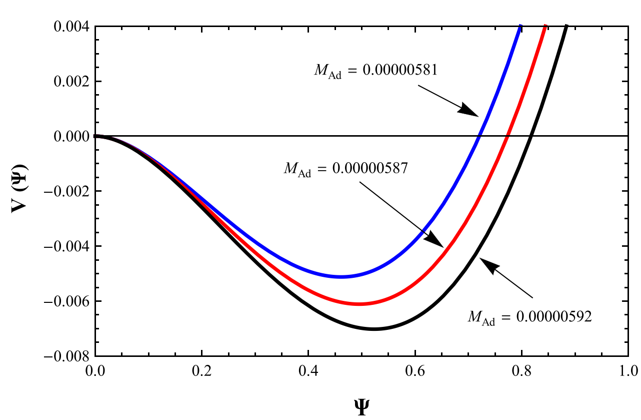

FIG. 8. Sagdeev potential $V(\Psi)$ versus Ψ for different values of M_{Ad} when $\theta = \pi/3$ and $n_{d0} = 10^3\ \mathrm{m}^{-3}$.

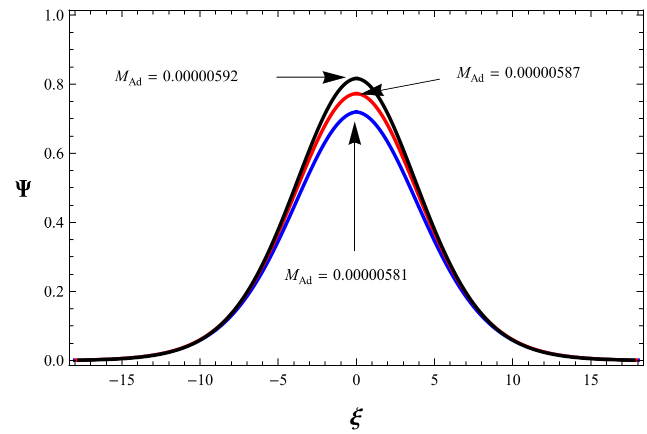

FIG. 9. Solitary structures corresponding to the Sagdeev potential $V(\Psi)$ shown in Fig. 7.

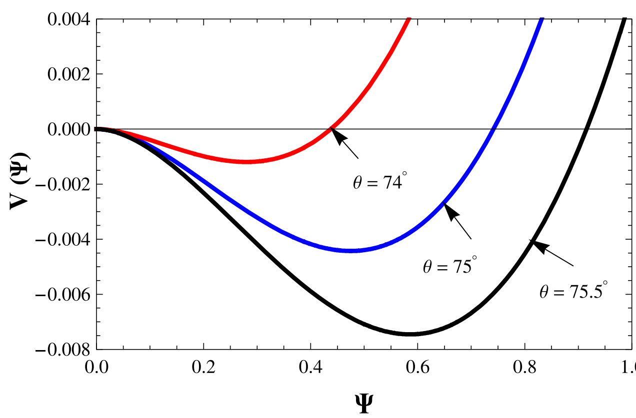

FIG. 10. Sagdeev potential $V(\Psi)$ versus Ψ for different values of θ when $M_{Ad} = 0.00000302$ and $n_{d0} = 10^3\ \mathrm{m}^{-3} n_{d0} = 10^3 m^{-3}$.

namely, that the spatial scale lengths over which the solitary structures form in dusty plasmas are much longer by comparison with their counterparts in the e-i plasmas.

VII. CONCLUSION

In this paper, we have studied the nonlinear coupling of kinetic Alfven waves with the acoustic waves in a self-

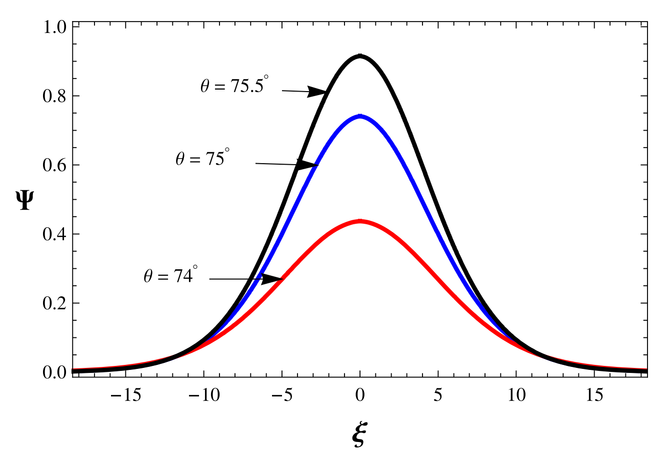

FIG. 11. Solitary structures corresponding to the Sagdeev potential $V(\Psi)$ shown in Fig. 9.

gravitating dusty plasma by considering separately, adiabatically trapped electrons and ions, and taking into account both positive and negative dust. The linear dispersion relation for the coupled dust kinetic Alfven-acoustic wave has been investigated by considering negatively as well as positively charged dust, and different limiting cases have also been discussed. It has been seen that for both negative and positive dust, the real frequency is larger for larger magnetic field strength; however, it mitigates with the increase in dust number density. Stability analysis of the coupled dust kinetic Alfven-acoustic wave has also been presented. It has been found that the increase in dust number density enhances the growth rate due to Jeans instability and also widens the range of wave numbers over which the wave grows. The Sagdeev potential has been obtained by ignoring the gravitational effects for both negative and positive dust and has also been numerically studied for different plasma parameters and the corresponding solitary structures have been shown. We have found that solitons can only be formed in the sub-Alfvenic regime for both negative and positive dust cases. The amplitude and width of the soliton increase with Mach number in both cases. Our study is applicable in a variety of space and astrophysical plasmas where positive and negatively charged dust is present such as those found in the planetary magnetospheres.

ACKNOWLEDGMENTS

This research was supported by the Higher Education Commission (HEC) Grant No. 20-2595/NRPU/R&D/HEC/13. We would like to express our gratitude to the anonymous referee for his constructive criticism which has helped us enormously to improve the manuscript.

[1]V. E. Zakharov, Sov. Phys. - JETP **35**, 908 (1972).
[2]R. Z. Sagdeev, *Review of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1996), Vol. 4.
[3]P. K. Shukla, N. N. Rao, M. Y. Yu, and N. L. Tsintsadze, Phys. Rep. **138**, 1 (1986).
[4]P. K. Shukla, N. L. Tsintsadze, and L. N. Tsintsadze, Phys. Fluids B **5**, 233 (1993).
[5]N. L. Tsintsadze, Astrophys. Space Sci. **191**, 151 (1992).
[6]V. I. Berezhiani, N. L. Tsintsadze, and P. K. Shukla, J. Plasma Phys. **48**, 139 (1992).
[7]N. L. Tsintsadze and P. K. Shukla, Phys. Lett., A **187**, 67 (1994).
[8]L. N. Tsintsadze and V. I. Berezhiani, Plasma Phys. Rep. **19**, 132 (1993).
[9]N. L. Tsintsadze, Phys. Scr. **50**, 413 (1994).
[10]S. Kartal, L. N. Tsintsadze, and V. I. Berezhiani, Phys. Rev. E **53**, 4225 (1996).
[11]M. Lontano, S. Bulanov, J. Koga, and M. Passoni, Phys. Plasmas **9**, 2562 (2002).
[12]N. L. Tsintsadze, Phys. Plasmas **2**, 4462 (1995).
[13]N. L. Tsintsadze, K. Nishikawa, T. Tajima, and J. T. Mendonca, Phys. Rev. E **60**, 7435 (1999).
[14]J. Vranjes, H. Saleem, and S. Poedts, Planet. Space Sci. **50**, 807 (2002).
[15]H. R. Prabhakara and V. L. Tanna, Phys. Plasmas **3**, 3176 (1996).
[16]J. B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).
[17]R. L. Merlino, A. Barkan, and N. D' Angelo, Phys. Plasmas **5**, 1607 (1998).
[18]V. E. Fortov, A. G. Khrapak, S. A. Khrapak, V. I. Molotkov, A. P. Nefedov, O. F. Petrov, and V. M. Torchinsky, Phys. Plasmas **7**, 1374 (2000).
[19]D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).
[20]P. K. Shukla, Phys. Plasmas **10**, 1619 (2003).
[21]P. K. Shukla, Phys. Plasmas **8**, 1791 (2001).
[22]O. Ishihara, J. Phys. D: Appl. Phys. **40**, R121 (2007).
[23]G. E. Morfill and A. Ivlev, Rev. Mod. Phys. **81**, 1353 (2009).
[24]U. De Angelis, V. Formisano, and M. Giordano, J. Plasma Phys. **40**, 399 (1988).
[25]U. De Angelis, Phys. Scr. **45**, 465 (1992).
[26]C. K. Goertz, Rev. Geophys. **27**, 271, doi:10.1029/RG027i002p00271 (1989).
[27]D. A. Gurnett, E. Grun, D. Gallagher, W. S. Kurth, and F. L. Scarf, Icarus **53**, 236 (1983).
[28]D. A. Gurnett, J. A. Ansher, W. S. Kurth, and L. J. Granroth, Geophys. Res. Lett. **24**, 3125, doi:10.1029/97GL03228 (1997).
[29]S. A. Collins, A. F. Cook, J. N. Cuzzi, G. E. Danielson, G. E. Hunt, T. V. Johnson, D. Morrison, T. Owen, J. B. Pollack, B. A. Smith, and R. J. Terrile, Nature **288**, 439 (1980).
[30]C. A. McGhee, R. G. French, L. Dones, J. N. Cuzzi, H. J. Salo, and R. Danos, Icarus **173**, 508 (2005).
[31]C. J. Mitchell, M. Horanyi, O. Havnes, and C. C. Porco, Science **311**, 1587 (2006).
[32]H. Kamaya and R. Nishi, Astrophys. J. **543**, 257 (2000).
[33]T. Nakano, R. Nishi, and T. Umebayashi, Astrophys. J. **573**, 199 (2002).
[34]V. Prudskikh and Yu. A. Shchekinov, Phys. Lett. **372**, 2671 (2008).
[35]D. A. Mendis, Astrophys. Space Sci. **176**, 163 (1991).
[36]T. Nakano, Astrophys, J. **494**, 587 (1998).
[37]J. Winter, Plasma Phys. Controlled Fusion **40**, 1201 (1998).
[38]E. Zweibel, Phys. Plasmas **6**, 1725 (1999).
[39]N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).
[40]P. K. Shukla and V. P. Silin, Phys. Scr. **45**, 508 (1992).
[41]J. Vranjes, Phys. Plasmas **18**, 084501 (2011).
[42]A. Barkan, R. L. Merlino, and N. D' Angelo, Phys. Plasmas **2**, 3563 (1995).
[43]N. Hayashi, Phys. Plasmas **8**, 3051 (2001).
[44]N. N. Rao, Planet. Space Sci. **41**, 21 (1993).
[45]N. N. Rao, J. Plasma Phys. **53**, 317 (1995).
[46]F. Verheest, P. K. Shukla, N. N. Rao, and P. Meuris, J. Plasma Phys. **58**, 163 (1997).
[47]F. Verheest, Space Sci. Rev. **77**, 267 (1996).
[48]H. Saleem, W. M. Moslem, and P. K. Shukla, J. Geophys. Res. **117**, A08220, doi:10.1029/2011JA017306 (2012).
[49]T. K. Baluku, M. A. Hellberg, and R. L. Mace, Phys. Plasmas **15**, 033701 (2008).
[50]A. Hasegawa and C. Uberoi, *The Alfven Wave* (Technical Information Centre, U. S. Department of Energy, Washington, DC, 1982).
[51]N. F. Cramer, *The Physics of Alfven Waves* (Wiley-VCH, Berlin, 2001).
[52]N. Rubab, N. V. Erkaev, D. Langmayr, and H. K. Biernat, Phys. Plasmas **17**, 103704 (2010).
[53]B. B. Kadomtsev, *Plasma Turbulence* (Academic Press, New York, 1965), p. 82.
[54]A. Hasegawa and L. Chen, Phys. Rev. Lett. **35**, 370 (1975).
[55]A. Hasegawa and K. Mima, Phys. Rev. Lett. **37**, 690 (1976).
[56]M. Y. Yu and P. K. Shukla, Phys. Fluids **21**, 1457 (1978).
[57]H. Kakati and K. S. Goswami, Phys. Plasmas **5**, 4229 (1998).
[58]C. Yinhua, L. Wei, and M. Y. Yu, Phys. Rev. E **61**, 809 (2000).
[59]A. V. Gurevich, Zh. Eksp. Teor. Fiz. **53**, 953 (1967) [Sov. Phys. - JETP **26**, 575 (1968) (in Russian)].
[60]L. D. Landau and E. M. Lifshitz, *Physical Kinetics* (Pergamon Press, New York, 1981).
[61]N. S. Erokhin, N. N. Zolnikova, and L. A. Mikhailovskaya, Fiz. Plazmy **22**, 137 (1996).
[62]R. Z. Sagdeev, *Review of Plasma Physics* (Consultants Bureau, New York, 1996), Vol. 4.
[63]C. Hansen, A. B. Reimann, and J. Fajans, Phys. Plasmas **3**, 1820 (1996).
[64]H. Abbasi, N. L. Tsintsadze, and D. D. Tshakaya, Phys. Plasmas **6**, 2373 (1999).
[65]A. Mushtaq and H. A. Shah, Phys. Plasmas **13**, 012303 (2006).
[66]H. Siddiqui, H. A. Shah, and N. L. Tsintsadze, J. Fusion Energy **27**, 216 (2008).
[67]M. Ayub, H. A. Shah, and M. N. S. Qureshi, Phys. Scr. **84**, 045505 (2011).
[68]H. A. Shah, W. Masood, and Z. Ali, Phys. Plasmas **20**, 032301 (2013).
[69]A. Sabeen, H. A. Shah, W. Masood, and M. N. S. Qureshi, Astrophys. Space Sci. **355**, 225 (2015).
[70]L. A. Pipes and L. R. Harvill, *Applied Mathematics for Engineers and Physicists* (McGraw-Hill, Japan, 1971).
[71]W. Masood, A. M. Mirza, A. Ijaz, and Q. Haque, Astrophys Space Sci. **349**, 829–834 (2014).
[72]H. A. Shah, M. N. S. Qureshi, and N. Tsintsadze, Phys. Plasmas **17**, 032312 (2010).