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Nonlinear density excitations in electron-positron-ion plasmas with trapping in a quantizing magnetic field

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In the present work, we have investigated the effect of trapping as a microscopic phenomenon on the formation of solitary structures in the presence of a quantizing magnetic field in an electron-positron-ion (e-p-i) plasma having degenerate electrons and positrons, whereas ions are taken to be classical and cold. We have found that positron concentration, quantizing magnetic field, and finite electron temperature effects not only affect the linear dispersion characteristics of the electrostatic waves under consideration but also have a significant bearing on the propagation of solitary structures in the nonlinear regime. Importantly, the system under consideration has been found to allow the formation of compressive solitary structures only. The work presented here may be beneficial to understand the propagation of nonlinear electrostatic structures in dense astrophysical environments and in intense-laser plasma interactions. *Published by AIP Publishing.*
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Three-component plasmas having ions with electrons and positrons are ubiquitous in stellar atmospheres. Multi-component plasmas exhibit different properties as compared to ordinary electron-ion plasmas. The conditions under which the positron annihilation process can be neglected were investigated by Ali *et al.*[1] It was found that, at a particular number density, the time for the annihilation process to take place is much longer than the collective interaction time for the charged particles. Such densities are found in white dwarf stars, and therefore, it is reasonable to neglect the annihilation of electrons and positrons.

Quantum or degenerate plasmas involving the collective interactions have attracted a lot of interest in the recent years owing to their potential applications in many situations of interest such as semiconductors, metals and microelectronics,[2] quantum wells, quantum dots, and carbon nanotubes.[3–5] Degenerate plasmas are found to play an important role in dense astrophysical objects like white dwarfs and neutron stars.[6] A substantial volume of literature related to quantum or degenerate plasmas is available, and linear and nonlinear propagation characteristics of electrostatic and electromagnetic modes have been discussed.[7–10] A vast majority of this work is based on the quantum hydrodynamic model.[11]

The effect of the strong ambient magnetic field has received some attention in degenerate plasmas. The properties of atoms, molecules, and condensed matter are qualitatively changed in the presence of the ambient strong magnetic field when the electron cyclotron energy is greater than the typical coulomb energy.[12] Due to the extreme confinement of electrons in the transverse direction, the Coulomb force becomes more effective in binding the electrons along the direction of the magnetic field. In this case, the Zeeman splitting of atomic energy levels is not considered as the perturbation effect in the strong field but instead, the Coulomb forces act as the perturbation to the magnetic forces.[13]

Landau diamagnetism or quantization is that the orbital motion of electrons in a magnetic field is quantized.[14] The gas is degenerate for temperature $T \ll \varepsilon_F$ (ε_F is the Fermi energy). The magnetic field responsible for Landau quantization of the electron motion is said to be quantizing.[15]

Bernstein, Green, and Kruskal showed that the nonlinear dynamics of the plasma gets significantly modified by the trapped particles while the trapping is considered by the wave itself.[16] A decade later, Gurevich[17] developed a new approach considering the effect of adiabatic trapping at the microscopic level. It was observed that the 3/2 power nonlinearity was introduced due to adiabatic trapping instead of quadratic nonlinearity without trapping. Trapping as a microscopic phenomenon was confirmed by experimental investigations[18] and computer simulations.[19] Trapping in quantum plasmas was considered by Shah *et al.*[20] using the Gurevich approach, and the ion acoustic solitary structures are investigated for fully and partially degenerate plasmas. Later on, this work was extended to relativistic degenerate plasmas and Landau quantization.[21–24]

In this brief communication, we derive an expression for the parallel propagating nonlinear electrostatic ion acoustic waves in the presence of adiabatic trapping in a quantizing magnetic field in electron-positron-ion plasmas. Electrons and positrons are treated quantum mechanically due to their tenuous masses by comparison with heavy ions. For the interest of readers, it is mentioned that the magnetic field (taken along the z-axis) is included here since we are interested in investigating the effect of Landau quantization. Using the standard calculations and Fermi-Dirac distribution function from the following Refs. 14 and 22, we obtain the following expression for the electron number density

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$$n_e = N_{e0}\left[\frac{3}{2}\eta(1+\Phi)^{\frac{1}{2}} + (1+\Phi-\eta)^{3/2} - \frac{\eta T^2}{2}(1+\Phi)^{-\frac{3}{2}} + T^2(1+\Phi-\eta)^{-1/2}\right]. \quad (1)$$

The normalizations used here are given by $N_{e0} = p_{Fe}^3/3\pi^2\hbar^3$, $\eta = \hbar\omega_{ce}/\varepsilon_{Fe}, \Phi = e\varphi/\varepsilon_{Fe}$, and $T = \pi T/2\sqrt{2}\varepsilon_{Fe}$. Here, n_e is the total electron number density. Simplification of Eq. (1) yields the following equations:

$$n_{e0} = N_{e0}\left\{(3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2}\right\}, \quad (2)$$

$$n_{e1} = N_{e0}\left\{(1+T^2)\frac{3}{4}\eta + \frac{3}{2}(1-\eta)^{1/2} - \frac{T^2}{2}(1-\eta)^{-3/2}\right\}\Phi, \quad (3)$$

here, n_{e0} is the equilibrium electron density in the absence of perturbation. And n_{e1} is the perturbed electron number density. Similarly, the total positron number density in the presence of the quantizing magnetic field is

$$n_p = N_{0p}\left[\frac{3}{2}\eta\delta^{-2/3}(1-\delta^{-2/3}\Phi)^{\frac{1}{2}} + \left(1-\delta^{-2/3}\Phi - \delta^{-2/3}\eta\right)^{3/2} - \frac{\delta^{-2}\eta T^2}{2}(1-\delta^{-2/3}\Phi)^{-\frac{3}{2}} + \delta^{-4/3}T^2 \times \left(1-\delta^{-2/3}\Phi-\delta^{-2/3}\eta\right)^{-1/2}\right], \quad (4)$$

here, $N_{p0} = p_{Fp}^3/3\pi^2\hbar^3$ and $\delta = N_{0p}/N_{0e}$. Similarly, the equilibrium and perturbed number densities for positrons are given by

$$n_{p0} = N_{0p}\left[(3-\delta^{-4/3}T^2)\frac{\eta\delta^{-2/3}}{2} + \left(1-\delta^{-2/3}\eta\right)^{3/2} + \delta^{-4/3}T^2\left(1-\delta^{-2/3}\eta\right)^{-1/2}\right], \quad (5)$$

$$n_{p1} = N_{0p}\left[-\frac{3}{4}\eta\delta^{-4/3}\left(1+\delta^{-4/3}T^2\right) - \frac{3}{2}\delta^{-2/3} \times \left(1-\delta^{-2/3}\eta\right)^{\frac{1}{2}} + \frac{\delta^{-2}T^2}{2}\left(1-\delta^{-2/3}\eta\right)^{-\frac{3}{2}}\right]\Phi. \quad (6)$$

Now, we find the linear dispersion relation for the ion acoustic wave by using the following set of equations:

$$\left[\frac{\partial\vec{v}_i}{\partial t} + (\vec{v}_i\cdot\vec{\nabla})\vec{v}_i\right] = -\frac{e}{m_i}\vec{\nabla}\varphi + \frac{e}{m_i}(\vec{v}_i\times\vec{B}), \quad (7)$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\cdot(n_i\vec{v}_i) = 0. \quad (8)$$

Equations (7) and (8) are the ion fluid momentum and continuity equations, respectively. The parallel propagating ion acoustic waves are not affected by the external magnetic field since we have taken the propagation along the magnetic field B_0, and hence, we can drop the Lorentz term in Eq. (7).

Finally, we include the Poisson's equation to close the set of equations

$$\nabla^2\varphi = 4\pi e(n_e - n_i - n_p). \quad (9)$$

Assuming sinusoidal perturbations, linearizing Equations (7)–(9), and using Equations (3) and (6), we arrive at the following linear dispersion relation for the electrostatic ion acoustic wave in an e-p-i plasma in the presence of the quantizing magnetic field and finite electron temperature effects

$$\frac{\omega}{k} = \sqrt{\frac{(\Gamma_1 - \Gamma_2\delta)C_{sF}^2}{(\chi_1-\delta\chi_2) + k^2\lambda_{TF}^2}}, \quad (10)$$

where $C_{sF} = \sqrt{\varepsilon_{Fe}/m_i}$ is the Fermi ion sound velocity and $\lambda_{TF} = \sqrt{\varepsilon_{Fe}/4\pi e^2 N_{e0}}$ is the screening length called the Thomas-Fermi length in the degenerate plasma. From the quasi-neutrality condition, $\delta = N_{p0}/N_{e0}$ and Γ_1, Γ_2, χ_1, and χ_2 are given by

$$\Gamma_1 = (3-T^2)\frac{\eta}{2} + (1-\eta)^{3/2} + T^2(1-\eta)^{-1/2},$$

$$\Gamma_2 = (3-\delta^{-4/3}T^2)\frac{\eta\delta^{-2/3}}{2} + \left(1-\delta^{-2/3}\eta\right)^{3/2} + \delta^{-4/3}T^2\left(1-\delta^{-2/3}\eta\right)^{-1/2},$$

$$\chi_1 = \left\{(1+T^2)\frac{3}{4}\eta + \frac{3}{2}(1-\eta)^{1/2} - \frac{T^2}{2}(1-\eta)^{-3/2}\right\},$$

$$\chi_2 = -\frac{3}{4}\eta\delta^{-4/3}\left(1+\delta^{-4/3}T^2\right) - \frac{3}{2}\delta^{-2/3}\left(1-\delta^{-2/3}\eta\right)^{\frac{1}{2}} + \frac{\delta^{-2}T^2}{2}\left(1-\delta^{-2/3}\eta\right)^{-\frac{3}{2}}.$$

The typical parameters for number densities for electrons and positrons and the magnetic fields found in the vicinity of white dwarfs are the following: The electron number density is of the order of $10^{27}\,\text{cm}^{-3}$, the positron number density is $10^{26}\,\text{cm}^{-3}$, and the magnetic field is approximately 10^{10} G. The electron Fermi temperature that corresponds to the aforementioned densities comes out to be 4.24291×10^7 K.[25,26]

We begin by examining the effects of the positron concentration and quantizing magnetic field expressed through δ and η, respectively, on the linear propagation of ion acoustic waves. Fig. 1 shows the dependence of normalized wave

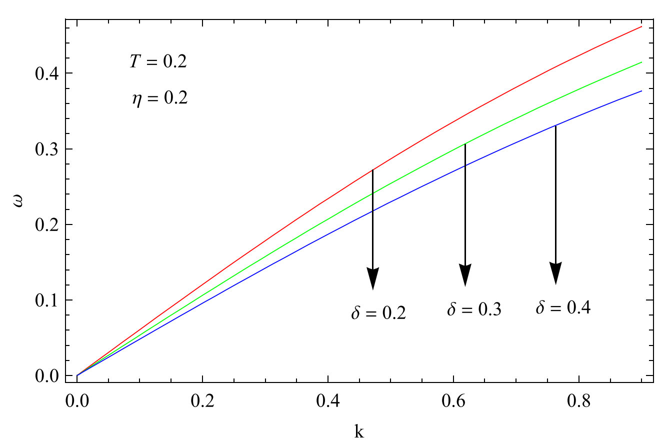

FIG. 1. Dispersion relation for ω versus k for different values of the positron concentration.

frequency ω (normalized by the ion plasma frequency) on the normalized wave number k (normalized by the Debye length) with the variation of positron number density keeping the temperature and magnetic field η fixed. It can be seen from the graphs that the wave frequency decreases with the increase in the positron concentration with constant temperature and η. Fig. 2 investigates the change in the frequency of the linear ion acoustic wave with the increase in the Landau quantization parameter η. It is observed that the frequency of the ion acoustic wave enhances with the increase in the Landau quantizing magnetic field. Note that the finite electron temperature effects also cause an increase in the frequency of the ion acoustic wave.

In order to study the nonlinear ion acoustic waves in the presence of degenerate electrons and positrons in the presence of a trapping and quantizing magnetic field, we proceed as follows: We consider the one dimensional case by shifting into the co-moving frame of reference and introducing the variable $\xi = z - ut$, where u is the velocity of propagation of perturbation. Taking the anti-derivatives of Equations (8) and (9) and making use of the following boundary conditions

$$\xi \to \infty, \quad \varphi, v_i \to 0, \quad and \quad n_i \to n_{i0},$$

we obtain the following nonlinear expression for the ion number density

$$n_i = N_{eo}(\Gamma_1 - \Gamma_2\delta)\left(1 - \frac{2\Phi}{M^2\alpha}\right)^{-1/2}, \tag{11}$$

where $n_{i0} = N_{eo}(\Gamma_1 - \Gamma_2\delta)$ is the background number density of ions. Here, "M" is the Mach number, which is defined as $M = \frac{u}{\omega/k}$, perturbation speed to the phase speed, and α is a constant given by

$$\alpha = \frac{(\Gamma_1 - \Gamma_2\delta)}{(\chi_1 - \delta\chi_2) + k^2\lambda_{TF}^2}.$$

Now, we derive an expression for the Sagdeev potential to study the formation of electrostatic solitary waves in the e-p-i plasma with trapping of electrons and positrons in a quantizing magnetic field and finite temperature effects. Following the procedure, we adopted to arrive at the expression of ion number density and using Equations (1), (4), and (11) in Eq. (9) yields the following expression

$$\begin{aligned}\frac{d^2\Phi}{d\xi^2} = 4\pi e^2 N_{e0}\Bigg[&\left\{\frac{3}{2}\eta(1+\Phi)^{\frac{1}{2}} + (1+\Phi-\eta)^{\frac{3}{2}} - \frac{\eta T^2}{2}(1+\Phi)^{-\frac{3}{2}} + T^2(1+\Phi-\eta)^{-\frac{1}{2}}\right\}\\ &-\delta\left\{\frac{3}{2}\eta\delta^{-2/3}(1-\delta^{-2/3}\Phi)^{\frac{1}{2}} + \left(1-\delta^{-2/3}\Phi-\delta^{-2/3}\eta\right)^{3/2} - \frac{\delta^{-2}\eta T^2}{2}(1-\delta^{-2/3}\Phi)^{-\frac{3}{2}} + \delta^{-4/3}T^2\right.\\ &\left.\times\left(1-\delta^{-2/3}\Phi-\delta^{-2/3}\eta\right)^{-1/2}\right\}\\ &-(\Gamma_1-\Gamma_2\delta)\left(1-\frac{2\Phi}{M^2\alpha}\right)^{-1/2}\Bigg],\end{aligned} \tag{12}$$

where ξ is normalized as $\xi = \xi/\lambda_{TF}$. Equation (12) can be expressed in the form of "energy integral" as follows to study the solitary structures.

$$\frac{1}{2}\left(\frac{d\Phi}{d\xi}\right)^2 + V(\Phi) = 0, \tag{13}$$

where V(Φ) is the Sagdeev potential, which is obtained by integrating the Eq. (12) and making use of the boundary condition that at $\xi \to \infty$, $\Phi = V(\Phi) = 0$, we obtain the final expression for the Sagdeev potential V(Φ) given by

$$\begin{aligned}V(\Phi) = \Bigg[&\left\{-\eta(1+\Phi)^{3/2} - \frac{2}{5}(1+\Phi-\eta)^{5/2} - \eta T^2(1+\Phi)^{-1/2} - 2T^2(1+\Phi-\eta)^{1/2}\right\}\\ &-\delta\left\{\eta(1-\delta^{-2/3}\Phi)^{3/2} + \frac{2}{5}\delta^{2/3}\left(1-\delta^{-2/3}\Phi-\delta^{-2/3}\eta\right)^{5/2} + \delta^{-4/3}T^2\eta(1-\delta^{-2/3}\Phi)^{-1/2}\right.\\ &\left.+2\delta^{-2/3}T^2\left(1-\delta^{-2/3}\Phi-\delta^{-2/3}\eta\right)^{1/2}\right\} - (\Gamma_1-\Gamma_2\delta)M^2\alpha\left(1-\frac{2\Phi}{M^2\alpha}\right)^{1/2}\\ &+(1+T^2)\eta + \frac{2}{5}(1-\eta)^{5/2} + 2T^2(1-\eta)^{1/2} + (\Gamma_1-\Gamma_2\delta)M^2\alpha\\ &+\delta\left\{\eta\left(1+\delta^{-4/3}T^2\right) + \frac{2}{5}\delta^{2/3}\left(1-\delta^{-2/3}\eta\right)^{5/2} + 2\delta^{-2/3}T^2\left(1-\delta^{-2/3}\eta\right)^{1/2}\right\}\Bigg].\end{aligned} \tag{14}$$

Equation (13) is considered as the "energy law" of the oscillating particle with the velocity $d\Phi/d\xi$ and position Φ in the potential V(Φ) having unit mass. In order to have the solitary wave solutions of Equation (14), we have (i) $(d^2V(\Phi)/d\Phi^2)_{\Phi=0} < 0$ so the fixed point is unstable at the origin and (ii) for compressive solitary waves $V(\Phi) < 0$ when $0 < \Phi < \Phi_{max}$ and $V(\Phi) < 0$, when $0 > \Phi > \Phi_{min}$ for rarefactive solitary waves. Where $\Phi_{max(min)}$ is the maximum (minimum) value of potential Φ for which $V(\Phi) = 0$.[22] From Equation (14), the range of the Mach number is given below

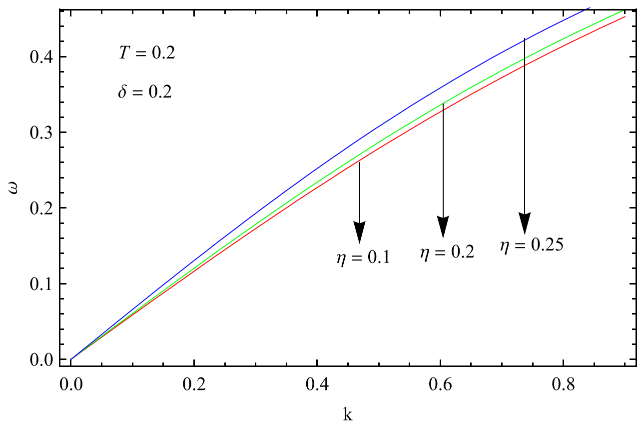

FIG. 2. Dispersion relation for ω versus k for different values of η.

$$1 \leq M < \left[\frac{2(\chi_1 - \delta\chi_2 + k^2\lambda_{TF}^2)}{(\Gamma_1 - \Gamma_2\delta)}\right]^{1/2}.$$

Note that the above range of Mach number defines the existence regimes of the formation of solitary structures. It can be seen that the presence of positrons enhances the upper limit of the existence regime of the formation of electrostatic solitary structures under investigation here. Now, we investigate the effect of the positron to electron concentrations ratio δ, quantizing magnetic field through η, finite electron temperature effect T, and Mach number M on the dynamical characteristics of nonlinear ion acoustic waves through the Sagdeev potential approach. We use Equation (14) to plot the different graphs. Fig. 3 shows a graph between Sagdeev potential V (Φ) and normalized potential Φ. It is seen from the graphs that the width and depth of Sagdeev potential are decreased as the positron concentration is increased keeping all the other parameters fixed. The solitary structures corresponding to the Sagdeev potential shown in Fig. 3 are shown in Fig. 4 with the same parameters used in Fig. 3. From Fig. 4, we can see that the amplitude of the solitary structure mitigates with the enhancement of the positron to electron concentration ratio, δ. It is apposite to mention here that the system under consideration admits only compressive solitary structures for the plasma parameters used in Figs. 3 and 4.

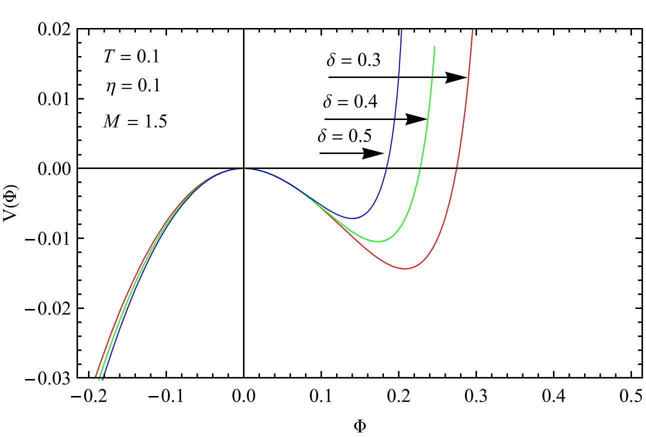

FIG. 3. Sagdeev potential V (Φ) versus Φ with the variation of concentration ratio δ at fixed T, η, and Mach number M.

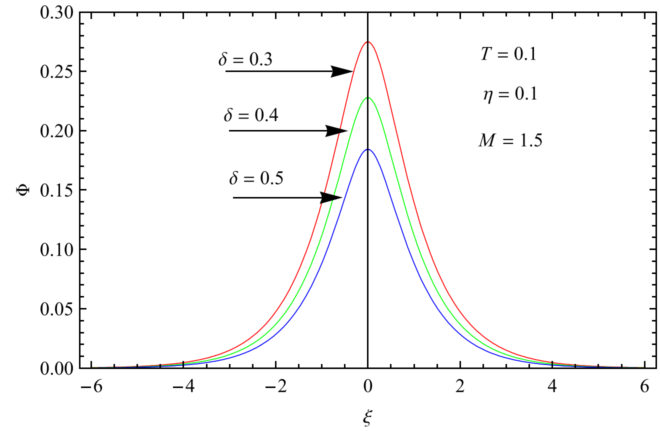

FIG. 4. Solitary wave amplitude Φ versus ξ, corresponding to the Sagdeev potential V(Φ) shown in Fig. 3.

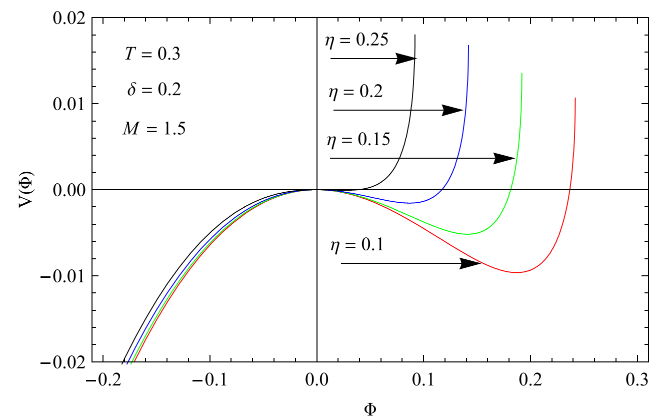

FIG. 5. Sagdeev potential V (Φ) versus Φ with the variation of η, keeping δ, T, and Mach number M fixed.

Next, we examine the effect of Landau quantization, η, on the formation of the Sagdeev potential. It is observed that the width and depth of the Sagdeev potential experience a significant decrease with the increasing values of η as can be seen from Fig. 5. The corresponding solitary structures of the Sagdeev potential drawn in Fig. 5 are shown in Fig. 6. It is interesting to note unlike the electron-ion plasma,[22] the

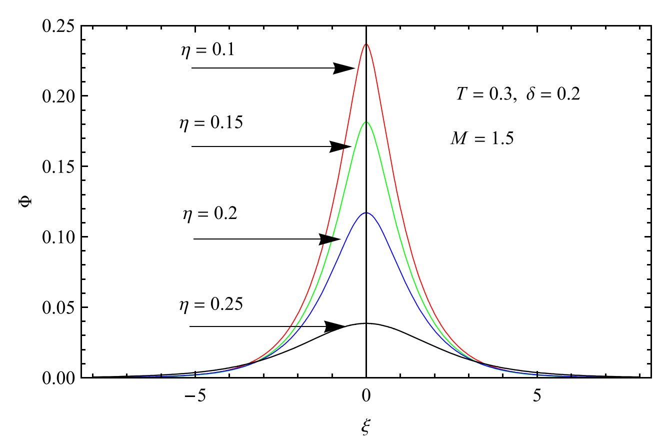

FIG. 6. Solitary wave amplitude Φ versus ξ, corresponding to the Sagdeev potential V(Φ) shown in Fig. 5.

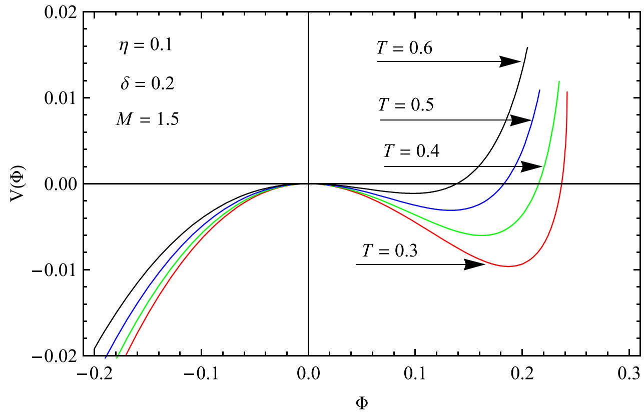

FIG. 7. Sagdeev potential V (Φ) versus Φ with the variation of T, keeping δ, η, and Mach number M fixed.

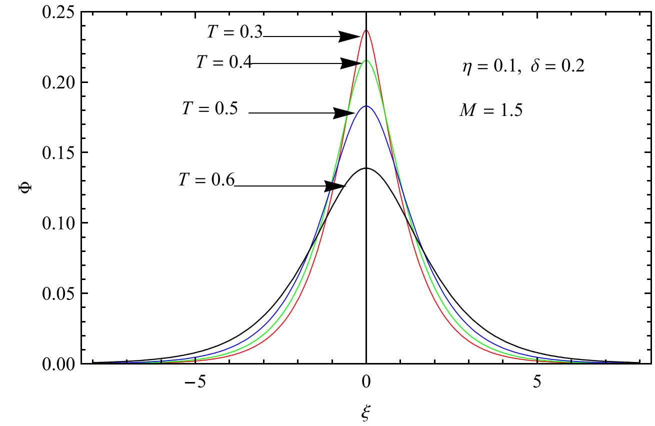

FIG. 8. Solitary wave amplitude Φ versus ξ, corresponding to the Sagdeev potential V(Φ) shown in Fig. 7.

amplitude as well as the width of the solitary structures mitigates in the e-p-i quantum plasmas.

Fig. 7 shows the plots of the Sagdeev potential V(Φ) versus Φ by varying the finite temperature correction effects in a partially degenerate plasma. The graph shows a similar trend to that observed in Figs. 3 and 5. The width and depth of the potential are observed to decrease with the increasing values of temperature correction effects. The corresponding solitary structures are shown in Fig. 8. Once again only the compressive solitary structures are observed for the plasma parameters that are used to plot the solitary structures. It is pertinent to mention here that the variation of the amplitude of the solitary structures with the plasma parameters such as Landau quantization and finite electron temperature effects is less in magnitude in the e-p-i plasma by comparison with the e-i case (see Ref. 22 for detailed comparison). Most importantly, the comparison of the solitary structures in this paper with the e-i one reveals that the spatial scale lengths over which the solitary structures form in the e-p-i plasma in the presence of Landau quantization and finite electron temperature corrections are shorter in comparison with their counterparts in the e-i plasma (see Ref. 22 for detailed comparison).

In the present work, we have examined the effect of trapping as a microscopic phenomenon on the formation of solitary structures in the presence of Landau quantization in the electron-positron-ion (e-p-i) plasma having degenerate electrons and positrons whereas ions have taken to be classical and cold. In this regard, we have derived the linear dispersion relation for the ion acoustic wave, which has been observed to be significantly modified by the positron concentration, quantizing magnetic field, and finite electron temperature effects. In the nonlinear case, the expression for the Sagdeev potential is procured for the e-p-i plasma taking Landau quantization into account. It has been observed that the system under consideration admits only compressive solitary structures. It has been shown that the inclusion of positrons increases the upper limit of the existence regime of the electrostatic structures under consideration here. It has also been found that the spatial extent of the solitary structures formed in e-p-i plasmas is shorter by comparison with their counterparts in e-i plasmas. The work presented here may be beneficial to understand the propagation of nonlinear electrostatic structures in dense astrophysical environments and in intense-laser plasma interactions.

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