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Alfvén solitary waves with effect of arbitrary temperature degeneracy in spin quantum plasma

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Nonlinear Alfvén waves are studied in a fluid model for nonrelativistic, magnetized spin-1/2 quantum plasmas with an arbitrary degeneracy effect. Following a local Fermi-Dirac distribution function, a modified equation of state is utilized which is applicable to both classical and degenerate limits. Using the fluid equations for Hall magnetohydrodynamics with quantum corrections due to statistical effects, Bohm potential, spin magnetization energy, and temperature degeneracy, a set of modified Zakharov equations are derived for circularly polarized nonlinear Alfvén waves. Ions are assumed to be cold, and the spin effects of electrons are incorporated through spin force along with spin magnetization current. A linear dispersion relation for finite amplitude Alfvén waves duly modified by spin magnetization and arbitrary temperature degeneracy effects is also obtained. Employing the Sagdeev potential approach, the properties of Alfvén solitary profiles in quantum plasmas with arbitrary degeneracy effects of electrons are analyzed. The amplitude of Sagdeev potential and of the associated soliton structure for both right and left-hand circularly polarized Alfvén waves is observed to decrease with the decrease in the value of the arbitrary temperature degeneracy factor G for the case of the nearly degenerate limit. Similarly, it is found that the amplitude of Sagdeev potential and of the related solitary profile increases for both kinds of circular polarized Alfvén waves with the increasing value of G in the case of the nearly non-degenerate limit. *Published by AIP Publishing.* https://doi.org/10.1063/1.5037649

I. INTRODUCTION

Over the last several years, the physics of low temperature and high density plasmas has rapidly grown beyond the conventional high temperature and low density plasmas commonly observed in space and laboratory. The interest in the field of quantum plasmas is due to their potential applications in high density astrophysical objects (interiors of white dwarf stars and neutron stars),[1,2] in thin metal films,[3] in quantum wells,[4] in ultra-cold plasmas,[5] in microelectronic devices,[6] and in strong laser produced plasmas.[7] In quantum plasmas, due to their high density and low temperature characteristics, the de Broglie wavelength of the electron becomes comparable to (or greater than) the average distance between them, i.e., $\lambda_{De}(=\hbar/m_e v_{te}) \geq \sqrt{n-3}$ (with n representing the equilibrium density, $\hbar$ the reduced Plank's constant, m_e the mass of electrons, and v_{te} the thermal speed of electrons), which leads to a number of novel quantum effects as a result of overlapping of wavefunctions associated with these electrons. Under such circumstances, the degeneracy pressure is larger than the usual thermal pressure, and the Fermi energy of the electrons exceeds their thermal energies due to which the electrons become degenerate. In order to consider the novel quantum effects arising due to the Pauli exclusion principle and Heisenberg's uncertainty principle, the quantum hydrodynamic (QHD) model has been extensively used in recent years.[8–10] The QHD model, a quantum analog of the classical fluid model,[11] generalizes the usual fluid model with the inclusion of the so-called Bohm potential,[12] a quantum correction term arising as a result of the quantum tunneling effect of electrons. This model also incorporates the quantum statistical effects through the inclusion of the Fermi pressure for degenerate electrons.

The most typical electromagnetic phenomena associated with plasmas in the presence of strong magnetic fields, i.e., Alfvén waves, are transverse magnetohydrodynamic (MHD) waves[13] propagating in a direction parallel to the external magnetic field. Various plasma regimes such as laboratory, space, and astrophysical plasmas are deeply influenced by the propagation of nonlinear Alfvén waves. These waves have been extensively studied in recent years due to their applications in plasma heating,[14] self-modulation in strongly magnetized plasmas,[15] reconnection,[16] interplanetary shocks,[17] and turbulence.[18] The ideal magnetohydrodynamic (MHD) model, commonly applied for the description of a magnetized plasma, was first used[19] for the derivation of Alfvén waves. In an ideal MHD model, the whole plasma is assumed to act like a single fluid as compared to that of the two-fluid model where the plasma is considered to consist of two intermingled fluids, i.e., electron fluid and ion fluid. The dispersion in the case of the two-fluid model arises as a result of the inclusion of the Hall term. The Hall-MHD description has been utilized by Brodin and Stenflo[20] to study the interactions caused by Alfvénic fluctuations. The propagation of linearly and circularly polarized Alfvén waves with the inclusion of their linear and nonlinear characteristics is a subject of prime focus in recent research activities.[21–23]

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In the case of plasmas subjected to strong magnetic fields, the propagation of waves is believed to be significantly influenced by the collective spin effects of charge particles. Owing to this fact, the MHD model has been extended to include the spin magnetization.[24,25] A modified Korteweg-de Vries (KdV) equation has been derived in Ref. 26 to study Alfvén solitary waves in non-degenerate (ND) quantum plasmas including the spin-1/2 effects of electrons. The electron spin effects are believed to play a vital role in the dynamics of low frequency modes as depicted in Refs. 25 and 27. Two dimensional nonlinear solitary profiles in degenerate quantum plasmas without spin[28] and in non-degenerate plasmas with the electron spin effects have been analyzed.[29] Similarly, magnetosonic solitary structures have also been analyzed for the case of one and two dimensional multicomponent quantum plasmas.[30,31] The propagation of nonlinear Alfvén waves in degenerate Fermionic quantum plasmas without and with electron spin effects was studied in Refs. 32 and 33, respectively.

Two main approaches, i.e., kinetic model and fluid model, have been extensively used to study the wave propagation in a degenerate quantum plasma. In the former approach, the electron distribution is characterized by employing a Fermi-Dirac (FD) function, whereas for the latter approach, the force balance equation is modified with the equation of state for a degenerate electron gas. For the classical ideal electron gas obeying the Maxwell-Boltzmann (MB) distribution, the energy distribution is solely determined by the thermodynamic temperature, and accordingly, the energy spread of a quantum degenerate electron gas is subjected to Fermi-Dirac (FD) distribution which is characterized by two parameters, i.e., thermodynamic temperature T_e and chemical potential μ. For the case of fully degenerate electron gas, the equation of state reduces to a function of only chemical potential μ. Thus, depending on the competition between thermal temperature and chemical potential, it is important to study the propagation of nonlinear waves in the intermediate regime by incorporating an arbitrary temperature degeneracy parameter $G\left(=\frac{Li_{\frac{5}{2}}(-\xi)}{Li_{\frac{3}{2}}(-\xi)}\right)$ (notations are defined in Sec. II), for the transition between thermal and ultra-cold cases. Inertial confinement fusion plasmas,[34] ultrasmall semiconductor devices,[35] and laboratory simulations of dense astrophysical scenarios (white dwarfs, neutron stars, etc.)[36] provide striking examples of such systems which are neither strongly degenerate (quantum) nor completely non-degenerate (classical).

Using classical kinetic theory, a general dispersion relation for ion-acoustic waves (IAWs) (low frequency) and Langmuir waves (high frequency) was previously obtained[37] in dense quantum plasmas with the effect of arbitrary temperature degeneracy of electrons. A longitudinal response function for a thermal electron gas has been calculated for arbitrary degeneracy in Ref. 38 by using quantum kinetic theory. Apart from these works which are restricted to linear waves only, nonlinear fluid evolution equations for high frequency plasma oscillations have also been derived[39] in a quantum plasma with the effect of arbitrary degeneracy of electrons. A Bernoulli pseudopotential approach has been utilized to analyze ion-acoustic waves (IAWs) in isothermal plasmas with arbitrary degeneracy effects. Linear and nonlinear characteristics of ion-acoustic waves with some degenerate degree of electrons were studied using the fluid model for nonrelativistic, unmagnetized, and magnetized quantum plasmas,[40,41] where the electron equation of state is modified by deriving the pressure tensor, solving the FD distribution in terms of the polylogarithm function with some arbitrariness in degeneracy.

In the present article, we study the propagation of nonlinear Alfvén waves nearly parallel to the ambient magnetic field, in spin quantum plasmas with the effect of arbitrary temperature degeneracy of electrons. Our main objective is to study the interaction of shear Alfvén waves and ion acoustic waves (IAWs), both of which predominantly propagate along the applied magnetic field with some sort of quantum and finite Larmor radius corrections.

In Sec. II, we present the basic equations for quantum Hall-magnetohydrodynamics (QHMD) and define a barotropic equation of state in terms of an arbitrary temperature degeneracy factor G. Using a two time scale perturbation approach, a set of modified nonlinear equations (called Zakharov equations) are obtained from the governing equations in Sec. III. We also derive a linear dispersion relation for Alfvén waves duly modified by spin effects of electrons and the arbitrary temperature degeneracy effect in the same section. In Sec. IV, the derivation of nonlinear nonrelativistic electromagnetic Sagdeev or pseudopotential is carried out, which is essential for the discussion of the soliton solution for circularly polarized spin-1/2 Alfvén waves. In Sec. V, the main results of this paper are presented. The summary of the work is given in Sec. VI.

II. BASIC EQUATIONS AND FORMULATION

We consider a two component electron-ion quantum magnetoplasma with arbitrary degeneracy of electrons including the spin effects of electrons. The ambient magnetic field is supposed to be along the z-axis in a Cartesian coordinate system $(\mathbf{B}_0 = B_0\hat{\mathbf{z}})$, while the propagation is assumed to be predominantly in a direction parallel to the external magnetic field, i.e., $\nabla = (0, 0, \partial_z)$ with small corrections of the transverse finite Larmor radius effect. In order to construct the one fluid quantum magnetohydrodynamic (QMHD) model, we start with the usual quantum hydrodynamic fluid equations for electrons and ions with the inclusion of quantum corrections. We have ignored the quantum effects of ions because of their large inertia as compared to that of electrons. The set of dynamic equations for nonlinear circularly polarized Alfvén waves in a spin-1/2 quantum plasma with arbitrary degeneracy effects is described as follows:

The force balance equation for inertial cold classical ions is

$$m_i \frac{d\mathbf{u}_i}{dt} = e(\mathbf{E} + \mathbf{u}_i \times \mathbf{B}), \tag{1}$$

and the continuity equation is

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0. \tag{2}$$

The electrons are considered inertialess owing to the low frequency nature of Alfvén waves, and thus, the inertialess equation of motion for electrons can be written as

$$0 = -e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \frac{\nabla P_e}{n_e} + \mathbf{F}_Q. \tag{3}$$

In the above equations, m_i represents the mass of ions, $\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{u}_i \cdot \nabla)$ is the usual convective fluid derivative, $\mathbf{u}_i$ and $\mathbf{u}_e$ are the symbols used for ion and electron fluid velocities, respectively, whereas n_i and n_e are the ions and electron number densities. $\pm|e|$ represents the magnitudes of the charge on ions (electron), $\mathbf{E}$ is the wave electric field vector, and $\mathbf{B}$ is the wave magnetic field vector. In Eq. (3), $P = P(n_e)$ is the electron fluid pressure which is specified by a barotropic equation of state, which is given as

$$P_e = \frac{G}{\beta'} n_e, \tag{4}$$

where $G = \frac{Li_{\frac{5}{2}}(-\xi)}{Li_{\frac{3}{2}}(-\xi)}$[38] is the arbitrary temperature degeneracy factor for the transition between thermal and ultra-cold cases, with $\xi = e^{\beta' \mu}, \beta' = \frac{1}{k_B T_e}$. Here, μ is the chemical potential which is the function of position r and time t, k_B is the usual Boltzmann constant, and T_e is the electron's temperature (constant). The moments of a local Fermi-Dirac (FD) distribution function of an ideal fermi gas lead[40] to the equation of state (4). The parameter ξ, which is a function of chemical potential μ and electron temperature T_e, describes the degeneracy. For the case of the nearly nondegenerate (NND) limit, we assume $\xi \ll 1$ ($\frac{\mu}{k_B T_e}$ is large and negative), whereas for the nearly degenerate (ND) limit, $\xi \gg 1$ ($\frac{\mu}{k_B T_e}$ is large and positive).[38] The equation of state (4) contains the polylogarithm function with index m, i.e., $Li_m(-\xi)$. For the case when $m > 0$, the polylogarithm function has the general form[42]

$$Li_m(-\xi) = -\frac{1}{\Gamma(m)} \int_0^\infty \frac{t^{m-1}}{\frac{e^t}{\xi} + 1} dt, \tag{5}$$

where $\Gamma(m)$ is the gamma function. The polylogarithm function (5) can be expanded under various assumptions. For the case of the nearly nondegenerate (NND) limit ($\xi \ll 1$), we get

$$Li_m(-\xi) = -\xi + \frac{\xi^2}{2^m}. \tag{6}$$

Putting the expanded values of $Li_{\frac{5}{2}}(-\xi)$ and $Li_{\frac{3}{2}}(-\xi)$, the arbitrary temperature degeneracy factor G becomes

$$G = 1 + \frac{\xi}{2^{\frac{5}{2}}}. \tag{7}$$

Similarly, for the case of the nearly degenerate (ND) limit ($\xi \gg 1$), Eq. (5) yields

$$Li_m(-\xi) = \frac{-(\ln \xi)^m}{\Gamma(m+1)}. \tag{8}$$

By using the values of $Li_{\frac{5}{2}}(-\xi)$ and $Li_{\frac{3}{2}}(-\xi)$, the arbitrary temperature degeneracy factor G takes the form

$$G = \frac{2}{5\delta}\left[1 - \frac{\pi^2}{12}\delta^2\right], \tag{9}$$

where $\delta = \frac{T_e}{T_F}$. It is important to discuss some limiting cases of the equation of state (4). For the dilute plasma limit, i.e., in the case of complete nondegeneracy (CND) ($\xi \to 0$), one get from Eq. (7) $G_{CND} = 1$, and thus, Eq. (4) reduces to the usual classical isothermal equation of state, i.e., $P = n_e k_B T_e$. On the other extreme for the dense plasma limit, i.e., in the case of complete degeneracy (CD) ($\xi \to \infty$), Eq. (9) yields $G_{CD} = \frac{2}{5}\frac{T_F}{T_e}$, which on putting in Eq. (4) gives the Fermi pressure for degenerate electrons, i.e., $P = \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e} n_e^{5/3}$.[43]

A general coupling parameter applicable to both nearly non-degenerate (NND) and nearly degenerate (ND) limits can be derived in terms of arbitrary temperature degeneracy factor G to show that our model does not take into account collisional damping.[40] The dimensionless coupling parameter Λ by definition is the ratio of the mean interaction energy per particle $\langle E_{int}\rangle$ to the corresponding mean kinetic energy $\langle E_{kin}\rangle$, i.e.,

$$\Lambda = \frac{\langle E_{int}\rangle}{\langle E_{kin}\rangle}.$$

For $\Lambda \geq 1$, the system is collision dominated, whereas the condition for low collisionality is that the average kinetic energy should be greater than the average electrostatic potential energy per particle, i.e., $\Lambda < 1$. For arbitrary degeneracy incorporating the values of electrostatic potential $\langle E_{int}\rangle \approx \left(\frac{4n_o}{3\pi^2}\right)^{\frac{1}{3}} \frac{e^2}{\varepsilon_o}$ (using the definition of the Wigner-Seitz ratio $r_s = \left(\frac{3}{4\pi n_o}\right)^{\frac{1}{3}}$) and kinetic energy $\langle E_{kin}\rangle = \frac{3}{2\beta'} \frac{Li_{\frac{5}{2}}(-\xi)}{Li_{\frac{3}{2}}(-\xi)}$ (with degenerate fugacity $\xi = e^{\beta'\mu}$), the coupling parameter can be expressed as

$$\Lambda = \frac{1}{6}\left(\frac{4n_o}{3\pi^2}\right)^{\frac{1}{3}} \frac{e^2\beta'}{\varepsilon_o} \frac{Li_{\frac{3}{2}}(-\xi)}{Li_{\frac{5}{2}}(-\xi)} = \frac{1}{6}\left(\frac{4n_o}{3\pi^2}\right)^{\frac{1}{3}} \frac{e^2\beta'}{\varepsilon_o G}, \tag{10}$$

where $G = \frac{Li_{\frac{5}{2}}(-\xi)}{Li_{\frac{3}{2}}(-\xi)}$. The relationship between the number density n_o and the chemical potential μ via a polylogarithm function can be written as[40,41]

$$-\frac{n_o}{Li_{\frac{3}{2}}(-\xi)}\left(\frac{m_e\beta'}{2\pi}\right)^{\frac{3}{2}} = 2\left(\frac{m_e}{2\pi\hbar}\right)^3.$$

The coupling parameter (10) in terms of the polylogarithm function can be deduced

$$\Lambda = -\frac{\sqrt{m_e\beta'/2}}{3^{\frac{4}{3}}\pi^{\frac{7}{6}}} \frac{e^2}{\varepsilon_o\hbar} \frac{\left(Li_{\frac{3}{2}}(-\xi)\right)^{\frac{4}{3}}}{Li_{\frac{5}{2}}(-\xi)}. \tag{11}$$

By simplifying Eqs. (11) and (10), we can express temperature T and number density n_0 in terms of the coupling parameter Λ as

$$T = \frac{m_e}{2k_B}\left[\frac{e^2\left(Li_{\frac{3}{2}}(-\xi)\right)^{\frac{4}{3}}}{3^{\frac{4}{3}}\pi^{\frac{7}{6}}\varepsilon_o\hbar\Lambda Li_{\frac{5}{2}}(-\xi)}\right]^2 \tag{12}$$

and

$$n_o = \frac{3\pi^2}{4}\left[\frac{6\varepsilon_o\Lambda Li_{\frac{5}{2}}(-\xi)}{e^2\beta' Li_{\frac{3}{2}}(-\xi)}\right]^3. \tag{13}$$

For the case of the nearly nondegenerate (NND) limit ($\xi \ll 1$), the expanded form of the polylogarithm function, i.e., Eq. (6), yields $Li_{\frac{3}{2}}(-\xi) = -\xi + \frac{(-\xi)^2}{2^{\frac{3}{2}}}$ and $Li_{\frac{5}{2}}(-\xi) = -\xi + \frac{(-\xi)^2}{2^{\frac{5}{2}}}$. Accordingly, for the case of the nearly degenerate (ND) limit ($\xi \gg 1$), the expanded form of the polylogarithm function using Eq. (8) takes the form $Li_{\frac{3}{2}}(-\xi) = \frac{-(\ln\xi)^{\frac{3}{2}}}{\Gamma\frac{5}{2}}$, $Li_{\frac{5}{2}}(-\xi) = \frac{-(\ln\xi)^{\frac{5}{2}}}{\Gamma\frac{7}{2}}$. In Eq. (3), the last term is the force (quantum) on electrons that can be expressed as

$$\mathbf{F}_Q = \frac{\hbar^2}{2m_e}\nabla\left[\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right] + \left(\frac{\beta'\mu_B^2 B}{G}\right)\nabla\mathbf{B},$$

where the first term is the quantum Bohm potential term that arises as a consequence of the tunneling effect of the electrons (degenerate), whereas the second term is the result of spin magnetization force in degenerate plasmas duly modified by the arbitrary temperature degeneracy factor G, $\hbar$ is the reduced Planck's constant, and $\mu_B (= \frac{e\hbar}{2m_e})$ is the Bohr magneton. The two relevant Maxwell equations, i.e., Ampere's law with the inclusion of spin magnetization and Faraday's law, are as follows:

$$\nabla\times\mathbf{B} = \mu_0(\mathbf{J}_p + \mathbf{J}_m), \tag{14}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}. \tag{15}$$

Here, μ_0 is the permeability of free space, $\mathbf{J_p} = e(n_i\mathbf{u}_i - n_e\mathbf{u}_e)$ is the plasma current density, and $\mathbf{J}_m = \nabla\times\mathbf{M}$ represents the electron magnetization spin current density, with $\mathbf{M} = \frac{\beta'\mu_B^2}{G}n\mathbf{B}$ the microscopic spin magnetization in a plasma in terms of temperature degeneracy factor G. In Eq. (14), we have ignored the displacement current as we consider low frequency waves only; furthermore, in any conducting medium, its value is negligible in comparison to the net current density J. Substituting $\mathbf{u}_e$ from (14) and Eq. (4) into (3), using the quasineutrality condition $n = n_i \simeq n_e$, we get

$$\mathbf{E} = -\left(\mathbf{u}_i - \frac{1}{en\mu_0}\nabla\times\mathbf{B} + \frac{1}{en}\nabla\times\mathbf{M}\right)\times\mathbf{B} - \frac{G}{en\beta'}\nabla n + \frac{\mathbf{F}_Q}{e}. \tag{16}$$

The normalized effective one fluid momentum equation can be obtained by eliminating E from (1) and (15) by using Eq. (16), which is

$$\frac{d\mathbf{u}_i}{dt} = \frac{1}{n}(\nabla\times\mathbf{B})\times\mathbf{B} - \frac{1}{n}(\nabla\times\mathbf{M})\times\mathbf{B} - \frac{\beta\delta G}{2}\nabla n + \frac{H_e^2}{2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{\varepsilon_o^2\beta}{2\delta G}B\nabla B, \tag{17}$$

where $\beta = \frac{c_{qs}^2}{V_A^2} = \frac{2\mu_0 n_0\mathcal{E}_{Fe}}{B_0^2}$ is a factor that measures the quantum statistical effects in a degenerate plasmas and is usually termed as plasma beta,[44] $\mathcal{E}_{Fe} = k_B T_F = \frac{(3\pi^2 n_e)^{2/3}\hbar^2}{2m_e}$ is the Fermi energy of degenerate electrons, which is the same as the equilibrium chemical potential in the fully degenerate case, $c_{qs} = (\frac{2\mathcal{E}_{Fe}}{m_i})^{1/2}$ is the quantum ion sound speed, $V_A = \frac{B_0}{(\mu_0 n_0 m_i)^{1/2}}$ is the Alfvén speed, and $H_e = \frac{\hbar\Omega_i}{\sqrt{m_e m_i}V_A^2}$ is a dimensionless parameter that arises as a result of the collective electron tunnelling effect through the so-called Bohm potential, with $\Omega_i = \frac{eB_0}{m_i}$ being the ion cyclotron frequency and $\varepsilon_0 = \frac{\mu_B B_0}{k_B T_F}$ being the Fermi-normalized Zeeman energy. Now, elimination of electric wave vector E between (1) and (15) leads to the magnetic field induction equation (normalized) with the inclusion of the Hall term, which is given as

$$\frac{\partial\mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}_i\times\mathbf{B}) - \nabla\times\left[\frac{1}{n}(\nabla\times\mathbf{B})\times\mathbf{B} - \frac{1}{n}(\nabla\times\mathbf{M})\times\mathbf{B}\right]. \tag{18}$$

In Eqs. (17) and (18), we have used the following rescaling:

$$\mathbf{r}\to\frac{\Omega_i\mathbf{r}}{V_A},\quad t\to\Omega_i t,\quad \mathbf{u}_i\to\frac{\mathbf{u}_i}{V_A},\quad \mathbf{B}\to\frac{\mathbf{B}}{B_0},\quad n\to\frac{n}{n_0},\quad \mathbf{M} = \frac{\mu_0\mathbf{M}}{B_0}.$$

The presence of the ion cyclotron effects (i.e., the wave frequency is comparable to the ion gyrofrequency Ω_i) in the QMHD model is guaranteed by the inclusion of the Hall term in (18). Equations (2), (17), and (18) are the set of basic equations of the QMHD model for a magnetized and collisionless plasma composed of classical cold ions and inertialess hot electrons with spin-1/2 effects.

III. NONLINEAR EVOLUTION EQUATIONS

Since we consider waves propagating along the direction of the external magnetic field (i.e., z-axis), the time derivative of x- and y-components of Eq. (18) can be written as

$$\frac{\partial B_x}{\partial t} = \frac{\partial}{\partial z}(u_x - u_z B_x) + \frac{\partial}{\partial z}\left[\frac{1}{n}\left(\frac{\partial B_y}{\partial z} - \frac{\partial M_y}{\partial z}\right)\right],$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial z}(u_y - u_z B_y) - \frac{\partial}{\partial z}\left[\frac{1}{n}\left(\frac{\partial B_x}{\partial z} - \frac{\partial M_x}{\partial z}\right)\right].$$

Here, u_z is the velocity of perturbation directed along the z-axis, B_x and B_y represent the components of the wave magnetic field, whereas $M_\mathbf{x}$ and $M_\mathbf{y}$ are the magnetization density magnitudes in the x and y directions, respectively. In order to deal with circularly polarized Alfvén waves, it is convenient to introduce a complex description of the transverse fields along with the perturbation velocity. The x and y components of Eq. (17) can be combined by using $u_\pm = u_x \pm i u_y, B_\pm = B_x \pm iB_y$, and $M_\pm = M_x \pm iM_y$ to obtain the following relation:

$$\frac{du_\pm}{dt} = \frac{1}{n}\frac{\partial B_\pm}{\partial z} - \frac{\varepsilon_o^2 \beta}{n}\frac{\partial (nB_\pm)}{\partial z}, \tag{19}$$

where we have used $M_\pm = \frac{\varepsilon_o^2 \beta n}{2\delta G} B_\pm$ obtained from the definition of $\mathbf{M} = \frac{\beta' \mu_B^2}{G} nB\hat{\mathbf{B}}$. Now, by using these relations, the x and y components of Eq. (18) can be combined to get

$$\begin{aligned}&\frac{\partial^2 B_\pm}{\partial t^2} + \frac{\partial}{\partial z}\left[u_z \frac{\partial B_\pm}{\partial t} + \frac{d}{dt}(u_z B_\pm)\right] \\ &\quad - \frac{\partial}{\partial z}\left[\frac{1}{n}\frac{\partial B_\pm}{\partial z} - \frac{\varepsilon_o^2 \beta}{2\delta G n}\frac{\partial (nB_\pm)}{\partial z}\right] \\ &\quad \pm i \frac{\partial}{\partial z}\left[\frac{d}{dt}\left(\frac{1}{n}\frac{\partial B_\pm}{\partial z}\right)\right] \mp i \frac{\varepsilon_o^2 \beta}{2\delta G}\frac{\partial}{\partial z}\left[\frac{d}{dt}\left(\frac{1}{n}\frac{\partial}{\partial z}\frac{\partial (nB_\pm)}{\partial z}\right)\right] = 0.\end{aligned} \tag{20}$$

Here, the upper and lower signs $(\pm)$ are used for right and left circularly polarized Alfvén waves (RCPAWs/LCPAWs) propagating along the z axis, respectively. By using the above definitions of $M_\pm$ and Eq. (19), the parallel component of (17) can be obtained as

$$\begin{aligned}\frac{du_z}{dt} &= -\frac{1}{2n}\frac{\partial |B_\pm|^2}{\partial z} + \frac{\varepsilon_o^2 \beta}{4\delta G}\frac{\partial |B_\pm|^2}{\partial z} + \frac{\varepsilon_o^2 \beta}{2\delta G n}|B_\pm|^2 \frac{\partial n}{\partial z} \\ &\quad - \frac{\beta \delta G}{2n}\frac{\partial n}{\partial z} + \frac{H_e^2}{2}\frac{\partial}{\partial z}\left[\frac{\frac{\partial^2}{\partial z^2}\sqrt{n}}{\sqrt{n}}\right].\end{aligned} \tag{21}$$

The quantities u_z and n are related via the continuity equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n u_z) = 0, \tag{22}$$

and operating the above equation by $\left(\frac{\partial}{\partial t} + u_z \frac{\partial}{\partial z}\right)$ and incorporating Eq. (21) yield

$$\begin{aligned}\frac{\partial^2 n}{\partial t^2} - \frac{\beta \delta G}{2}\frac{\partial^2 n}{\partial z^2} &= u_z \frac{\partial^2 (n u_z)}{\partial z^2} + \frac{1}{2}\frac{\partial^2 |B_\pm|^2}{\partial z^2} - \frac{\varepsilon_o^2 \beta}{4\delta G}\frac{\partial}{\partial z}\left[n \frac{\partial |B_\pm|^2}{\partial z}\right] \\ &\quad - \frac{\varepsilon_o^2 \beta}{2\delta G}\frac{\partial}{\partial z}\left[|B_\pm|^2 \frac{\partial n}{\partial z}\right] - \frac{\partial}{\partial z}\left(u_z \frac{dn}{dt}\right) \\ &\quad - \frac{H_e^2}{2}\frac{\partial}{\partial z}\left[n \frac{\partial}{\partial z}\left(\frac{1}{\sqrt{n}}\frac{\partial^2 \sqrt{n}}{\partial z^2}\right)\right].\end{aligned} \tag{23}$$

Equations (20), (22), and (23) form a complete set of nonlinear equations in a Fermionic spin quantum plasma with an arbitrary degeneracy effect relating $B_\pm$, n, and the fluid velocity along the direction of the ambient magnetic field, i.e., u_z. For a brief study of the linear dispersion relation for Alfvén waves in spin quantum plasmas, we can linearize Eq. (20). Since in the linear approximation the quantities n and u_z are treated as constants, Eq. (20) in dimensional form can be written as

$$\left[\frac{\partial^2}{\partial t^2} - \vartheta V_A^2 \frac{\partial^2}{\partial z^2} \pm i\vartheta \frac{V_A^2}{\Omega_i}\frac{\partial^3}{\partial z^2 \partial t}\right]B_\pm = 0, \tag{24}$$

where $\vartheta = 1 - \frac{\varepsilon_o^2 \beta}{2\delta G}$. Using the plane wave solution, the above equation yields

$$\omega^2 \pm \frac{\omega \omega_A^2}{\Omega_i} - \omega_A^2 = 0. \tag{25}$$

For the case of low frequency waves ($\frac{\omega_A}{\Omega_i} \ll 1$), Eq. (25) reduces to

$$\omega_\pm = \omega_A \left(1 \mp \frac{\omega_A}{2\Omega_i}\right), \tag{26}$$

which gives the linear dispersion relation for finite amplitude spin-1/2 Alfvén waves modified by the spin magnetization along with the arbitrary temperature degeneracy effect. The quantity ω_A is the Alfvén wave frequency (modified) with associated wave number k_A such that $\omega_A = k_A V_A (1 - \frac{\varepsilon_o^2 \beta}{2\delta G})^{\frac{1}{2}}$. For classical plasmas, i.e., in the case of complete non-degeneracy, Eq. (26) reduces to the same expression as obtained in Ref. 45, with $\omega_A = k_A V_A$. The same result[26] is also obtained in Ref. 33 for the case of complete degenerate plasmas with $\omega_A = k_A V_A \sqrt{(1 - \varepsilon_o^2 \beta)}$. It is important to note that the dispersion relation given by Eq. (26) represents a wave propagating towards right, whereas the wave travelling in the opposite direction,

$$\boldsymbol{\omega}_\pm = -\boldsymbol{\omega}_A \left(1 \mp \frac{\omega_A}{2\Omega_i}\right),$$

is not considered here. In order to determine the changes in transverse magnetic field $B_\pm$, number density n, and parallel fluid velocity u_z (Alfvénic fluctuations), we linearize equations (20), (22), and (23) by putting $v_z = \delta v$ and $n = 1 + \delta n$ to get the following set of equations:

$$\begin{aligned}&\frac{\partial^2 B_\pm}{\partial t^2} + \frac{\partial}{\partial z}\left[\delta v \frac{\partial B_\pm}{\partial z} + \frac{d}{dt}(\delta v B_\pm)\right] - \frac{\partial}{\partial z}\left[(1 - \delta n)\frac{\partial B_\pm}{\partial z}\right] \\ &\quad + \frac{\varepsilon_o^2 \beta}{2\delta G}\frac{\partial}{\partial z}\left[(1 - \delta n)\frac{\partial}{\partial z}[(1 + \delta n)B_\pm]\right] \\ &\quad \pm i \frac{\partial}{\partial z}\frac{d}{dt}\left[(1 - \delta n)\frac{\partial B_\pm}{\partial z}\right] \mp i \frac{\varepsilon_o^2 \beta}{2\delta G}\frac{\partial}{\partial z}\frac{d}{dt} \\ &\quad \times \left[(1 - \delta n)\frac{\partial}{\partial z}[(1 + \delta n)B_\pm]\right] = 0,\end{aligned} \tag{27}$$

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\beta \delta G}{2}\frac{\partial^2}{\partial z^2} + \frac{H_e^2}{2}\frac{\partial^4}{\partial z^4}\right)\delta n = \frac{1}{2}\left(1 - \frac{\varepsilon_o^2 \beta}{2\delta G}\right)\frac{\partial^2 |B_\pm|^2}{\partial z^2}, \tag{28}$$

$$\frac{\partial\,\delta n}{\partial t}+\frac{\partial}{\partial z}(\delta v)=0. \tag{29}$$

Equations (27)–(29) constitute a set of modified Zakharov equations for Alfvén solitary waves propagating in spin quantum plasmas with the arbitrary temperature degeneracy effect. For the case of complete degenerate plasmas ($2\delta G\sim 1$), the above set of equations reduces to the results obtained in Ref. 33. Similarly, by ignoring the spin effect, i.e., by substituting $\varepsilon_o=0$, we obtain the Zakharov equations of Ref. 32. Equation (28) is similar to one of the quantum Zakharov equations obtained for Langmuir envelope solitons,[46] where the term on the right hand side is the ponderomotive force, showing that density fluctuations are caused by the transverse magnetic pressure gradient for Alfvén wave solitons in spin quantum plasmas. To proceed with the analysis, we begin by inserting an Alfvén wave of the form

$$\mathbf{B}_{\pm}=b(z,t)\,\exp\left[i(k_Az-\omega_{\pm}t)\right] \tag{30}$$

into Eqs. (27) and (28), with $b(z,t)$ being the real amplitude, $\omega_{\pm}$ the frequency, and k_A the wavenumber of the carrier wave. Introducing the inequality $\omega_A/\Omega_i\ll 1$, with the assumption of a set of stretched time variable τ, such that $1/\tau\ll\omega_{\pm}\approx\omega_A$,[32,45] we finally obtain the following set of equations with

$$i\frac{\partial b}{\partial t}+iv_g\frac{\partial b}{\partial z}+\frac{\omega_A^2}{2}\delta nb-k_A\delta vb\mp\frac{1-\alpha}{2}\frac{\partial^2 b}{\partial z^2}=0, \tag{31}$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\beta\delta G}{2}\frac{\partial^2}{\partial z^2}+\frac{H_e^2}{2}\frac{\partial^4}{\partial z^4}\right)\delta n=\frac{1}{2}(1-\alpha)\frac{\partial^2|b|^2}{\partial z^2}, \tag{32}$$

and

$$\frac{\partial\,\delta n}{\partial t}+\frac{\partial}{\partial z}(\delta v)=0, \tag{33}$$

where $v_g=(1-\alpha)(1\mp k_A)$ is the group velocity (normalized) of the Alfvén wave with $k_A\sqrt{1-\alpha}\approx\omega_A$ and $\alpha=\frac{\varepsilon_o^2\beta}{2\delta G}$. The above set of equations, i.e., (31)–(33), are the nonlinear evolution equations describing the variations in the perturbed magnetic field $b(z,t)$, the plasma number density $\delta n(z,t)$, and the velocity $\delta v(z,t)$ for a nonlinear wavepacket propagating along the ambient magnetic field through a spin quantum plasma with the arbitrary temperature degeneracy effect of electrons taken into account. The set of nonlinear evolution equations for a classical plasma[45] can be retrieved from Eqs. (31)–(33) by ignoring terms, i.e., H_e and ε_o, with $\beta=c_s^2/V_A^2$ (c_s being the ion acoustic speed given by $c_s=(\frac{k_BT_e}{m_i})^{1/2}$). For classical plasmas, for the limit of $|b|\to 0$, Eq. (32) reduces to the quantum acoustic wave equation, whereas by ignoring the nonlinear coupling in the system, Eq. (31) becomes the Schrödinger equation for a free particle with $\delta v(z,t),\ \delta n(z,t)\to 0$.

IV. SAGDEEV POTENTIAL FOR SPIN ALFVÉN WAVES WITH THE EFFECT OF ARBITRARY TEMPERATURE DEGENERACY

In the current section, we apply the Sagdeev potential approach for the spin-1/2 Alfvén wave by neglecting the Bohm potential term while retaining the Fermi pressure and magnetization energy terms duly modified with the arbitrary temperature degeneracy effect of electrons in Eq. (32). In order to obtain an energy integral equation (nonlinear differential equation) from the set of Eqs. (31)–(33), we choose a transformed coordinate ξ by shifting to a comoving frame of reference such that

$$\xi=z-v_ot, \tag{34}$$

where v_o represents a constant speed normalized by the Alfvén velocity V_A, called the Alfvénic Mach number. Now, by using Eq. (34) in Eq. (32), we obtain $\delta n=\frac{1-\alpha}{2(v_o^2-\gamma)}|b|^2$, with $\gamma=\frac{\beta\delta G}{2}$. Here, we have ignored the dimensionless parameter H_e that arises due to the collective electron tunnelling effect through the so-called Bohm potential. Similarly, putting Eq. (34) in Eq. (33) yields $\delta v=v_o\delta n$. Now, substituting the values of δn and δv in Eq. (31), we get the following expression:

$$-i(v_o-v_g)\frac{\partial b}{\partial\xi}\mp\frac{1-\alpha}{2}\frac{\partial^2 b}{\partial\xi^2}+\frac{k_A(1-\alpha)}{2(v_o^2-\gamma)}(t-v_o)b|b|^2=0, \tag{35}$$

where $t=\frac{\sqrt{1-\alpha}}{2}$. Furthermore, by using the expression for the complex perturbed magnetic field, i.e., $b=A(\xi)e^{i\phi(\xi)}$ in Eq. (35), and separating the resulting equation into real and imaginary parts, we obtain

$$(v_o-v_g)A\frac{\partial\phi}{\partial\xi}\mp\frac{1-\alpha}{2}\left[\frac{\partial^2 A}{\partial\xi^2}-A\left(\frac{\partial\phi}{\partial\xi}\right)^2\right]+\frac{k_A(1-\alpha)}{2(v_o^2-\gamma)}(t-v_o)A^3=0, \tag{36}$$

$$-(v_o-v_g)\frac{\partial A}{\partial\phi}\mp\frac{1-\alpha}{2}\left[2\frac{\partial A}{\partial\xi}\frac{\partial\phi}{\partial\xi}+A\frac{\partial^2\phi}{\partial\xi^2}\right]=0. \tag{37}$$

Here, A and ϕ are real quantities whose values will be determined presently. Integrating Eq. (37) yields[47] $\phi=-\mp\frac{1}{2F}\times(M_A-Vg)\xi$. Substituting this value into (36), we obtain the nonlinear differential equation

$$\frac{d^2A}{d\xi^2}+\frac{d}{dA}V(A)=0, \tag{38}$$

where the Sagdeev potential (pseudopotential) is defined as

$$V(A)=\mp\frac{k_A(t-v_o)}{4(v_o^2-\gamma)F}A^4-\frac{3(v_o-v_g)}{2(1-\alpha)^2}A^2. \tag{39}$$

A straight forward procedure can be applied to investigate the Sagdeev potential. For instance, from the roots of $V(A)=0$, we obtain $A=0$ and

$$A=\pm\sqrt{\mp\frac{6(v_o-v_g)^2(v_o^2-\gamma)}{k_A(1-\alpha)^2(t-v_o)}}, \tag{40}$$

where the upper negative sign inside the square root corresponds to the right-handed circularly polarized Alfvén wave

(RCPAW), while the lower positive sign corresponds to the left-handed circularly polarized Alfvén wave (LCPAW). We may obtain various conditions for the formation of solitary structures through Sagdeev potential for different values of quantum plasma beta and magnetization energy with the effects of arbitrary temperature degeneracy of electrons, by using the well-established conditions.[48]

In order to have a real amplitude A of the pseudopotential $V(A)$, given by Eq. (40), the following conditions must be met for the formation of RCPAW:

(1) $v_o^2 > \gamma \Rightarrow v^2 > \delta G c_{qs}^2$ (super-quantum acoustic) and $t < v_o \Rightarrow \frac{k_A V_A}{2} < v$ (super-spin Alfvénic)
(2) $v_o^2 < \gamma \Rightarrow v^2 < \delta G c_{qs}^2$ (sub-quantum acoustic) and $t > v_o \Rightarrow \frac{k_A V_A}{2} > v$ (sub-spin Alfvénic)

These conditions show that the solitary structures for the RCPAW will form when the wave is either super-quantum acoustic and super-spin Alfvénic or sub-quantum acoustic and sub-spin Alfvénic.

Now, for the LCPAW, the conditions that lead to the real value of A are as follows:

(1) $v_o^2 > \gamma \Rightarrow v^2 > \delta G c_{qs}^2$ (super-quantum acoustic) and $t > v_o \Rightarrow \frac{k_A V_A}{2} > v$ (sub-spin Alfvénic)
(2) $v_o^2 < \gamma \Rightarrow v^2 < \delta G c_{qs}^2$ (sub-quantum acoustic) and $t < v_o \Rightarrow \frac{k_A V_A}{2} < v$ (super-spin Alfvénic)

Thus, the set of conditions obtained for solitary wave propagation are different in this case as compared to the case for the RCPAW. These conditions predict that the solitary structures are formed for LCPAW when the wave is super-quantum acoustic and sub-spin Alfvénic or when it is sub-quantum acoustic and super-spin Alfvénic.

V. RESULTS AND DISCUSSION

In this section, we are going to analyze the main results of Sec. IV, i.e., the general expression obtained for the Sagdeev potential of Eq. (39), and discuss parametrically the associated solitary profile in spin quantum plasmas. To obtain the conditions for the existence of nonrelativistic nearly degenerate (ND) and nearly non-degenerate (NND) electron quantum fluid, we use typical numerical values relevant to the dense plasmas of compact astrophysical objects. In such dense astrophysical scenarios, like the interiors of massive white dwarf stars and the atmosphere of neutron stars, the density, magnetic field, and temperature vary over a wide range of values. For example, the magnetic field in some white dwarf stars (neutron stars) is estimated to be varying from the fraction of a tesla to few megatesla (teratesla).[49,50] For such regimes, the density lies in the range 10^{29}–10^{35} m^{-3}, and the temperature is estimated to lie in the range 10^5–10^7 K. Since the parameters such as quantum plasma beta β, magnetization energy ε_o, and Fermi energy $\mathcal{E}_{Fe}$ are the functions of particle density n_o, magnetic field B_o, and temperature T_{Fe}, any change in these parameters will consequently alter the wave dynamics. Using the numerical values of constants in SI units, the quantum statistical parameter (quantum plasma beta) can be written as $\beta \sim 10^{-28}\,(n_o^{5/3} T_{Fe}/B_o^2)$, and the normalized Zeeman energy due to electron spin is $\varepsilon_o = \frac{\mu_B B_o}{\mathcal{E}_{Fe}}$, with $T_{Fe} = \frac{(3\pi^2 n_o)^{2/3}\hbar^2}{2k_B m_e}$. For the above-mentioned dense astrophysical objects, the quantum statistical parameter β has some finite values, and the normalized Zeeman energy due to the electron spin effect ε_o is of the order of unity or less. To check the validity of the quantum plasma model devised in Secs. II and III, we will calculate the numerical values of the relevant plasma parameters of a white dwarf with a system temperature of $T \approx 10^6$ K. Our theory is more relevant in the intermediate regimes, where the Fermi temperature must be comparable to the thermal temperature of electrons, i.e., $T_F \approx T$; otherwise, the fully degenerate and fully nondegenerate limits are well established and are sufficiently accurate.

A. Nearly degenerate (ND) case

For the discussion of the nearly degenerate case, the parameters chosen are $n_o = 10^{33}\mathrm{m}^{-3}$, $B_o = 10^6$T, $m_e = 9.1 \times 10^{-31}$kg, and $m_i = 1.67 \times 10^{-27}$kg. The Fermi temperature of the degenerate electrons is a function of the plasma density, i.e., $T_{Fe} \propto n_o^{\frac{2}{3}}$, which for the above-mentioned plasma density comes out to be 3.8×10^7 K. Similarly, for the chosen parameters, the values of the plasma beta and Zeeman energy turns out to be $\beta = 3.8$ and $\varepsilon_o = 0.016$, respectively. Since the value of the Fermi temperature is slightly larger than the system temperature, the condition $T_F > T$, for nearly degenerate electrons in plasma, is justified. The ions remain nondegenerate in the present model due to the fact that their Fermi temperature $T_{Fi} = 2 \times 10^4$K is much smaller than the system temperature $(T_{Fi} \ll T_i)$. The condition for degenerate electrons to be in the non-relativistic regime is $k_B T_F \ll m_e c^2$, where $k_B T_F$ is the Fermi energy and $m_e c^2$ is the rest mass energy associated with the degenerate electrons. In our model, the Fermi energy comes out to be 5.2×10^{-16}J, which is clearly less than the rest mass energy of the degenerate electrons 8.2×10^{-14}J, justifying the above condition. Furthermore, the value of the magnetization energy, i.e., $\mu_B B_o = 8.8 \times 10^{-17}$J, in the current model is comparable to the Fermi energy of the electrons, thus leading to the justification of the inclusion of the spin effects of electrons in the model. For the case when $\mu_B B_o$ (magnetization energy) $\ll k_B T_F$ (Fermi energy), the spin effects of the degenerate electrons can be ignored.

The general expression obtained for Sagdeev potential given by Eq. (39), for right-handed circularly polarized Alfvén waves (RCPAWs) in nearly degenerate (ND) spin-1/2 quantum plasma with arbitrary degeneracy effects of electrons, is plotted at different values of $\delta\,(=\frac{T_e}{T_F})$, as shown in Fig. 1. The corresponding solitary profile variation with respect to δ is depicted in Fig. 2, where we have plotted the soliton amplitude A against the space coordinates η. It can be seen form Figs. 1 and 2 that the amplitudes of Sagdeev potential and of the associated soliton structure decrease with the increasing values of δ. Increasing the value of δ correspondingly decreases the value of the arbitrary temperature degeneracy factor $G\,(=\frac{2}{5\delta}[1-\frac{\pi^2}{12}\delta^2])$ for the nearly degenerate (ND) case, and since that is proportional to the amplitude, as a result, the amplitude decreases. Decreasing

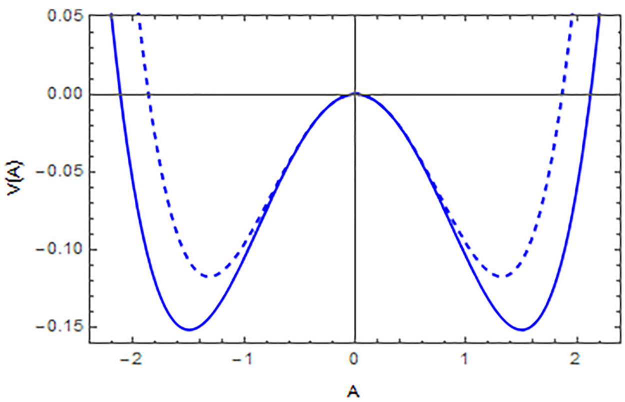

FIG. 1. Variation of Sagdeev potential $V(A)$ versus amplitude A for right circularly polarized Alfvén waves in a nearly degenerate regime for different values of δ such that $\delta = 0.1$ (solid blue line) and $\delta = 0.5$ (dashed blue line) with other physical parameters $\beta = 3.8$ and $\varepsilon_o = 0.016$.

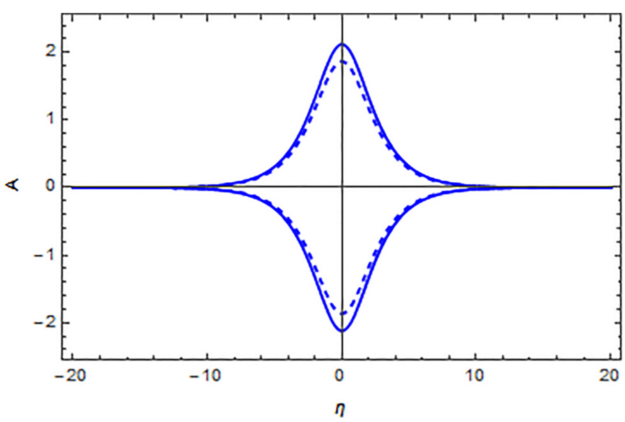

FIG. 2. Variation of soliton amplitude A versus coordinates η for right circularly polarized Alfvén waves in a nearly degenerate regime with the same numerical values as in Fig. 1.

degeneracy means that the number density minifies, which resultantly reduces the system energy and hence the amplitude of soliton. Figures 3 and 4 show the same pattern as observed in Figs. 1 and 2, where the Sagdeev potential of (39) and related soliton solution for left-handed circularly polarized Alfvén waves (LCPAWs) are plotted for two values of δ. However, for the case of LCPAW, the decrease in the amplitude is more accompanied by the decrease in the width of the soliton.

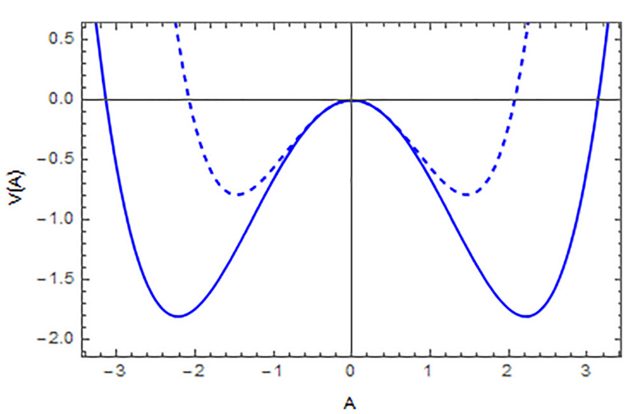

FIG. 3. The plot of Sagdeev potential $V(A)$ as a function of amplitude A for various values of δFor left circularly polarized Alfvén waves in a nearly degenerate limit such that $\delta = 0.1$ (solid blue line) and $\delta = 0.5$ (dashed blue line) with $\beta = 3.8$ and $\varepsilon_o = 0.016$.

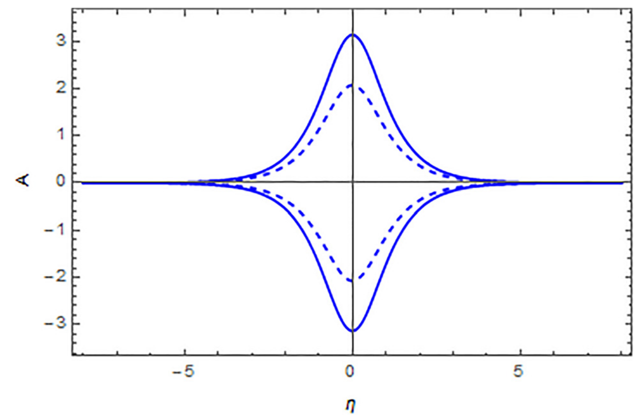

FIG. 4. Corresponding soliton amplitude variation with respect to coordinates η for left circularly polarized Alfvén waves in a nearly degenerate limit with the same numerical parameters as in Fig. 3.

B. Nearly non-degenerate (NND) case

In order to investigate the propagation of spin-1/2 Alfvén waves in nearly non-degenerate regimes, we chose the plasma parameters as $n_o = 10^{29}\mathrm{m}^{-3}, B_o = 10^3\mathrm{T}, m_e = 9.1 \times 10^{-31}\mathrm{kg}$, $m_i = 1.67 \times 10^{-27}\mathrm{kg}$, and $\delta = 10$. For these typical parameters of the dilute degenerate plasma limit, the values of Fermi temperature T_F, quantum plasma beta β, and Zeeman energy ε_o turn out to be $8.2 \times 10^4\,\mathrm{K}$, 0.82 and 0.008, respectively. Thus, the condition $T_F < T$ holds for the nearly non-degenerate (NND) electrons because the Fermi temperature is slightly smaller than the system temperature. For the chosen plasma density, the value of the ion Fermi temperature is 44.7 K, which is again much smaller than the system temperature $(T_{Fi} \ll T_i)$, and thus, ions remain nondegenerate. The condition for electrons to be in the non-relativistic regime, i.e., $k_B T_F \ll m_e c^2$, is also valid in nearly nondegenerate limits.

Figures 5 and 6 show plots of Sagdeev potential given by Eq. (39) and the corresponding soliton structure variation for RCPAW in nearly nondegenerate spin-1/2 quantum

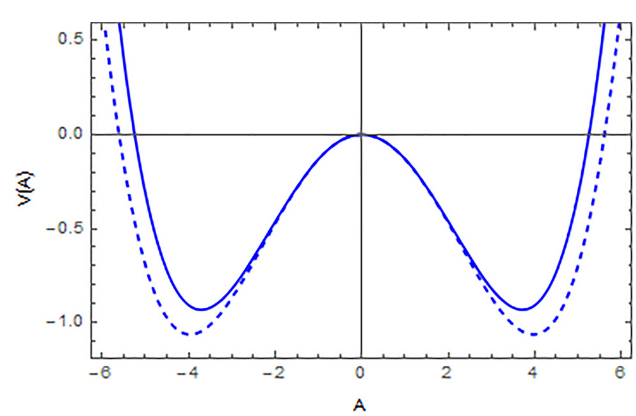

FIG. 5. Variation of Sagdeev potential $V(A)$ versus amplitude A for right circularly polarized Alfvén waves in a nearly non-degenerate regime for two values of ζ such that $\xi = 0.1$ (solid blue line) and $\xi = 0.9$ (dashed blue line) with $\beta = 0.82$, $\varepsilon_o = 0.008$, and $\delta = 10$.

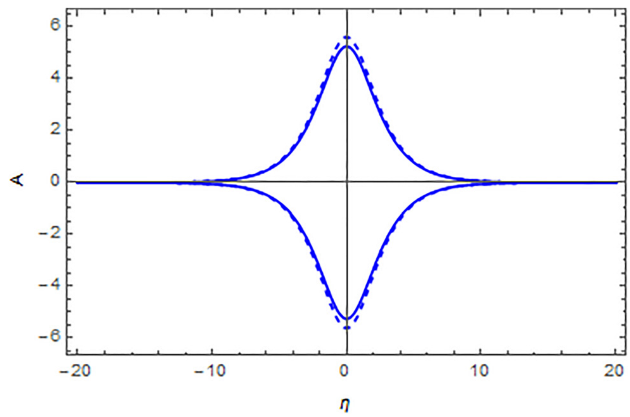

FIG. 6. Variation of soliton amplitude A versus coordinates η for right circularly polarized Alfvén waves in a nearly non-degenerate regime with the same numerical values as in Fig. 5.

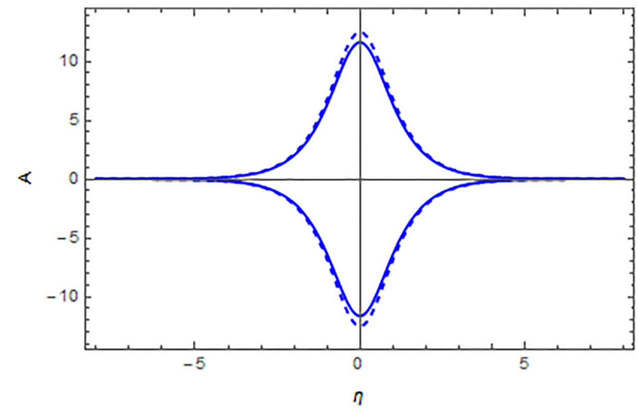

FIG. 8. Corresponding soliton amplitude variation with respect to coordinates ξ for left circularly polarized Alfvén waves in a nearly non-degenerate limit with the same numerical parameters as in Fig. 7.

plasma for two values of $\xi\,(=e^{\beta'\mu}, \beta'=\frac{1}{k_BT_e})$. The parameter ξ, which is a function of chemical potential μ and electron temperature T_e, describes the degeneracy. For the case of the nearly nondegenerate (NND) limit, $\xi \ll 1$ ($\frac{\mu}{k_BT_e}$ is large and negative), whereas for the nearly degenerate (ND) limit, $\xi \gg 1$ ($\frac{\mu}{k_BT_e}$ is large and positive). In Fig. 5, the amplitude of Sagdeev potential $V(A)$ is observed to enhance with the increasing values of parameter ξ. The amplitude of the associated solitary profile, which is a function of the space coordinates η, also increases for the given parameters, as shown in Fig. 6. For the case of the nearly nondegenerate limit, the increase in the value of parameter ξ is accompanied by the increase in the arbitrary temperature degeneracy factor $G\,(=1+\frac{\xi}{2^{\frac{5}{2}}})$, and since G is proportional to the amplitude, consequently, the amplitude increases. From 0 to higher values of ξ (less than one) means that in the non-degenerate system, some degenerate levels add due to the higher values of number density, which consequently increases the amplitude of Sagdeev potential and of soliton. The same increasing trend is depicted in Figs. 7 and 8 where the Sagdeev potential and the corresponding soliton structure are plotted with the same numerical values for left-handed circularly polarized Alfvén waves (LCPAWs).

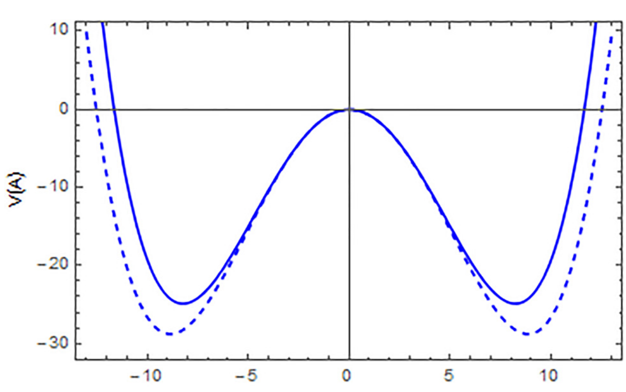

FIG. 7. Effect of temperature degeneracy on Sagdeev potential $V(A)$ as a function of amplitude A for left circularly polarized Alfvén waves in a nearly non-generate regime such that $\xi = 0.1$ (solid blue line) and $\xi = 0.9$ (dashed blue line). Other physical parameters are chosen as $\beta = 0.82$, $\varepsilon_o = 0.008$, and $\delta = 10$.

VI. SUMMARY AND CONCLUSIONS

In this paper, we have investigated the propagation of nonlinear circularly polarized Alfvén waves in a nonrelativistic, magnetized spin-1/2 quantum plasma with arbitrary degeneracy effects of electrons. We have assumed the ambient magnetic field to be in the z-direction, and the propagation was also considered in the parallel direction. Employing a local Fermi-Dirac distribution function, a modified equation of state was utilized which is applicable for both the dilute and dense cases. Besides degeneracy, the spin effect of electrons was also incorporated through the inclusion of the spin force along with the spin magnetization current. In this regard, a linear dispersion relation for finite amplitude Alfvén waves duly modified by spin magnetization and arbitrary temperature degeneracy effects of electrons was derived. A modified set of Zakharov equations have been derived by using a two time scale perturbation approach. The nonlinear coupling of Alfvén waves and quantum acoustic waves in the presence of arbitrary temperature degeneracy effects of electrons in both nearly degenerate (ND) and nearly non-degenerate (NND) quantum plasmas has also been investigated. The soliton solutions for nearly degenerate (ND) and nearly non-degenerate (NND) spin-1/2 quantum plasmas have been discussed through the well-known Sagdeev potential (pseudopotential) approach. The effects of the values of arbitrary temperature degeneracy factor G for both the right-hand and left-hand circularly polarized Alfvén waves in ND and NND limits were discussed. In the case of the nearly degenerate (ND) limit, the amplitude of Sagdeev potential and of the related solitary profile has been found to diminish with the increasing values of $\delta\,(=\frac{T_e}{T_F})$ (decreasing G) for both RCPAW and LCPAW. Decreasing the value of arbitrary temperature degeneracy factor G means that the number density minifies, which resultantly reduces the system energy and hence the amplitude of soliton. However, for the case of the nearly non-degenerate (NND) limit, the amplitude of Sagdeev potential and of the associated soliton structure was observed to enhance with the increase in the value of G for both kinds of circularly polarized Alfvén waves. In this case, increasing the value of G means that in the NND system, some degenerate levels add due to the higher values of number density which consequently increased the amplitude of Sagdeev potential and of soliton.

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