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Nonlinear ion acoustic waves in a relativistic degenerate plasma with Landau diamagnetism and electron trapping

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Abstract. We consider the effects of trapping in a relativistic degenerate plasma with quantizing magnetic field. The linear dispersion relation for such an ion acoustic wave is derived and the propagation characteristics of these waves are discussed. The Sagdeev potential for the formation of arbitrary amplitude solitary structures is obtained and it is seen that only compressive solitary structures are formed for the relativistic degenerate quantized magnetoplasma. The dependence of the linear and nonlinear propagation characteristics of ion acoustic waves on different plasma parameters is explored. A comparison is also made with the previous studies. The useful applications of the present investigation in dense astrophysical environments like white dwarf stars and neutron stars, in semiconductor physics, in high-energy density physics, in inertial confinement fusion and in next generation laser–plasma interaction experiments are pointed out.

1 Introduction

Nonlinear effects have been investigated for more than three decades for classical plasmas in the relativistic regime. Demchenko and El-Naggar [1] made the early investigations in this field and it was shown that the nonlinear forced oscillations have substantial effect in relativistic plasmas. It was demonstrated that owing to the relativistic nature of the oscillations, the nonlinear terms that appear in the equation of motion lead to parametric instability besides limiting the oscillation amplitude in the response region. The studies of different nonlinear structures like solitary and shock structures, double layers etc. have experienced a great deal of interest in a relativistic degenerate dense plasma in recent times on account of their important applications in astrophysical objects like white dwarf stars, neutron stars and pulsar magnetospheres [2–4]. Compact astrophysical objects like white dwarf stars comprise of degenerate electrons and ions of heavy elements are supported by the observational evidence and theoretical analyses [5,6]. In dense Fermi plasmas, the properties of electron oscillations in the linear regime have been well studied in references [7–10] (and the references therein) where a quantum dispersion equation was derived to study the propagation of small longitudinal perturbations in electron–ion collisionless plasmas. In quantum plasmas, the quantization of orbital motion and the spin of electrons have their effect on the propagation of longitudinal waves which is reported in reference [11]. The two magnetic effects in the Fermion gas that are known to appear in the presence of a strong magnetic field are Pauli paramagnetism and Landau diamagnetism. Here, former is due to the spin of the electrons and latter corresponds to the quantization of the orbital motion of the electrons in the magnetic field, and is also known as the Landau quantization [12]. In reference [11] a dispersion relation for the longitudinal wave propagating parallel to the ambient magnetic field was derived and the dispersion relation was shown to have a strong dependence on the magnetic field in a degenerated plasma in the nonrelativistic regime. The thermodynamic properties of the medium and the propagation characteristics of the plasma waves are influenced by the presence of the strong and super strong magnetic fields and this has become an important area in the study of neutron stars, supernovae, the convective zone of the sun, the evolution of the universe in the early pre-stellar period and in laboratory plasmas and laser–matter interaction. It is observed that the interior and surface magnetic fields for neutron stars are different and the interior fields can have values of around 10^{15} G. Due to quantum effects at such large values of magnetic fields, the dynamical and thermodynamic characteristics of waves can be unusual. This is verifiable when the electron characteristic energy approaches the chemical

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potential on a Landau level. Thus, for the nonrelativistic limit $\mu_e = \varepsilon_{\mathrm{Fe}} = \frac{\hbar|e|B_0}{2m_e c}$, $B_0 = B_S \frac{v_F^2}{C^2}$, where Schwinger's magnetic field is $B_S = \frac{m_e^2 c^3}{\hbar|e|}$, and v_F is the Fermi velocity of electrons [13].

The effect of Landau quantization on the equations of state in degenerate plasma in the presence of a strong magnetic field has been derived and discussed by Eliezer et al. [14]. Quantum or degenerate plasmas have found applications in microelectronics [15], metals, semiconductors, carbon nanotubes, quantum dots and quantum wells [16–18] and in space and astrophysical objects like white dwarf stars and neutron stars [19]. The propagation of electrostatic and electromagnetic modes in the linear and nonlinear regimes has been presented in references [13,20–22].

The plasma dynamics for both unmagentized and magnetized, classical and quantum plasmas in nonrelativistic and relativistic regimes have been discussed in detail by Melrose [23]. The quantum hydrodynamic model has been used extensively to investigate wave propagation in degenerate plasma in both linear and nonlinear regimes [24,25]. The strong ambient magnetic field qualitatively changes the properties of the atoms, molecules and condensed matter whenever the electron cyclotron energy becomes larger than the typical Coulomb energy [26]. Because of the extreme confinement of electrons in the transverse direction the Coulomb force effectively binds the electrons in a direction parallel to the external magnetic field. So, in this case the Coulomb forces act as a perturbation to the magnetic forces while in the strong magnetic field the normal effect of Zeeman splitting of the energy levels is not taken to be a perturbation effect [27].

When trapping of the particles is considered, the nonlinear wave dynamics is modified as was shown by the Bernstein, Green and Kruskal model, where particle trapping was taken in the wave potential [28]. Later, Gurevich [29] explored trapping at the microscopic level and showed that adiabatic trapping produces a 3/2 power nonlinearity instead of quadratic nonlinearity in the absence of trapping. It has been observed that whenever the phase velocity increases, the trapped electron extracts energy from the wave and experiences acceleration. This phenomenon leads to generation of high energy particles in a media like plasma, whose energies can be estimated [30]. If the intensity of incident high frequency EM waves induced in a plasma, is high, the number density of plasma will be modulated by the high frequency pressure force resulting in the production of low-frequency longitudinal wave field which may trap the electrons [31]. In plasmas, the trapping of electrons by the longitudinal electric field not only effects the propagation of the nonlinear waves but can also change their existence probability. Heidari et al. [32] included the electron trapping to study the electrosound solitary waves for high intensity laser propagation in relativistic plasmas. The role of trapped particles has been studied experimentally for kinetic modelling of electromagnetic space tethers [33]. Many experiments and computer simulations have alluded to the significance of microscopic particle trapping [34]. The rapid development in the field of high-intensity lasers has drawn the attention of researchers to explore the light–matter interactions in the relativistic regime, where lighter particles like electrons can be accelerated close to the speed of light. Recent development in laser physics and laser plasma interactions has opened possibilities to investigate nonlinear effects in the relativistic regime [35–39]. This field has generated a lot of excitement and attention because of the possibility of examining many novel relativistic plasma physics issues, ranging from compact particle accelerators [40–44] to high energy density laboratory astrophysics [45–47] and fast ignition inertial fusion [48,49]. Electron trapping has also been found to play a role in electron–cyclotron maser emission from white dwarf pairs and white dwarf planetary systems [50].

In quantum or degenerate plasmas, trapping was first introduced by Shah et al. [51] by applying the Gurevich model for fully and partially degenerate plasma by investigating ion acoustic solitary structures in nonrelativistic regime. This technique was further applied in relativistic and ultra-relativistic fully and partially degenerate plasmas without the Landau diamagnetism effect in reference [52]. In references [53,54], the authors incorporated the trapping effect in partially and fully degenerate plasma in the presence of quantizing magnetic field leading to Landau quantization for both electron–ion and electron–positron–ion plasmas in the nonrelativistic regime, respectively.

In the present article, we investigate the effect of trapping in a fully relativistic degenerate plasma in the presence of Landau diamagnetism. In this scenario, we study the propagation of linear and nonlinear ion acoustic waves. The results of previous studies in degenerate plasmas with the effect of trapping are retrieved in the appropriate limiting cases.

The layout of the present work is as follows: in Section 2, starting from the basic set of equations, we derive the linear dispersion relation of ion acoustic wave for a relativistic degenerate plasma incorporating trapping and Landau diamagnetism. The nonrelativistic and ultra-relativistic limits with and without Landau diamagnetism are also derived. In Section 3, the nonlinear evolution equations are obtained and the ensuing Sagdeev potential is arrived at to consider soliton formation. Here too the limiting cases are investigated. In Section 4, we analyse and discuss our results. Finally, in Section 5, we recapitulate the main findings of paper.

2 Basic set of equations

We start with the derivation for the expression of electron number density which is relativistic and degenerate by considering adiabatic trapping of electrons in the electrostatic potential with Landau diamagnetism effect. We follow the method outlined in references [53,55] and obtain the expression for the relativistic energy of the electrons in an electrostatic potential with Landau quantization:

$$\left(\frac{\varepsilon_e^{\ell,\sigma} + e\varphi}{m_e c^2}\right)^2 = 1 + \frac{p_Z^2}{m_e^2 c^2} + \frac{\hbar\omega_{ce}}{m_e c^2}\left(2\ell + 1 - \sigma\right). \quad (1)$$

Here, $\omega_{ce} = eB_0/m_e c$ is the electron cyclotron frequency in the external magnetic field $\mathbf{B}_0 = B_0\hat{z}$, where $\hat{z}$ and B_0 are the orientation and magnitude of the ambient magnetic field in the Cartesian coordinates respectively, e and m_e are the charge and rest mass of electron, $\hbar = h/2\pi$ where h is the Planck's constant and φ is the electrostatic potential in which the electrons are trapped, and p_Z is the momentum parallel to the ambient magnetic field. The electrons in plasma in the presence of magnetic field arrange themselves in different levels with quantized energy levels (called Landau level) given by ℓ, and σ is the operator the z-component of which describes the spin orientation $\vec{s} = \frac{1}{2}\vec{\sigma}$, ($\sigma = \pm 1$).

From above equation one sees that the energy spectrum for electrons consists of lowest Landau levels $\ell = 0$, $\sigma = 1$ and pairs of degenerate levels with opposite polarization, $\sigma = -1$. Thus, each value with $\ell = 0$ occurs once whereas, it occurs twice for $\ell \neq 0$ [13]. Thus, the expression for the total quantized relativistic energy of electrons in the quantizing magnetic field can be written as

$$\varepsilon_e^\ell = m_e c^2 \left\{ \sqrt{1 + D\cdot\ell + u^2} - \frac{e\varphi}{m_e c^2} \right\}, \tag{2}$$

where $D = \frac{2\hbar\omega_{ce}}{m_e c^2}$ which shows the effect of Landau diamagnetism and $u^2 = \frac{p_Z^2}{m_e^2 c^2}$.

The number of quantum states of a particle in a volume V and interval dp_z is given by [12]

$$\frac{2V\,|e|\,H}{(2\pi\hbar)^2 c} dp_z = \frac{V\cdot D}{(2\pi)^2 X_c^3} du \tag{3}$$

where $X_c = \frac{\hbar}{m_e c}$. Using equation (3), the expression for the total occupation number for the relativistic degenerate electrons is given by

$$n_{eR} = \frac{D}{(2\pi)^2 X_c^3} \sum_{\ell=0}^{\infty} \int_{-\infty}^{\infty} du f_{\mathrm{Fe}}(u,\ell,\varphi). \tag{4}$$

Here, $f_{\mathrm{Fe}}(u,\ell,\varphi)$ is the Fermi–Dirac distribution function $f_{\mathrm{Fe}} = \frac{1}{e^{\frac{\varepsilon_e^\ell - \mu_e}{T_e}}+1}$, ε_e^ℓ is the quantized energy of the electrons, T_e is the thermal temperature and μ_e is the chemical potential or the limiting energy for the electron gas. Trapping takes place when the condition $\varepsilon_e^\ell = 0$ is satisfied and the energies $\varepsilon_e^\ell > 0$ and $\varepsilon_e^\ell < 0$ correspond to free and trapped electrons, respectively. In the limit $T_e \to 0$, the Fermi–Dirac distribution is best set out by the Heaviside step function $f_{\mathrm{Fe}} = H\left(\mu_e - \varepsilon_e^\ell\right)$ [13]. Following references [12,53], the summation may be replaced by integration and we get the expression for the total electron number density as follows:

$$n_{eR} = \frac{1}{2\pi^2 X_c^3} \frac{1}{\epsilon_R^3} \left\{ D\epsilon_R^2 \sqrt{(1+\Phi_R)^2 - \epsilon_R^2} + \frac{2}{3}\left((1+\Phi_R)^2 - \epsilon_R^2(1+D)\right)^{3/2} \right\}. \tag{5}$$

Here, $\epsilon_R = \frac{m_e c^2}{\mu_e}$ is the ratio of rest mass energy to the chemical potential and Φ_R is the normalized relativistic potential, i.e. $\Phi_R = \frac{e\varphi}{\mu_e}$ (subscript R denotes relativistic). The total chemical potential $\mu_e = \varepsilon_{\mathrm{Fe}} + m_e c^2$ is the sum of the Fermi energy and the rest mass energy. The expression for the Fermi energy is $\varepsilon_{\mathrm{Fe}} = \frac{\hbar^2}{2m_e}(3\pi^2 N_{e0})^{2/3}$. In the absence of the trapping potential $\Phi_R = 0$, the expression for the electron number density given by equation (5) shows

$$n_{e0,R} = \frac{1}{2\pi^2 X_c^3 \epsilon_R^3} \left\{ D\epsilon_R^2 \sqrt{1-\epsilon_R^2} + \frac{2}{3}\left(1 - \epsilon_R^2(1+D)\right)^{3/2} \right\}. \tag{6}$$

Here $n_{e0,R}$ is the equilibrium number density for electrons in the relativistic limit. From equation (6), it is observed that the equilibrium number density increases with the increase of the Landau diamagnetism effect (magnetic field). In other words, the occupation number in equilibrium state gets enhanced in the presence of magnetic field. From above equation, we see that the upper limit of the magnetic field is obtained by ensuring that the second term on the right-hand side of equation (6) remains real and this is given by $B_0 \leq (\frac{1}{\epsilon_R^2} - 1)B_s$, where B_s, the Schwinger's magnetic field, is given in the introduction.

The ions are treated as nondegenerate, nonrelativistic and cold due to their heavy mass in comparison with the lighter electrons. The ion equation of motion and the continuity equation are as follows:

$$\frac{\partial \vec{v}_i}{\partial t} + \left(\vec{v}_i \cdot \vec{\nabla}\right)\vec{v}_i = -\frac{e}{m_i}\vec{\nabla}\varphi + \frac{e}{m_i}\left(\vec{v}_i \times \vec{B}\right), \tag{7}$$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\cdot(n_i \vec{v}_i) = 0. \tag{8}$$

The ion Lorentz force term does not contribute since we consider motion parallel to the external magnetic field. The system of equations is closed by employing Poisson's equation:

$$\nabla^2\varphi = 4\pi e\,(n_e - n_i)\,. \tag{9}$$

Now linearizing equations (7)–(9) and applying the plane wave solution to the above equations, we arrive at the following linear dispersion relation for ion acoustic wave with relativistic degenerate electrons and Landau diamagnetism effect:

$$\frac{\omega^2}{k^2} = \frac{C_{SR}^2}{\Gamma_R + k^2\lambda_{\mathrm{TF},R}^2}. \tag{10}$$

Here, ω and k are the frequency and wave number of ion acoustic wave, while C_{SR} and $\lambda_{\mathrm{TF},R}$ are the relativistic Fermi ion sound velocity and the relativistic Thomas–Fermi length, respectively, and are given by the expressions. $C_{SR} = \sqrt{\frac{\mu_e}{m_i}}$, $\lambda_{\mathrm{TF},R} = \sqrt{\frac{\mu_e}{4\pi e^2 n_{e0,R}}}$. Also, the expression for Γ_R is given by

$$\Gamma_R = \frac{1}{y_e}\left(\frac{D\epsilon_R^2}{\sqrt{1-\epsilon_R^2}} + 2\left(1-\epsilon_R^2(1+D)\right)^{1/2}\right), \quad \text{where,}$$
$$y_e = D\epsilon_R^2\sqrt{1-\epsilon_R^2} + \frac{2}{3}\left(1-\epsilon_R^2(1+D)\right)^{3/2}.$$

It can be seen from equation (10) that the phase velocity of ion acoustic wave has a strong dependence on the magnetic field through Landau diamagnetism in a quantum-degenerate plasma and has no classical equivalent. In the absence of quantizing magnetic field, i.e., $D = 0$, equation (10) reduces to the expression for linear dispersion relation for fully degenerate relativistic electron–ion plasmas [52].

Limiting cases

Case I: Nonrelativistic case

In the case when $m_e c^2 \gg \varepsilon_{\mathrm{Fe}}$, we get the nonrelativistic limit and in this case, we retrieve the results presented in reference [53].

Case II: Ultra-relativistic case

In the ultra-relativistic case, we have $m_e c^2 \ll \varepsilon_{\mathrm{Fe}}$, thus $\mu_e \approx \varepsilon_{\mathrm{Fe}}$. In this case, the expression of electron number density (given by Eq. (5)) derived for the relativistic case reduces to the following expression for the ultra-relativistic limit:

$$n_{eU} = \frac{1}{2\pi^2 X_c^3}\left\{\epsilon_U D\left(1+\Phi_U\right) + \frac{2}{3}\left(\epsilon_U^2\left(1+\Phi_U\right)^2 - (1+D)\right)^{3/2}\right\}. \quad (11)$$

Here, U stands for ultra-relativistic and $\epsilon_U = \frac{\varepsilon_{\mathrm{Fe}}}{m_e c^2}$. In this case, the Fermi-energy should be calculated from the ultra-relativistic density. The Fermi energy, in general, for the ultra-relativistic case is replaced by the Fermi momentum P_{F} as $\varepsilon_{\mathrm{Fe}} = cP_{\mathrm{F}} = \hbar c\left(3\pi^2 N_{e0}\right)^{1/3}$. In the absence of the electrostatic potential which is the trapping potential, the expression for the electron number density reduces to

$$n_{e0,U} = \frac{1}{2\pi^2 X_c^3}\left\{\epsilon_U D + \frac{2}{3}\left(\epsilon_U^2 - (1+D)\right)^{\frac{3}{2}}\right\}. \quad (12)$$

From the above equation, the limiting magnetic field for ultra-relativistic case is given by $B_0 \leq (\epsilon_U^2 - 1)\frac{B_s}{2}$. It is observed that an extra factor appears with the Schwinger's limit for both relativistic and ultra-relativistic cases [13]. The limiting value of magnetic field is maximum for ultra-relativistic, intermediate for relativistic and minimum for nonrelativistic cases. To calculate the linear dispersion relation for ion acoustic wave in the ultra-relativistic case, we linearize the set of equations (7), (8) and (11). Applying the sinusoidal wave solution, we obtain the following linear dispersion relation for the ultra-relativistic regime:

$$\frac{\omega^2}{k^2} = \frac{C_{sU}^2}{\Gamma_U + k^2\lambda_{\mathrm{TF},U}^2}. \quad (13)$$

Here, $C_{SU} = \sqrt{\frac{\varepsilon_{Fe,U}}{m_i}}$ is the ultrarelativistic sound velocity and $\lambda_{\mathrm{TF},U} = \sqrt{\frac{\varepsilon_{Fe,U}}{4\pi e^2 n_{e0}}}$ is the ultra-relativistic Thomas–Fermi length. The factor Γ_U is given by

$$\Gamma_U = \frac{1}{y_e'}\left[\epsilon_U D + 2\epsilon_U^2\left(\epsilon_U^2 - (1+D)\right)^{1/2}\right], \quad \text{where}$$
$$y_e' = \epsilon_U D + \frac{2}{3}\left(\epsilon_U^2 - (1+D)\right)^{\frac{3}{2}}.$$

To study the nonlinear properties of the ion acoustic waves, we again consider the equation of motion and continuity equation for ions given by equations (7) and (8). Further, we consider the one-dimensional case such that the propagation of the ion acoustic wave is along the z-direction. We shift to a co-moving frame of reference by taking $\xi = z - vt$, here v is the propagation velocity of the nonlinear perturbation. To obtain the density of ions, we integrate equations (7) and (8) and then apply the boundary conditions such that all the perturbations vanish at infinity, i.e. $\xi \to \infty$, φ, or $\Phi_R, v_i \to 0$, and $n_i \to n_{i0}$, where n_{i0} is the equilibrium number density of ions such that $n_{e0,R} = n_{i0}$. We have the following expression for the nonlinear density of ions:

$$n_i = n_{e0,R}\left(1 - \frac{2\Phi_R}{M_R^2\alpha_R}\right)^{-1/2}. \quad (14)$$

Here, M_R is the relativistic Mach number which is defined as the ratio of perturbation speed to the phase speed of a wave, i.e. $M_R = \frac{v}{\omega/k}$ and α_R is given by $\alpha_R = \frac{1}{\Gamma_R + k^2\lambda_{\mathrm{TF},R}^2}$.

3 Sagdeev potential

In this section, we derive an expression for the Sagdeev potential in the relativistic case and arrive at the conditions for the existence of the solitary structures in the presence of Landau diamagnetism. We use equations (5) and (14) for electron and ion nonlinear number densities in the Poisson's equation (9) and obtain

$$\frac{d^2\Phi_R}{d\xi^2} = \frac{1}{y_e}\left\{D\epsilon_R^2\sqrt{\left(1+\Phi_R\right)^2 - \epsilon_R^2} + \frac{2}{3}\left(\left(1+\Phi_R\right)^2 - \epsilon_R^2(1+D)\right)^{3/2}\right\} - \left(1 - \frac{2\Phi_R}{M_R^2\alpha_R}\right)^{-\frac{1}{2}}. \quad (15)$$

Here, $\xi = \xi/\lambda_{\mathrm{TF},R}$. The above equation can be recast in the form of an energy integral using the analogy of a

$$V_R(\Phi_R)=\frac{-1}{\mathrm{y_e}}\left\{\frac{D\epsilon_R^2}{2}(1+\Phi_R)\sqrt{(1+\Phi_R)^2-\epsilon_R^2}+\frac{2}{3}\sqrt{(1+\Phi_R)^2-\epsilon_R^2(1+\mathrm{D})}\left(\frac{1}{4}(1+\Phi_R)^3-\frac{5}{8}\epsilon_R^2(1+\mathrm{D})(1+\Phi_R)\right)\right.$$
$$\left.-\frac{\mathrm{D}\epsilon_R^4}{2}\ln\left(1+\Phi_R+\sqrt{(1+\Phi_R)^2-\epsilon_R^2}\right)+\frac{\epsilon_R^4(1+D)^2}{4}\ln\left((1+\Phi_R)+\sqrt{(1+\Phi_R)^2-\epsilon_R^2(1+D)}\right)\right\}-M_R^2\alpha_{\mathrm{R}}\left(1-\frac{2\Phi_R}{M_R^2\alpha_{\mathrm{R}}}\right)^{\frac{1}{2}}+M_R^2\alpha_{\mathrm{R}}$$
$$+\frac{1}{y_e}\left\{\frac{D\epsilon_R^2}{2}\sqrt{1-\epsilon_R^2}+\frac{2}{3}\sqrt{1-\epsilon_R^2(1+D)}\left(\frac{1}{4}-\frac{5\epsilon_R^2}{8}(1+D)\right)-\frac{D\epsilon_R^4}{2}\ln\left(1+\sqrt{1-\epsilon_R^2}\right)+\frac{\epsilon_R^4(1+D)^2}{4}\ln\left(1+\sqrt{1-\epsilon_R^2(1+D)}\right)\right\} \tag{17}$$

$$V_U(\Phi_U)=-\frac{1}{y_e'}\left[\epsilon_U D\left(\Phi_U+\frac{{\Phi_U}^2}{2}\right)+\frac{2}{3}\left(\epsilon_U^2(1+\Phi_U)^2-(1+D)\right)^{\frac{1}{2}}\left\{\frac{\epsilon_U^2}{4}(1+\Phi_U)^3-\frac{5}{8}(1+D)(1+\Phi_U)\right\}\right.$$
$$\left.+\frac{1}{4\epsilon_U}(1+D)^2\ln\left[\epsilon_U^2(1+\Phi_U)+\epsilon_U\sqrt{\epsilon_U^2(1+\Phi_U)^2-(1+D)}\right]\right]-M_U^2\alpha_U(1-\frac{2\Phi_U}{M_U^2\alpha_U})^{1/2}$$
$$+\frac{1}{y_e'}\left[\frac{2}{3}(\epsilon_U^2-(1+D))^{\frac{1}{2}}\left\{\frac{\epsilon_U^2}{4}-\frac{5}{8}(1+D)\right\}+\frac{1}{4\epsilon_U}(1+D)^2\ln\left[\epsilon_U^2+\epsilon_U\sqrt{\epsilon_U^2-(1+D)}\right]\right]+M_U^2\alpha_U \tag{18}$$

particle in a potential well given by

$$\frac{1}{2}\left(\frac{d\Phi_R}{d\xi}\right)^2+V_R(\Phi_R)=0, \tag{16}$$

where $V_R(\Phi_R)$ is the Sagdeev potential for the relativistic case and is given by equation (17).

See equation (17) above.

We have evaluated the integration constants using the boundary conditions given earlier. The conditions necessary for the formation of the solitary waves given in references [5] and [51,53] (and the references therein) must be met. The range of the Mach number for the existence of the solitary wave structures is given by

$$1\le M_R^2<2\left(\frac{\frac{D\epsilon_R^2}{\sqrt{1-\epsilon_R^2}}+2\left(1-\epsilon_R^2(1+D)\right)^{1/2}}{D\epsilon_R^2\sqrt{1-\epsilon_R^2}+\frac{2}{3}(1-\epsilon_R^2(1+D))^{3/2}}\right).$$

Now we will discuss the Sagdeev potential expressions for nonrelativistic and ultra-relativistic cases.

For nonrelativistic limit, when $m_ec^2\gg\varepsilon_{\mathrm{Fe}}$, we retrieve the results of reference [53].

Similarly, in the limit $m_ec^2\ll\varepsilon_{\mathrm{Fe}}$, the expression for the Sagdeev pseudo-potential in the ultra-relativistic regime is calculated and is given by equation (18).

See equation (18) above.

Here V_U and M_U are the Sagdeev potential and the Mach number for the ultra-relativistic degenerate plasma respectively. Again, the ultra-relativistic Mach number is defined the same way as earlier for the relativistic case i.e. it is the ratio of the perturbation speed to the phase speed of the wave. Also, $\alpha_U=\frac{1}{\Gamma_U+k^2\lambda_{\mathrm{TF},U}^2}$.

Solitary structures would form under the specific conditions already discussed above in the relativistic case. Now in this ultra-relativistic limit, the range of the Mach number is given by

$$1\le \mathrm{M}_U^2<2\left(\frac{\epsilon_U D+2\epsilon_U^2\left(\epsilon_U^2-(1+D)\right)^{1/2}}{\epsilon_U D+\frac{2}{3}(\epsilon_U^2-(1+D))^{\frac{3}{2}}}\right).$$

4 Results and discussion

In this section, we discuss our results for both linear and nonlinear cases numerically. This is done since the expression of Sagdeev potential is quite cumbersome and cannot be investigated analytically. The parameters used in our numerical analysis pertain to dense astrophysical objects like white dwarf stars, neutron stars and pulsars. The typical electron number density ranges from 10^{26} to 10^{29} cm^{-3} for relativistic case and for ultra-relativistic case are of the order of 10^{30}–10^{32} cm^{-3} [56]. The magnetic field in the white dwarf stars can have values of upto 10^9 G while in the neutron star it is found that stronger magnetic fields may exist, and on the basis of the data provided by astrophysical observations, the magnetic field value on the surface of the neutron star can be of the order of $H=10^{11}$–10^{13} G, and the magnetic field in the interior of the neutron star can reach 10^{15} G or even higher [2,13,57–59]. The value of the magnetic field may increase by an additional factor of 10^3–10^4 G due to the presence of the rotation of the stars as shown by Bisnovati-Kogan [60]. The thermodynamic properties and the wave dynamics in degenerate plasmas are expected to be quite different and governed by quantum effects in such strong magnetic fields.

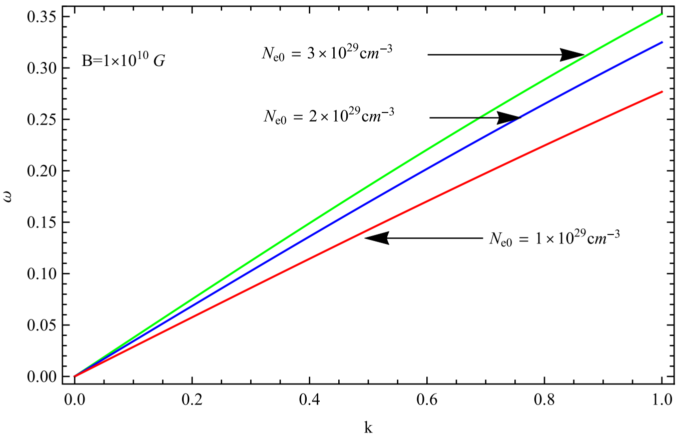

Fig. 1. Dispersion curves ω vs. k for varying plasma density, keeping the quantizing magnetic field constant.

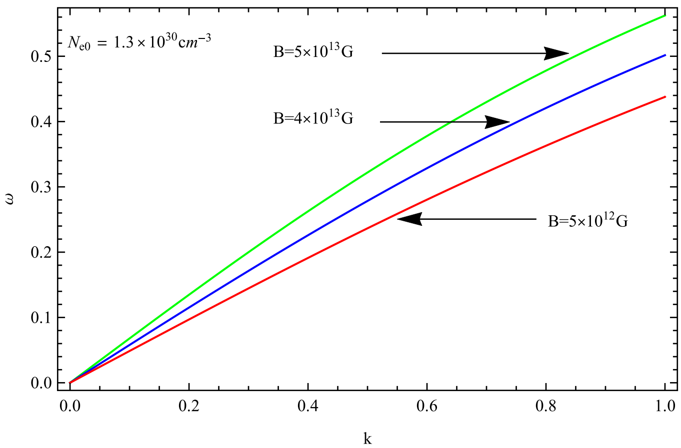

Fig. 2. Dispersion curves ω vs. k for varying quantizing magnetic field, keeping the plasma density constant.

4.1 Linear dispersion for the relativistic case

We note that the presence of Landau diamagnetism significantly modifies the expression of linear dispersion relation equation (10) for the ion acoustic waves with relativistic degenerate electrons. It is appropriate to mention here that unlike classical plasmas, trapping contributes in the linear regime through the coefficient Γ_R for quantum degenerate plasmas.

Figure 1 shows the plots of normalized frequency of the ion acoustic wave (normalized by the ions plasma frequency) versus the normalized wave number (normalized by relativistic Thomas–Fermi length $\lambda_{\mathrm{TF},R}$, equivalent to Debye length in classical plasma) for different values of ϵ_R which essentially means that we are studying the effects of variation in electrons number density. It is found that decreasing ϵ_R (or increasing the number density) enhances the frequency or phase speed of the ion acoustic waves in a relativistic-degenerate plasma.

Figure 2 explores the effect of Landau diamagnetism effect through parameter D which implies the effect of the variation of ambient magnetic field. It is found that enhancing the Landau diamagnetism effect enhances the frequency of the ion acoustic waves in a relativistic-degenerate plasma as opposed to the nonrelativistic case [53]. The quantization enhances the linear frequency by an order of magnitude in relativistic degenerate plasmas. It is, however, pertinent to mention here that the change in the frequency of the ion acoustic waves is significant for values of magnetic fields greater than 10^{12} Gauss which means that this effect will be important in neutron stars and pulsars where such magnetic fields are present.

4.2 Linear dispersion for ultra-relativistic case

In this section we consider ultra-relativistic case in the linear regime and investigate the effect of plasma number density and quantizing magnetic field on the frequency of ultra-relativistic ion acoustic waves, (see Eq. (13)). It is found once again that increasing ϵ_U (increasing the number density) enhances the frequency of the ion acoustic waves in ultra-relativistic degenerate plasmas. However, the frequency of the ion acoustic wave in the ultra-relativistic case is higher than its relativistic counterpart as can be seen from Figure 1. Now by keeping the factor ϵ_U fixed, we consider the effect of quantizing magnetic field through the factor D for the ultra-relativistic degenerate plasmas. We see that the frequency of the ion acoustic wave increases by increasing the quantizing magnetic field (Landau diamagnetism effect) through the factor D. Here we notice that for the ultra-relativistic degenerate case, the frequency of the ion acoustic wave is slightly lower than the relativistic case discussed earlier and shown in Figure 2. The values of the magnetic fields used here give strong quantization effect and are customarily found in super-dense astrophysical objects like neutron stars and pulsars [57–60].

4.3 Nonlinear results for relativistic case

Figure 3 shows the plots of Sagdeev potential $V_R(\Phi_R)$ versus the potential Φ_R given by equation (17) for the relativistic case and this figure exhibits the effect of decreasing ϵ_R (i.e. increasing number density) on the Sagdeev potential while all the other factors, i.e. relativistic Mach number and Landau quantization effect, are kept constant. It is found that the decreasing values of ϵ_R (or increasing number density) increases the width and depth of the Sagdeev potential. Figure 4 shows the corresponding solitary structures for nonlinear ion acoustic waves. It is found that the decreasing ϵ_R increases both the amplitude and width of the solitary waves.

Figure 5 shows the plots of the relativistic Sagdeev potential $V_R(\Phi_R)$ (i.e. Eq. (17)) versus relativistic potential Φ_R for different values of Landau quantization factor D keeping ϵ_R (number density) and relativistic Mach number M_R constant. It is observed that by increasing the values of quantizing magnetic field, i.e. Landau diamagnetism affects the width and the depth of the Sagdeev potential both showing an increase. The Sagdeev potential is deeper for larger values of magnetic fields implying that the Sagdeev potential minima becomes more negative and manifests greater sensitivity to the quantization effect at $B > 5 \times 10^{12}$ G. It is found that enhancing the

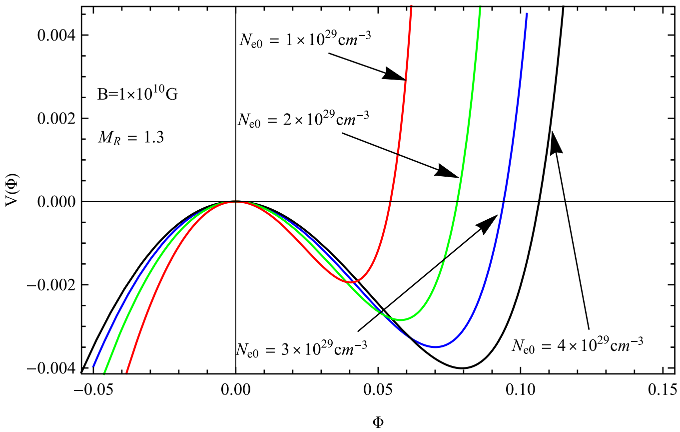

Fig. 3. Sagdeev potential curves $V_R(\Phi_R)$ vs. Φ_R for plasma density, keeping B and M_R constant.

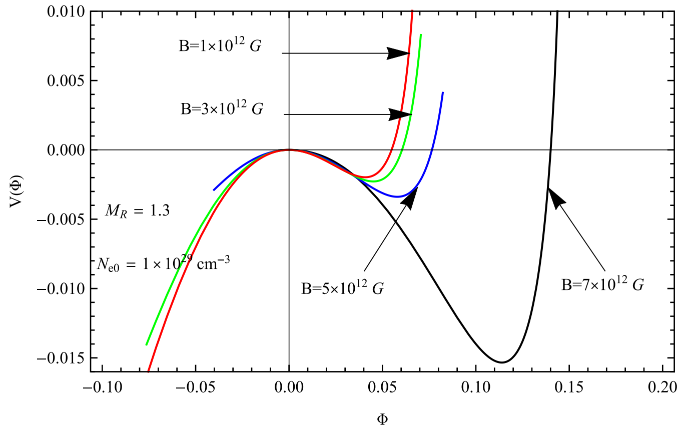

Fig. 5. Sagdeev potential curves $V_R(\Phi_R)$ vs. Φ_R for quantizing magnetic field, keeping plasma density and M_R constant.

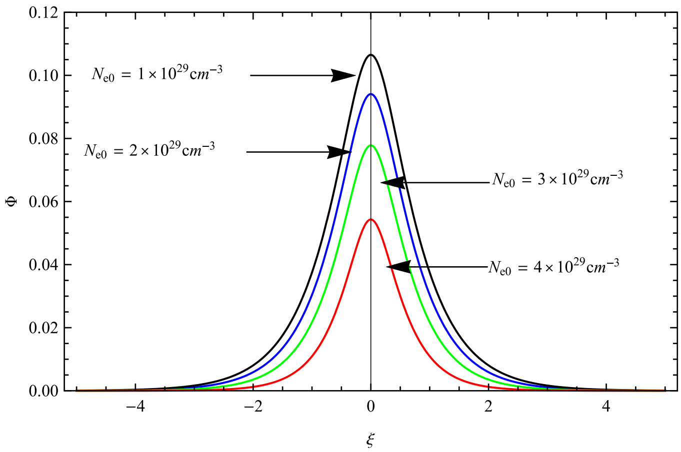

Fig. 4. Solitary wave amplitude Φ_R versus ξ, corresponding to the Sagdeev potential $V_R(\Phi_R)$ shown in Figure 3.

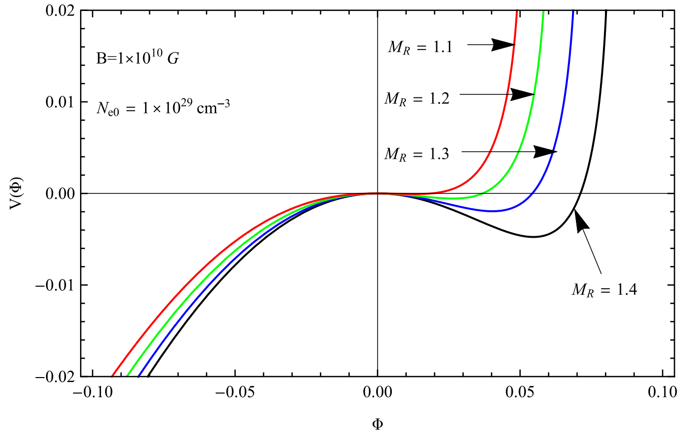

Fig. 6. Sagdeev potential $V_R(\Phi_R)$ vs. Φ_R curves for Mach number M_R, keeping plasma density and quantizing magnetic field constant.

Landau diamagnetism effects result in the increase of the amplitude Φ_R of the solitons. Initially the increment in the amplitude of the solitary structures is not as significant as the field is increased by regular intervals, but for high field value of the order of $B = 7 \times 10^{12}$ G there is an abrupt peak in the amplitude of the solitons. This may be because Landau quantization reinforces the nonlinearity in the system.

Figure 6 shows the plots for Sagdeev potential $V_R(\Phi_R)$ versus relativistic potential Φ_R. These curves are for different values of Mach number while the Landau diamagnetism effect (quantizing magnetic field) and the number density (relativistic energy) are kept fixed. It can be seen from the graphs that the Sagdeev potential becomes deeper at the higher values of the Mach number. Here in this case the minimum of the Sagdeev potential is decreased. The amplitude of the solitary structures increases as the value of the Mach number is increased keeping all the other parameters constant. We obtain the solitary structures similar to those obtained in Figure 4. The width of the solitary profiles decreases as the height or amplitude increases. The gradual change is observed in Sagdeev potential and corresponding soliton amplitude with the variation of relativistic Mach number unlike the case without quantization effects. Furthermore, the spatial extension against which the potential Φ_R is plotted is observed to be less as compared to our previous results in relativistic degenerate unmagnetized plasma [52].

4.4 Nonlinear results for ultra-relativistic case

In this section, we discuss the results numerically for the ultra-relativistic case. First, we observe the effect of variation of number density through the factor ϵ_U (or ultra-relativistic energy) on equation (18) keeping Landau diamagnetism factor D and ultra-relativistic Mach number M_U fixed. The value of the quantizing field that we have used in this case is $B = 1 \times 10^{10}$ G. It is observed that the depth of the Sagdeev potential and width of the corresponding potential increases as the relativistic energy is increased. In this case, we make a comparison of the depth and width of the Sagdeev potential with its counterpart in the relativistic case. Again, the values of the amplitudes are significantly higher than the amplitudes shown in Figure 4 for the relativistic case with the similar parameters used. Further, we plot the

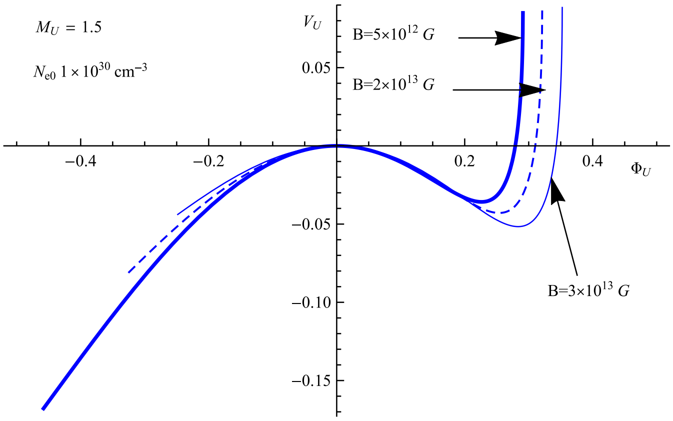

Fig. 7. Ultra-relativistic Sagdeev potential $V_U(\Phi_U)$ versus Φ_U curves for Quantizing magnetic field, keeping density and M_U constant.

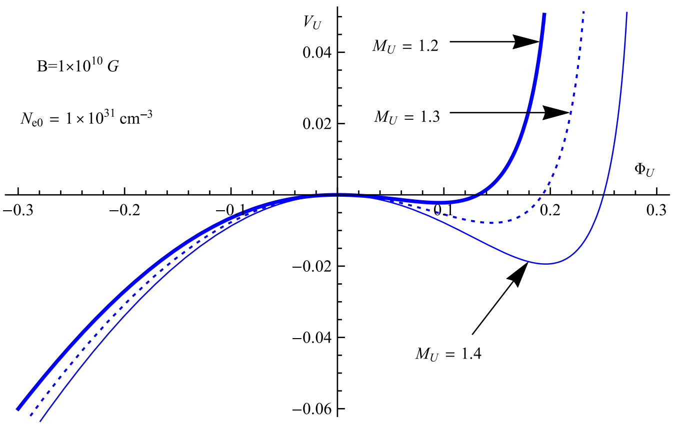

Fig. 8. Ultra-relativistic Sagdeev potential $V_U(\Phi_U)$ versus Φ_U curves for Mach number M_U, keeping plasma density and B constant.

ultra-relativistic Sagdeev potential against the potential for different values of Landau diamagnetism effect, keeping all the other factors fixed like number density ϵ_U (ultra-relativistic energy) and Mach number M_U as shown in Figure 7. The trend of these curves is like the relativistic case as discussed earlier in Figure 5. Increase in the width and depth of the Sagdeev potential is noticed in these plots with the higher values of the magnetic field which are of the order of 10^{13} G, such values are known to exist in the neutron stars and pulsars [13].

As we increase the value of the quantizing magnetic field through the Landau diamagnetic factor, the amplitude of the solitons tends to increase. Finally, we plot the curves between the Sagdeev potential $V_U(\Phi_U)$ and the potential Φ_U in the ultra-relativistic regime as shown in Figure 8. These are plotted for different values of the Mach number while the Landau diamagnetism factors D and ϵ_U including the effect of quantizing magnetic field and number density or the ultra-relativistic energy respectively are remained constant. We can obtain the solitary structures similar to those obtained in Figure 4.

Finally, it is appropriate to mention here that values of the amplitudes of solitary structures with Landau diamagnetism have appeared to be least for relativistic, intermediate for ultra-relativistic and highest for nonrelativistic degenerate plasma regimes. The same trend in amplitudes of the solitons has been observed for all three regimes with and without the quantization the effect, however, it has been found that amplitude of soliton is higher with the quantization effect in all three regimes by comparison with its counterpart without the quantization effect. Moreover, as was seen earlier [51,52] in partially degenerate nonrelativistic, relativistic and ultra-relativistic plasmas cases, we obtain both compressive and rarefaction solitons. However, in the fully degenerate case, we obtain only compressive solitary waves without and with the inclusion of the Landau quantization in these three plasma regimes. It is seen that the nonlinear results for both relativistic and ultra-relativistic cases show complicated dependence on the plasma parameters.

5 Conclusion

In conclusion, we have investigated the formation of ion acoustic solitary wave structures for both the relativistic and ultra-relativistic degenerate magnetoplasmas. The effect of trapping of electrons with Landau diamagnetism is incorporated for the first time to the best of our knowledge in a relativistic degenerate plasma. This work represents the most general framework that contains all the previous cases in certain limits but deviates significantly in its main result. We have derived the modified linear dispersion relation for ion acoustic wave for both relativistic and ultra-relativistic regimes and it is observed that the parallel propagating wave has strong dependence on the Landau quantization. Linear and nonlinear theoretical results have been investigated numerically for different parameters. We have presented our results numerically and the plots show the formation of solitary structures and their dependence on different plasma parameters. We have shown that only compressive solitons are formed for relativistic and ultra-relativistic plasmas. These results may play an important role in the description of complex phenomena that may be observed in dense astrophysical objects like white dwarfs and neutron stars and in the ultra-strong femtosecond laser–plasma interactions.

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Author contribution statement

M.J. Iqbal did the theoretical calculations and wrote the manuscript; H.A. Shah gave guidance in the progress of the calculation and results; W. Masood helped in the numerical work and discussion; N.L. Tsintsadze suggested the problem and gave some initial guidance. All the authors contributed in writing and editing the manuscript.

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