

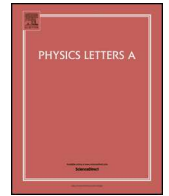


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# Energy behavior of spin electron cyclotron wave in a spin polarized plasma

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## ARTICLE INFO

### Article history:

Received 12 December 2018

Received in revised form 2 April 2019

Accepted 4 June 2019

Available online xxxx

Communicated by F. Porcelli

### Keywords:

Spin polarization

Energy flow

Longitudinal waves

Bohm potential

Solid state plasma

Degenerate plasma

## ABSTRACT

In degenerate quantum plasma the energy behavior of electrostatic modes propagating perpendicular to the external magnetic field is studied by employing the separated spin evolution quantum hydrodynamic (SSE-QHD) model. This model reveals that spin electron cyclotron wave (SECW) appears additionally with the upper hybrid wave (UHW). In case of SECW, the curves for the energy flow speed at different levels of spin polarization effect flip over at a particular value of wave number. The spin polarization effect enhances the energy flow speed before this value of wave number and then suppresses it afterward. The energy flow speed is enhanced by spin polarization effect in the entire range of wave number for the propagation of UHW. The Bohm potential effect drastically increases the energy flow speed at high wave number domain in both the waves. This study may find its applications to understand the energy behavior in spin polarized solid state plasmas

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## 1. Introduction

In recent times there has been great interest in the study of energy transport and energy flow speed of various waves in plasmas. The energy flow speed depends upon the energy flux density contributed by various types of energy sources [1–3]. For electromagnetic waves, Poynting theorem enables us to find the energy flux density and thus to determine the energy flow rate [4]. In case of electrostatic waves, the energy transfer is shared by the electric potential energy, kinetic energy and, for dense plasmas, quantum interaction energy. Recently, several works on this subject have been executed for electrostatic and electromagnetic waves propagating through three dimensional (bulk) and two dimensional (layered) electron-ion, magnetized and unmagnetized quantum Fermi plasmas [5–7].

Last two decades have witnessed intense activity on quantum plasma due to its numerous applications in laboratory plasma (in microelectronic devices, laser produced plasma, nano-systems) [8–10] and in extremely dense astrophysical objects (like white dwarfs, neutron stars, pulsars and magnetars) [11–13]. The cold and highly dense plasmas qualify for quantum treatment. The statistical quantum effects are included through the Fermi pressure

and the quantum interaction (diffraction and tunneling effects) are incorporated through the Bohm force term in the momentum equation. Another quantum effect which is commonly present in strongly magnetized plasmas is the intrinsic spin property of the electrons. The electron spin effect in the plasma was introduced by deriving the quantum hydrodynamic (QHD) model for spin-1/2 particles in Refs. [14–16]. Later on generalized form of QHD model for spin-1/2 particles was presented by Andreev [17] in which the author considered that the electrons of spin-up and spin-down are two different fluids. This model is called separate spin evolution quantum hydrodynamic (SSE-QHD) model. The QHD model for spin-1/2 particles has received great deal of attention due to emergence of some new wave phenomena and its effect on instabilities [18,17,19–24]. Subsequently, the kinetic quantum model for spin-1/2 quantum plasmas was developed and applied to study the dispersion properties of different plasma waves [25–29]. The SSE has unique features which give birth to new wave phenomena [17,19,20,23]. The non linear analysis of these newly investigated spin dependent waves has been investigated in electron-ion and electron-positron-ion plasma [30–32]. Employing the SSE-QHD model some new wave features and Langmuir wave instability have been investigated inside the cylindrical waveguide. It is found that electron spin effects the geometry effects have opposite effects on the growth rate and wavenumber-domain of the instability [33]. Further, SSE predicted two new waves solutions in the spectrum of

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<https://doi.org/10.1016/j.physleta.2019.06.005>

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obliquely propagating extraordinary waves [34]. Recently, spin polarization effect on the Raman three-wave interaction and hybrid wave instabilities have been discussed in Refs. [35,37,36].

The energy densities and energy flow speed for the electrostatic and electromagnetic waves have been investigated in quantum plasma in which only Bohm potential effect was taken into account. Spin of the electrons is also responsible for interaction of particles in the presence of external magnetic field. Thus the energy density is changed due to the spin effect and consequently the energy flow speed. So the study of energy flow speed for spin polarized plasma is very much in order. However to the best of our knowledge energy behavior of high frequency electrostatic waves have yet not been discussed in spin polarized plasma. In this manuscript, we describe the energy densities and energy flow speed for upper hybrid waves and SECW in a magnetized electron-quantum degenerate plasmas on the basis of SSE.

The manuscript is arranged as follows: In section 2, the mathematical formulation for the derivation of energy densities and energy flow speed for perpendicular propagating electrostatic waves is presented. Results and discussion are given in section 3. Section 4 is devoted for the conclusion.

## 2. Mathematical formulation

We consider an electron-ion magnetized quantum degenerate plasma in which high frequency longitudinal waves propagating perpendicular to the magnetic field are to be investigated. The ions are assumed to be stationary lying in the back ground. The electrons are supposed to form a degenerate gas for which quantum statistical and particle dispersive effects are included through the Fermi pressure and Bohm potential terms respectively. For governing the dynamics of electrons, we consider the SSE-QHD model which was developed in Ref. [17]. In this model the spin-up and spin-down electrons are considered as separate fluids. Therefore, the continuity equation with spin projection of each species is presented as

$$\partial_t n_s + \nabla \cdot (n_s \mathbf{v}_s) = (-1)^{i_s} T_z, \quad (1)$$

where  $s = u, d$  for the spin-up and spin-down state of particles,  $n_s$  and  $\mathbf{v}_s$  are the number density and velocity field of electrons being in the spin state  $s$ ,  $T_z = \frac{\gamma_e}{\hbar} (B_x S_y - B_y S_x)$  is the  $z$ -projection of spin torque,  $\gamma = -\mu_B$ ,  $\mu_B$  is the Bohr magneton,  $i_s = 2, 1$  for up and down species respectively,  $S_x$  and  $S_y$  are spin density projections. In this model, the  $z$ -projection of the spin density  $S_z$  is not an independent variable, it is a combination of concentrations i.e.,  $S_z = n_u - n_d$ . The momentum equation for electrons is given as

$$\begin{aligned} mn_s (\partial_t + \mathbf{v}_s \cdot \nabla) \mathbf{v}_s + \nabla P_s - \frac{\hbar^2}{4m} n_s \nabla \left( \frac{\Delta n_s}{n_s} - \frac{(\nabla n_s)^2}{2n_s^2} \right) \\ = -en_s \left( \mathbf{E} + \frac{1}{c} [\mathbf{v}_s, \mathbf{B}] \right) + (-1)^{i_s} \gamma n_s \nabla B_z \\ + \frac{\gamma}{2} (S_x \nabla B_x + S_y \nabla B_y) + (-1)^{i_s} m (\tilde{\mathbf{T}}_z - \mathbf{v}_s T_z). \end{aligned} \quad (2)$$

Here  $s = u, d$  where  $u$  is for up-spin and  $d$  is for down-spin.  $P_s = (6\pi^2)^{2/3} n_s^{5/3} \hbar^2 / 5m$  is the degenerate pressure for both the species of electrons, term proportional to  $\hbar^2$  is Bohm potential. On the right hand side the of momentum Eq. (2) is the Lorentz force,  $\tilde{\mathbf{T}}_z = \frac{\gamma}{\hbar} (\mathbf{J}_{(M)x} B_y - \mathbf{J}_{(M)y} B_x)$ , which is the torque current, where  $\mathbf{J}_{(M)x} = (\mathbf{v}_u + \mathbf{v}_d) S_x / 2$ , and  $\mathbf{J}_{(M)y} = (\mathbf{v}_u + \mathbf{v}_d) S_y / 2$  are the convective parts of the spin current tensor. For the study of propagation of electrostatic waves ( $B_1 = 0$ ) in electron-ion plasma the set of equations for SSE-QHD model is closed by taking the Poisson's equation as

$$\nabla \cdot \mathbf{E} = 4\pi e (n_i - n_u - n_d). \quad (3)$$

For the electrostatic waves the perturbation in magnetic field is zero ( $\mathbf{B}_1 = 0$ ). We assume that the perturbations are  $\sim e^{i(\mathbf{kz} - \omega t)}$  and by using  $E_z = -ik\Phi$  in Eq. (1) to Eq. (3) we obtain the expressions for perturbed velocity and number density of electrons as

$$v_{1s} = - \frac{e\omega k\Phi}{m(\omega^2 - \omega_{ce}^2 - v_{Fs}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})}, \quad (4)$$

$$n_{1s} = - \frac{en_{0s} k^2 \Phi}{m(\omega^2 - \omega_{ce}^2 - v_{Fs}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})}. \quad (5)$$

By using Eq. (3) and Eq. (5), we obtain the dispersion relation for perpendicularly propagating electrostatic waves as

$$\begin{aligned} \epsilon = 1 - \frac{\omega_{pu}^2}{\omega^2 - \omega_{ce}^2 - k^2(v_{Fu}^2 + \frac{\hbar^2 k^2}{4m^2})} \\ - \frac{\omega_{pd}^2}{\omega^2 - \omega_{ce}^2 - k^2(v_{Fd}^2 + \frac{\hbar^2 k^2}{4m^2})}. \end{aligned} \quad (6)$$

Here  $\omega_{ce} = eB_0/mc$  is the electron cyclotron frequency,  $\omega_{p(u,d)}^2 = (1 \mp \eta)\omega_{pe}^2/2$ , where  $\omega_{pe} = (4\pi n_0 e^2/m)^{1/2}$  is the electron plasma frequency,  $v_{F(u,d)}^2 = v_F^2 (1 \mp \eta)^{2/3} / 3$ ,  $v_F = (3\pi^2 n_0)^{1/3} \hbar / m_e$  is the Fermi velocity of electrons. The spin polarization factor can be expressed as  $\eta = (n_{0u} - n_{0d})/n_0 = -(3\mu_B B_0 / 2\epsilon_F)$ , where  $\epsilon_F = (3\pi^2 n_0)^{2/3} \hbar^2 / 2m$  is the Fermi energy of the electrons. The equilibrium number density of spin-up and spin-down electrons in terms of unpolarized equilibrium number density can be expressed as  $n_{0(u,d)} = n_0 (1 \mp \eta) / 2$ . Eq. (6) is an equation of second degree relatively  $\omega^2$  which means it has two wave solutions. However, if we treat the electrons as a single fluid ( $v_{Fu}^2 = v_{Fd}^2$ ) we get only one wave solution i.e., upper hybrid wave modified with quantum Bohm potential effect. Further in the limit of small magnetic field Eq. (6) admits two possible solutions given as

$$\begin{aligned} \omega^2 = \omega_{pe}^2 + \omega_{ce}^2 + \frac{1}{2} k^2 (v_{Fu}^2 + v_{Fd}^2 + \frac{\hbar^2 k^2}{2m^2}) \\ + \frac{k^4 (v_{Fu}^2 - v_{Fd}^2)^2 + 2k^2 (v_{Fu}^2 - v_{Fd}^2) (\omega_{pu}^2 - \omega_{pd}^2)}{4(\omega_{pu}^2 + \omega_{pd}^2)} \end{aligned} \quad (7)$$

$$\begin{aligned} \omega^2 = \omega_{ce}^2 + \frac{1}{2} k^2 (v_{Fu}^2 + v_{Fd}^2 + \frac{\hbar^2 k^2}{2m^2}) \\ - \frac{k^4 (v_{Fu}^2 - v_{Fd}^2)^2 - 2k^2 (v_{Fu}^2 - v_{Fd}^2) (\omega_{pu}^2 - \omega_{pd}^2)}{4(\omega_{pu}^2 + \omega_{pd}^2)}. \end{aligned} \quad (8)$$

Eq. (7) and Eq. (8) are the dispersion relations of UHW and SECW. The latter one is an additional wave which arises due to the pressure difference of the spin-up and spin-down electron fluids. It is obvious that for a single electron fluid only UHW is observed. On the other hand, the separate spin evolution reveals its second partner SECW in the spin polarized plasma. In this case, the energy is stored and carried away by both the waves. To examine this we exploit the Poynting theorem which can be expressed for one dimensional case, in the electrostatic approximation, as follows

$$\frac{\partial \Gamma}{\partial x} = - \frac{\partial}{\partial t} \left( \frac{1}{2} \epsilon_0 E_x^2 \right) - E_x J_x. \quad (9)$$

Where  $\Gamma$  is the energy flux density and  $J_x$  is the current density which can be expressed as

$$J_x = -e(n_{0u}v_{xu} + n_{0d}v_{xd}). \tag{10}$$

We use Eq. (2) and Eq. (10) to evaluate  $E_x J_x$ :

$$E_x J_x = \sum_{s=u,d} \left( \left(1 - \frac{\omega_{ce}^2}{\omega^2}\right) \frac{m n_{0s}}{2} \frac{\partial v_{xs}}{\partial t} + \frac{m(v_{Fs}^2 + \frac{\hbar^2 k^2}{4m^2})}{2n_{e0s}} \frac{\partial n_s^2}{\partial t} + m(v_{Fs}^2 + \frac{\hbar^2 k^2}{4m^2}) \frac{\partial (n_s v_{xs})}{\partial x} \right). \tag{11}$$

Detailed derivation up to Eq. (11) is given in appendix. Now we use Eq. (4), Eq. (5), Eq. (9) and Eq. (11) to express the energy conservation law as follows

$$\frac{\partial \Gamma}{\partial x} + \frac{\partial \varepsilon}{\partial t} = 0, \tag{12}$$

where  $\Gamma = \Gamma_{xE} + \Gamma_{xQ}$  and  $\varepsilon = \varepsilon_E + \varepsilon_K + \varepsilon_Q$  is energy density. The symbols  $E$ ,  $K$  and  $Q$  in the subscripts represent the electrostatic, kinetic and quantum aspects respectively. We can easily identify the various types of energy densities and energy flow densities from the energy conservation law (Eq. (12)) as

$$\varepsilon_E = \frac{1}{2} \epsilon_0 k^2 \Phi_0^2 \sin^2(kx - \omega t), \tag{13}$$

$$\varepsilon_K = \frac{1}{2} \epsilon_0 k^2 \Phi_0^2 \left( \frac{\omega_{pu}^2 (\omega^2 - \omega_{ce}^2)}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (\omega^2 - \omega_{ce}^2)}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right) \cos^2(kx - \omega t), \tag{14}$$

$$\varepsilon_Q = \frac{1}{2} \epsilon_0 k^2 \Phi_0^2 \left[ \frac{\omega_{pu}^2 (v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right] \cos^2(kx - \omega t), \tag{15}$$

$$\Gamma_{xE} = 0, \tag{16}$$

$$\Gamma_{xQ} = \epsilon_0 \omega k \Phi_0^2 \left( \frac{\omega_{pu}^2 (v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right) \cos^2(kx - \omega t), \tag{17}$$

where  $E_x = -\partial \Phi / \partial x$  and we assume that  $\Phi = \Phi_0 \cos(kx - \omega t)$ . Now we find time average values of energy densities and energy flow densities as below:

$$\langle \varepsilon_E \rangle = \frac{1}{4} \epsilon_0 k^2 \Phi_0^2, \tag{18}$$

$$\langle \varepsilon_K \rangle = \langle \varepsilon_E \rangle \left[ \frac{\omega_{pu}^2 (\omega^2 - \omega_{ce}^2)}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (\omega^2 - \omega_{ce}^2)}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right], \tag{19}$$

$$\langle \varepsilon_Q \rangle = \langle \varepsilon_E \rangle \left[ \frac{\omega_{pu}^2 (v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right] \tag{20}$$

$$\langle \Gamma_{xE} \rangle = 0, \tag{21}$$

$$\langle \Gamma_{xQ} \rangle = \langle \varepsilon_E \rangle \frac{2\omega}{k} \left[ \frac{\omega_{pu}^2 (v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right]. \tag{22}$$

Finally, we find the energy flow speed in degenerate plasma, given as

$$v = \frac{\langle \Gamma_x \rangle}{\langle \varepsilon \rangle}, \tag{23}$$

where  $\langle \Gamma_x \rangle = \langle \Gamma_{xE} \rangle + \langle \Gamma_{xQ} \rangle$  and  $\langle \varepsilon \rangle = \langle \varepsilon_E \rangle + \langle \varepsilon_K \rangle + \langle \varepsilon_Q \rangle$ . Thus by using Eqs. (18)–(22) we get

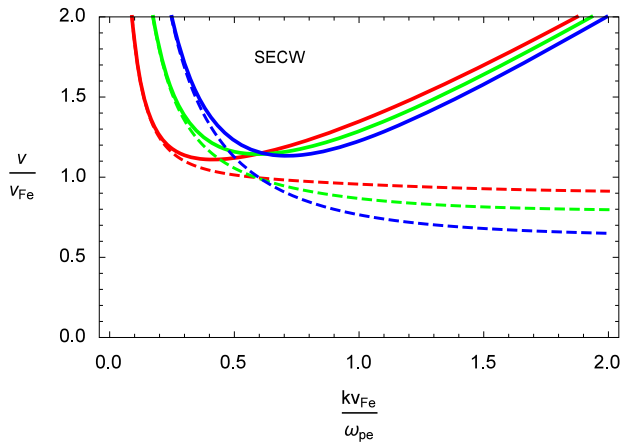
$$\langle \Gamma_x \rangle = \langle \varepsilon_E \rangle \frac{2\omega}{k} \left[ \frac{\omega_{pu}^2 (v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right], \tag{24}$$

$$\langle \varepsilon \rangle = \langle \varepsilon_E \rangle \left[ 1 + \frac{\omega_{pu}^2 (\omega^2 - \omega_{ce}^2 + v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} + \frac{\omega_{pd}^2 (\omega^2 - \omega_{ce}^2 + v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2})}{(\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2} \right]. \tag{25}$$

By using Eq. (24) and Eq. (25) in Eq. (23) we obtain the energy flow speed as following,

$$v = \frac{2\omega}{k} \frac{\left[ \omega_{pu}^2 (v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2}) (\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2 + \omega_{pd}^2 (v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2}) (\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2 \right]}{\left[ (\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2 (\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2 + \omega_{pu}^2 (\omega^2 - \omega_{ce}^2 + v_{Fu}^2 k^2 + \frac{\hbar^2 k^4}{4m^2}) (\omega^2 - \omega_{ce}^2 - v_{Fu}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2 + \omega_{pd}^2 (\omega^2 - \omega_{ce}^2 + v_{Fd}^2 k^2 + \frac{\hbar^2 k^4}{4m^2}) (\omega^2 - \omega_{ce}^2 - v_{Fd}^2 k^2 - \frac{\hbar^2 k^4}{4m^2})^2 \right]}. \tag{26}$$

Eq. (26) can be analyzed for the energy flow speed for the propagation of UHW and SECW using their dispersion relation given by Eq. (6). If we consider the electrons as a single fluid ( $v_{Fu}^2 = v_{Fd}^2$ ), we obtain the result for energy flow speed modified by quantum Bohm potential for the propagation of only UHW.



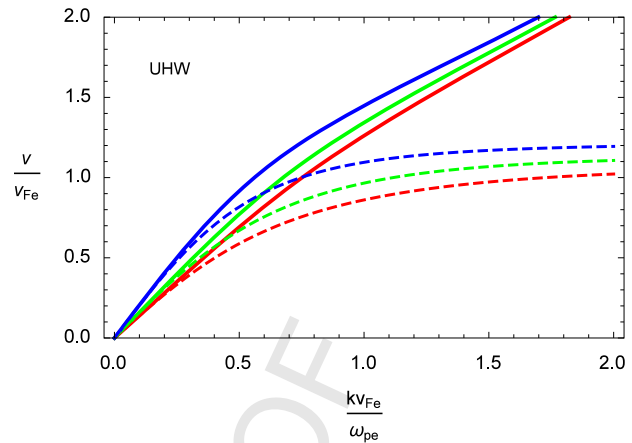
**Fig. 1.** The figure shows the effect of spin polarization on energy flow speed, for the propagation of SECW in the plasma. In this figure thick colored curves shows variation of flow speed for different values of spin polarization factor  $\eta$  in the presence of Bohm potential and dashed colored curves without taking account of Bohm potential. In figure  $\eta = 0.25$  (Red thick and dashed),  $\eta = 0.25$  (Green thick and dashed) and  $\eta = 0.25$  (Blue thick and dashed). The other parameters are used as:  $n_{0e} = 1 \times 10^{22} \text{ cm}^{-3}$  and  $B_0 = 5 \times 10^7 \text{ G}$ ,  $1 \times 10^8 \text{ G}$  and  $1.5 \times 10^8 \text{ G}$ . (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

### 3. Results and discussion

In this section we present the numerical analysis of energy flow speed for the propagation of SECW and UHW. First we discuss the energy flow speed for SECW. Fig. 1 shows the energy flow speed plotted against the wave number with and without the Bohm potential effect. The bold curves illustrate the variation of energy flow speed with  $k$  when the Bohm potential effect is taken into account where as dashed lines represent the same when it is not included. We chose the different values of the spin polarization effect, typically i.e.  $\eta = 0.25, 0.50, 0.80$ , for plotting the curves. A couple of observations are in order. The curves of energy flow speed at different values of  $\eta$  flip over at a particular value of wave number  $0.52$ . The spin polarization enhances the energy flow speed for  $k < 0.52$  and suppresses it for  $k > 0.52$ . In the absence of Bohm potential the energy flow speed decreases exponentially with  $k$  and tends to go for a constant value. It is evident that Bohm potential plays an important role at larger values of  $k$  where it enhances the flow speed. The upward trend begins around  $k = 0.52$ . The Bohm potential effect is not just a small correction but changes the energy flow speed significantly. The corresponding results for UHW are summarized in Fig. 2 which clearly demonstrates that the spin polarization effect enhances the energy flow speed over the whole range of  $k$  in the absence of the Bohm potential effect. The Bohm potential effect plays a similar role here to keep the upward trend almost linearly in the upper range of  $k$  as well. Thus the Bohm potential effect plays an important role and enables both the waves to transport energy at high speed in the upper range of wave number which otherwise stays at a constant level. In the low  $k$  domain, however, the energy transport due to SECW is quite significant.

### 4. Summary and conclusion

In this work, using the SSE-QHD model, we have investigated energy behavior in the electrostatic longitudinal waves propagating perpendicular to the external magnetic field in the spin polarized quantum plasma. This model generates a new wave SECW which arises due to spin polarization. Regarding its energy flow speed, it is observed that for different values of  $\eta$  the curves flip over at a particular value of the wave number. In the absence of Bohm potential effect the energy flow speed decreases exponentially to



**Fig. 2.** Fig. 1: The figure shows the effect of spin polarization on energy flow speed, for the propagation of upper hybrid in the plasma. In this figure thick colored curves shows variation of flow speed for different values of spin polarization factor  $\eta$  in the presence of Bohm potential and dashed colored curves without taking account of Bohm potential. In figure  $\eta = 0.25$  (Red thick and dashed),  $\eta = 0.25$  (Green thick and dashed) and  $\eta = 0.25$  (Blue thick and dashed). The other parameters are used as:  $n_{0e} = 1 \times 10^{22} \text{ cm}^{-3}$  and  $B_0 = 5 \times 10^7 \text{ G}$ ,  $1 \times 10^8 \text{ G}$  and  $1.5 \times 10^8 \text{ G}$ .

a constant value. On the other hand, in the presence of Bohm potential effect, the energy flow speed increases almost linearly for wave numbers beyond the flip-point. It is concluded that the Bohm potential effect enables the wave to transmit energy with high speed at larger values of  $k$ . It may also be mentioned here that, in the low  $k$  domain, the flow speed falls very rapidly for both with or without Bohm potential. Moreover, we have studied energy flow speed of UHW. First we note that the speed is enhanced as we increase  $\eta$ . Further, without the Bohm potential effect the energy flow speed initially increases with  $k$  and then becomes almost constant. But with Bohm potential effect the energy flow speed increases with  $k$  almost linearly at higher values. Usually, Bohm potential effect is seen to be a small correction in most of quantum treatments but here its role is domineering as it drastically enhances the energy flow speed at higher values of  $k$  in both the waves. From the application point of view, we have chosen the parameters of spin-polarized plasma having equilibrium number density  $n_0 \approx 10^{22} \text{ cm}^{-3}$  yielding Fermi temperature of the order  $T_F \approx 2 \times 10^4 \text{ K}$ . These parameters lie in the domain of metallic structures and magnetically ordered metals (like Fe, Py, Co, Ni, or MnAs) which are spin polarized and are used in the development of spintronic devices [38]. Further for most practical purposes, metallic structures can be thought of as operating effectively at room temperature ( $T = 300 \text{ K}$ ). Moreover, the electron dynamics in these metallic structures is governed by the plasma effects [39]. Thus quantum statistical and electron tunneling effects should be included because the Fermi temperature is greater than the room temperature and the de Broglie wavelength is comparable or larger than the inter-particle distance [40]. Spin polarization of carriers in solid state materials is a precondition for spintronic devices and magnetoelectronic devices, in which spin rather than charge is carrier of information. Further the spin polarized carriers can also be produced by injecting polarized particles or by optical pumping [41,42] or through interaction of plasma with strong ultrashort laser pulses [43,44]. Although the spintronics has benefited from these magnetic materials there is an immense need for fundamental studies of these materials before the potential of spintronics applications can be fully realized. We investigated energy flow speed in these spin polarized metallic structure materials because in most of the solid state devices the energy flow speed determines the rate of information (data) transfer. Thus our results may be relevant to understand the energy behavior in spin

polarized metallic structure materials which are used in the development of spintronic devices.

## 5. Appendix

### Detailed derivation up to Eq. (11).

For the perpendicular electrostatic waves the perturbation of magnetic field is zero ( $\mathbf{B}_1 = 0$ ). So the above Eqs. (1) and (2) can be rewritten in a simplified form as

$$\partial_t n_{es} + \nabla \cdot (n_{es} \mathbf{v}_{es}) = 0 \quad (27)$$

$$\begin{aligned} mn_s (\partial_t + \mathbf{v}_s \cdot \nabla) \mathbf{v}_s + \nabla P_{eu} - \frac{\hbar^2}{4m} n_s \nabla \left( \frac{\Delta n_s}{n_s} - \frac{(\nabla n_s)^2}{2n_s} \right) \\ = -en_s \left( \mathbf{E} + \frac{1}{c} [\mathbf{v}_s, \mathbf{B}] \right) \end{aligned} \quad (28)$$

Here we have used  $E_x = -\partial_x \Phi$ . The  $x$ -component of Eq. (28)

$$\partial_x \Phi = \frac{m}{e} \partial_t v_{1xs} + \frac{m\omega_c}{e} v_{1ys} + \frac{m}{en_{0s}} (v_{Fs}^2 + \frac{\hbar^2 k^2}{4m^2}) \partial_x n_s \quad (29)$$

The  $y$ -component of Eq. (28)

$$\partial_t v_{1ys} = \omega_c v_{1xs} \quad (30)$$

on substituting  $v_{1ys}$  in above equation, we obtain,

$$\partial_x \Phi = \frac{m}{e} \left( 1 - \frac{\omega_c^2}{\omega^2} \right) \partial_t v_{1xs} + \frac{m}{en_{0s}} (v_{Fs}^2 + \frac{\hbar^2 k^2}{4m^2}) \partial_x n_s \quad (31)$$

As the Poynting theorem for the electrostatic energy flow is

$$\frac{\partial \Gamma}{\partial x} + \frac{\partial}{\partial t} \left( \frac{1}{2} \epsilon_0 E_x^2 \right) = (\partial_x \Phi) J_x, \quad (32)$$

where  $J_x = -e(n_{0u} v_{1xu} + n_{0d} v_{1xd})$ . Multiply Eq. (31) by  $J_x$  and the product  $(\partial_x \Phi) J_x$  may be expressed as

$$\begin{aligned} (\partial_x \Phi) J_x = \frac{m}{e} \left( 1 - \frac{\omega_c^2}{\omega^2} \right) (-en_{0s} v_{1xs}) \partial_t v_{1xs} \\ + \frac{m}{en_{0s}} (v_{Fs}^2 + \frac{\hbar^2 k^2}{4m^2}) (-en_{0s} v_{xs}) \partial_x n_s \end{aligned} \quad (33)$$

By using continuity equation for spin up electrons, we rearrange the above equation as

$$\begin{aligned} (\partial_x \Phi) J_{xu} = -\frac{mn_{0u}}{2} \left( 1 - \frac{\omega_c^2}{\omega^2} \right) \partial_t v_{1xu}^2 - m \left( v_{Fu}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_x (v_{1xu} n_u) \\ - \frac{m}{2n_{0u}} \left( v_{Fu}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_t n_u^2 \end{aligned} \quad (34)$$

Similarly for spin-down electrons

$$\begin{aligned} (\partial_x \Phi) J_{xd} = -\frac{mn_{0d}}{2} \left( 1 - \frac{\omega_c^2}{\omega^2} \right) \partial_t v_{1xd}^2 - m \left( v_{Fd}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_x (v_{1xd} n_d) \\ - \frac{m}{2n_{0d}} \left( v_{Fd}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_t n_d^2 \end{aligned} \quad (35)$$

By adding the above equations to get

$$\begin{aligned} (\partial_x \Phi) J_x \\ = -\frac{m}{2} \left( 1 - \frac{\omega_c^2}{\omega^2} \right) (n_{0u} \partial_t v_{xu}^2 + n_{0d} \partial_t v_{xd}^2) \end{aligned} \quad (36)$$

$$\begin{aligned} -m \left[ \left( v_{Fu}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_x (v_{1xu} n_u) + \left( v_{Fd}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_x (v_{1xd} n_d) \right] \\ - \frac{m}{2} \left[ \frac{1}{n_{0u}} \left( v_{Fu}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_t n_u^2 + \frac{1}{n_{0d}} \left( v_{Fd}^2 + \frac{\hbar^2 k^2}{4m^2} \right) \partial_t n_d^2 \right] \end{aligned}$$

By substituting  $v_{xu}$ ,  $v_{xd}$ ,  $n_u$  and  $n_d$  from Eq. (4) and Eq. (5) we get the next equations of the paper.

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**Highlights**

- Studied the energy flow speed for the propagation spin electron cyclotron wave (SECW) and upper hybrid wave.
- Examined how the spin polarization and Bohm potential influence the energy flow speed.
- Results are applicable to environments like solid state plasma.

UNCORRECTED PROOF

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