Received: 12 June 2019 | Revised: 3 August 2019 | Accepted: 3 September 2019
DOI: 10.1002/ctpp.201900065

ORIGINAL ARTICLE

Contributions to Plasma Physics

Investigation of cubic non-linearity-driven electrostatic structures in the presence of double spectral index distribution function

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Funding information
Higher Education Commission, Pakistan, 7558/Punjab/NRPU/R&D/HEC/2017; National Natural Science Foundation of China, 41674145

Abstract
In this article, modified Korteweg-de Vries equation (which involves cubic non-linearity) was derived to study non-linear ion acoustic waves in a plasma in which electrons follow the double spectral index distribution function. The double spectral index distribution successfully apes the distribution functions that have been frequently observed in space plasmas. The spectral index r moulds the distribution function at low energy and by increasing its value, flatness of the distribution enhances. The spectral index r can also have negative values due to which distribution becomes spiky at low energies. The index q, on the other hand, controls the shape of the tail of distribution function. It has been shown that propagation of the solitary structures gets significantly altered by the choice of the double spectral indices, namely, r and q. A comparison was also made, using the parameters of the auroral zone, between the quadratic and cubic non-linearities-driven non-linear structures and it was shown that the solitary structures form on a much shorter scale for cubic non-linearity compared to their quadratic counterpart.

KEYWORDS
(r, q) distribution, KdV equation, modified KdV, nonlinear ion acoustic waves, nonthermal electrons

1 | INTRODUCTION

Ion acoustic solitary waves have been the most widely studied electrostatic modes both in theoretical and experimental plasmas.[1–7] Soliton is one of the solitary waves, which represents non-linear disturbance in the medium that manifests a spectacular feature of retaining its shape after collision. The idea of non-linear waves in a dispersive medium with exact soliton solution was first presented by Korteweg and de Vries[8] after the observation of solitary structure by Russel in 1834. In laboratory, ion acoustic soliton was experimentally observed for the first time by Ikezi[9] and then further studied experimentally using a double plasma device.[10] In the last century, soliton's significance has been realized in many fields such as solid state physics, fluid dynamics,[11,12] medical[13] astrophysics,[14] space physics[9,15] etc., and still retains the importance in plasma physics on account of its importance in the energy transport problem.

Ion acoustic solitons have been studied via reductive perturbation technique or fully non-linear pseudo-potential approach in unmagnetized and magnetized plasmas.[9,16–18] In the small amplitude limit, the Korteweg-de Vries (KdV) equation or the modified Korteweg-de Vries (mKdV) equation can be derived for quadratic and cubic non-linearities, respectively. The KdV equation was derived to study the effect of ion temperature on the propagation characteristics of ion acoustic solitary waves.[19] Later, the mKdV equation was derived to study ion acoustic waves in cylindrical and spherical coordinate systems assuming cold ions and isothermal electrons and their numerical solitary solutions were compared.[20,21] Since then, mKdV has been studied extensively in different plasma scenarios.[22,23]

It has been found that shape of the particle distribution functions changes the characteristics of the non-linear wave structures and play an important role in the wave dynamics.[24,25] In space plasmas, particle distribution functions often deviate from the Maxwellian distribution function either due to the presence of plenty of superthermal particles or the presence of flat top at low energies.[26–31] Cairns et al[32] proposed a non-thermal electron distribution function for their plasma model to interpret the density depletion structures observed by Freja and Viking satellites,[33] which could not be anticipated on the basis of Maxwellian distribution. Whenever particle distribution has an abundance of superthermal particles, it is modelled in a better fashion by employing kappa distribution function. However, when the distribution function deviates from the Gaussian peak and has flat top, it is more appropriately modelled by the generalized (r, q) distribution function (Qureshi et al[34]). Tahir et al[25] employed bi-product (r, q) distribution for adiabatically trapped electrons to study obliquely propagating Alfven waves. Khalid et al[35] studied the non-linear kinetic Alfven waves by employing the (r, q) distribution and found good agreement with the observed non-linear kinetic Alfven waves structures observed by Freja and Fast satellites. In a recent article, non-linear electron acoustic wave has been studied by considering the (r, q) distributed hot electrons and it has been found that polarity of the solitons is dependent on the low-energy part of the distribution function.[36] This aspect could not be investigated by distributions containing high-energy tails only. It is, therefore, imperative to study different plasma modes based on the double spectral index distribution function to better understand the distribution behaviour especially at low energies.

In this article, we investigate the propagation characteristics of the ion acoustic waves by deriving the mKdV equation. This article is organized as: Section 2 is devoted to the distribution function and basic model equations. In section 3, we derive the mKdV equation and discuss its soliton solution. We present and discuss the numerical results in section 4 followed by conclusion in section 5.

2 | DISTRIBUTION AND BASIC MODEL

We consider unmagnetized, collisionless plasma containing singly charged ions and electrons that follow the generalized (r, q) distribution. Our aim is to study the non-linear ion acoustic waves and derive the mKdV equation by considering (r, q) distributed electrons. The generalized (r, q) distribution in the presence of potential has the following form[7,35]

$$f_{rq}(v) = \frac{3\,\Gamma[q]\,(q-1)^{-3/(2+2\,r)}}{4\,\pi\,\beta^{3/2}\,(2T_e/m_e)^{3/2}\,\Gamma\left[q-\frac{3}{2+2\,r}\right]\Gamma\left[1+\frac{3}{2+2\,r}\right]}\left[1+\frac{1}{q-1}\left(\frac{v^2-2e\phi/m_e}{\beta\,(2T_e/m_e)}\right)^{r+1}\right]^{-q} \tag{1}$$

where

$$\beta = \frac{3\,(q-1)^{-1/(1+r)}\,\Gamma\left(q-\frac{3}{2+2\,r}\right)\Gamma\left(\frac{3}{2+2\,r}\right)}{2\,\Gamma\left(q-\frac{5}{2+2\,r}\right)\Gamma\left(\frac{5}{2+2\,r}\right)} \tag{2}$$

Here, Γ is the gamma function, ϕ is the electrostatic potential, $(2T_e/m_e)^{1/2}$ is the thermal velocity, T_e is the electron temperature, r and q are the spectral indices representing the shoulders and tail in the profile of the distribution function, which are required to fulfil the conditions $q > 1$ and $q(r+1) > 5/2$. In their pioneering work, Qureshi et al[28] presented the generalized (r, q) distribution function to study Alfven waves in space plasmas. The (r, q) distribution function is the only distribution that not only fits the flat tops or shoulders at low energies but spiky distributions at low energies along with the high energy tails in the profile of the distribution. In the limiting cases when (a) $r = 0$ and $q \to \infty$ and (b) $r = 0$ and $q \to (\kappa + 1)$, the generalized (r, q) distribution function reduces to the Maxwellian and generalized Lorentzian distributions, respectively.[7,28] First reported observations of flat top distributions came from Vela 4 and ISEE observations around the Earth's bow shock.[37–39] Subsequent observations of flat top electron distributions were reported by Cluster observations from magnetosheath.[30,31] Kiran et al[40] employed the (r, q) distribution to fit the solar wind proton distributions and successfully explained the solar wind heating mechanism using the fitting parameters r and q, which could not be explained earlier when idealized Maxwellian distribution was assumed. Later on, electron velocity distributions from magnetosheath were fitted with the (r, q) distribution and successfully interpreted the observations of Lion roars on the basis of fitting parameters r and q, which was also not explained when the idealized Maxwellian distribution was employed to interpret the lion roars or whistler waves with electron temperature anisotropy in the magnetosheath.[34]

Integrating the (r, q) distribution function (1) over velocity space and expanding the result in the limit $\varphi \ll 1$, we obtain the total electron number density given by

$$n_e = n_0\left(1 + A\varphi + B\varphi^2 + C\varphi^3\right) \tag{3}$$

where $\varphi = e\phi/T_e$, and

$$A = \frac{(q-1)^{-1/(1+r)}\,\Gamma\left(q - \frac{1}{2+2\,r}\right)\Gamma\left(\frac{1}{2+2\,r}\right)}{2\,\beta\,\Gamma\left(q - \frac{3}{2+2\,r}\right)\Gamma\left(\frac{3}{2+2\,r}\right)} \tag{4}$$

$$B = \frac{3\,(q-1)^{-2/(1+r)}\,\Gamma\left(q + \frac{1}{2+2\,r}\right)\Gamma\left(1 - \frac{1}{2+2\,r}\right)}{8\,\beta^2\,\Gamma\left(q - \frac{3}{2+2\,r}\right)\Gamma\left(1 + \frac{3}{2+2\,r}\right)} \tag{5}$$

$$C = -\frac{(-1+q)^{-\frac{3}{1+r}}\,\Gamma\left(q - \frac{3}{2+2\,r}\right)\Gamma\left(q + \frac{3}{2+2\,r}\right)}{16\,\beta^3\,\Gamma\left(q - \frac{3}{2+2\,r}\right)\Gamma\left(1 + \frac{3}{2+2\,r}\right)} \tag{6}$$

In the limit $r = 0$ and $q \to \infty$, the constants in Equations (4), (5), and (6) become $A = 1$ and $B = 1/2$ and $C = 1/6$. Also when $r = 0$ and $q \to (\kappa + 1)$, we get $A = \frac{\kappa-1/2}{\kappa-3/2}$, $B = \frac{(\kappa-1/2)(\kappa+1/2)}{2\,(\kappa-3/2)^2}$ and $C = \frac{(4\,\kappa^2-1)(3+2\kappa)}{6\,(2\kappa-3)^3}$.

The model equations that describe the propagation of ion acoustic waves for singly charged ions (one specie) and (r, q) distributed electrons in one dimension are the continuity, momentum, and Poisson's equations, given, respectively, as

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{7}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + v\frac{\partial v}{\partial x} + \frac{\partial \varphi}{\partial x} = 0 \tag{8}$$

$$\frac{\partial^2 \varphi}{\partial x^2} - n_e + n = 0 \tag{9}$$

In the above equations, n, t, x, and v are normalized by n_0, ω_{pi}^{-1}, λ_{De}, and c_s, respectively, where $\omega_{pi} = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m_i}}$, $\lambda_{De} = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_0 e^2}}$ and $c_s = \sqrt{\frac{k_B T_e}{m_i}}$.

3 | THE mKdV EQUATION

To develop the non-linear mKdV equation, the following stretched coordinates are used:

$$\zeta = \varepsilon(x - V_0\,t) \quad \text{and} \quad \tau = \varepsilon^3 t \tag{10}$$

where V_0 is the phase velocity of the solitary wave and ε is the small parameter that represents the strength of non-linearity. Introducing the reductive perturbation technique, the quantities n, v, and φ in the perturbed form are written as:

$$n = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \cdots \tag{11}$$

$$v = \varepsilon v_1 + \varepsilon^2 v_2 + \varepsilon^3 v_3 + \cdots \tag{12}$$

$$\varphi = \varepsilon \varphi_1 + \varepsilon^2 \varphi_2 + \varepsilon^2 \varphi_2 + \cdots \tag{13}$$

Using Equation (10) and above perturbed quantities in Equations (7)–(9), the next higher order in "ε" can be written as

$$V_0 n_2 = n_1 v_1 + v_2 \tag{14}$$

$$\varphi_2 = V_0 v_2 - \frac{1}{2} v_1^2 \tag{15}$$

$$V_0{}^4 = \frac{3}{2B} \tag{16}$$

The next higher order in ε yields the following set of equations

$$\frac{\partial n_1}{\partial \tau} - V_0 \frac{\partial n_3}{\partial \zeta} + \frac{\partial (n_2 v_1)}{\partial \zeta} + \frac{\partial (n_1 v_2)}{\partial \zeta} + \frac{\partial v_3}{\partial \zeta} = 0 \tag{17}$$

$$\frac{\partial v_1}{\partial \tau} - V_0 \frac{\partial v_3}{\partial \zeta} + v_1 \frac{\partial v_2}{\partial \zeta} + v_2 \frac{\partial v_1}{\partial \zeta} + \frac{\partial \varphi_3}{\partial \zeta} = 0 \tag{18}$$

$$\frac{\partial^2 \varphi_1}{\partial \boldsymbol{\zeta}^2} = -n_3 + A\,\varphi_3 + 2B\,\varphi_1 \varphi_2 + C\,\varphi_1{}^3 \tag{19}$$

By combing Equations (17)–(19), we finally arrive at the mKdV equation given by

$$\frac{\partial \varphi_1}{\partial \tau} + D \frac{\partial \varphi_1{}^3}{\partial \zeta} + E \frac{\partial^3 \varphi_1}{\partial \zeta^3} = 0 \tag{20}$$

where $D = \frac{1}{2}\left(\frac{5}{2V_0{}^3} - V_0{}^3 C\right)$ and $E = \frac{V_0{}^3}{2}$. The stationary solution of the above mKdV equation can be written by considering a comoving frame $\boldsymbol{\xi} = (\boldsymbol{\zeta} - \boldsymbol{u\tau})$ and obtain[41]

$$\varphi_1 = \sqrt{\frac{6u}{D}} \operatorname{sech}\left[\frac{\xi}{\Delta}\right] \tag{21}$$

Here, u is the velocity of non-linear structure and "Δ" is the width of the soliton that is given by

$$\Delta = \sqrt{\frac{u}{E}} \tag{22}$$

4 | NUMERICAL RESULTS

Figure 1 is plotted for different values of q when $r = 0$, which corresponds to the kappa-like distribution, that is, where the behaviour at low energies is Maxwellian with the presence of a modified tail. In this case, we obtain compressive solitary

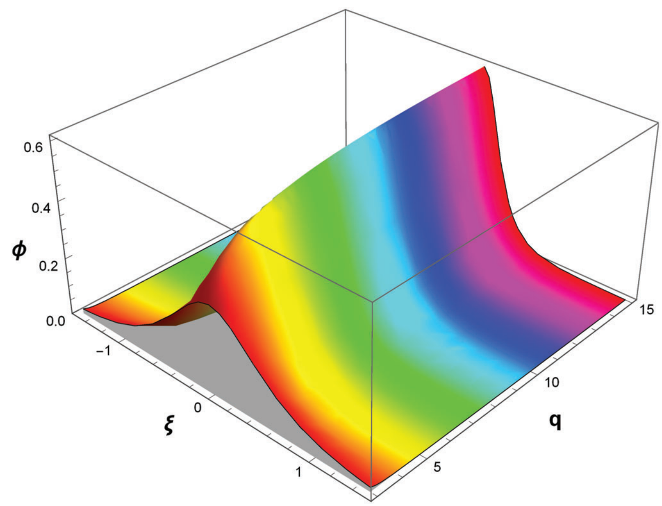


FIGURE 1 Soliton structures for the mKdV equation for the limiting cases of (r, q) distribution function, that is, when $r = 0$ and different q. Here $u = 0.025$

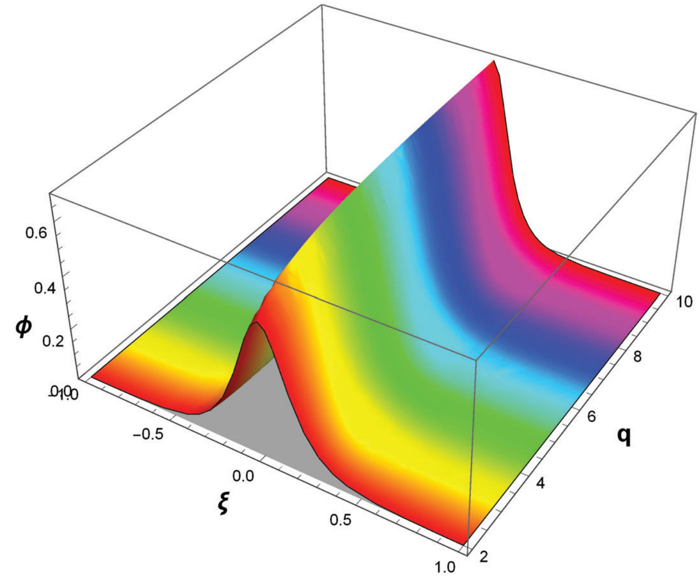


FIGURE 2 Solitary structures for the mKdV equation for different values of q when $r = 1$ and $u = 0.025$

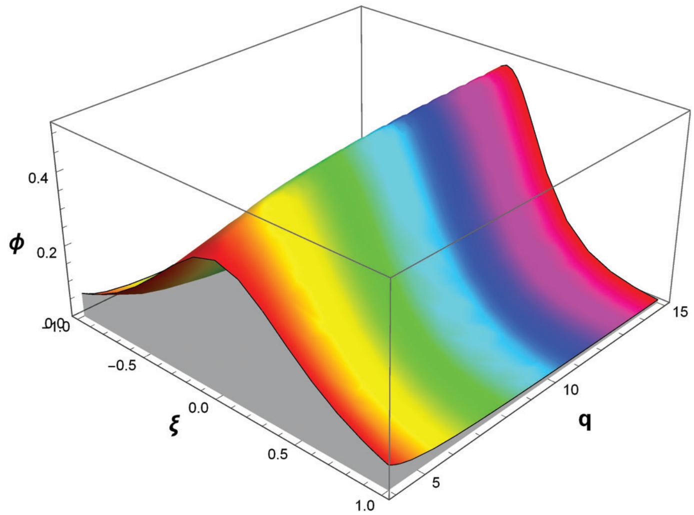


FIGURE 3 Solitary structures for the mKdV equation for different values of q when $r = -0.15$ and $u = 0.025$

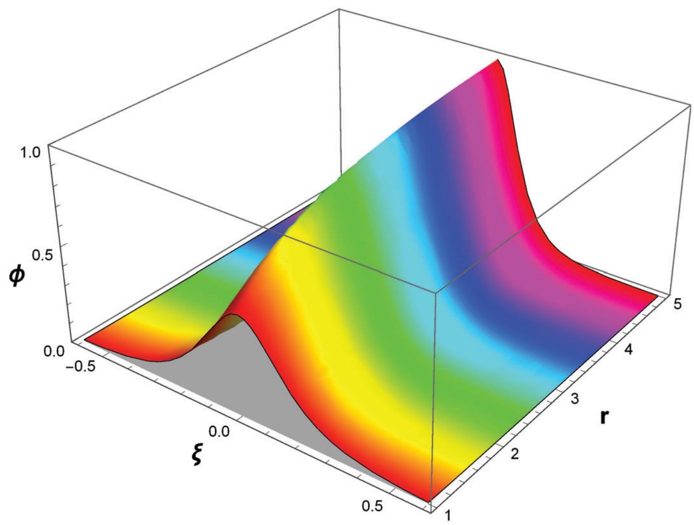


FIGURE 4 Solitary structures for the mKdV equation for different values of r when $q = 2$ and $u = 0.025$

structures. As the parameter q increases, high-energy tail decreases and the distribution tends to become Maxwellian. From Figure 1, we can see that decreasing the population of the energetic electrons (i.e., increasing q) in the tail of the distribution function gives rise to an increase in amplitude whereas width of the solitary structure experiences a decrease.

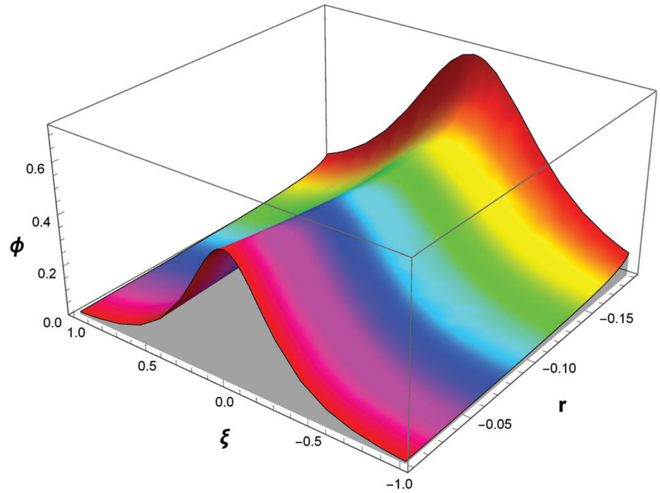


FIGURE 5 Solitary structures for the mKdV equation for different negative values of r when $q = 5$ and $u = 0.025$

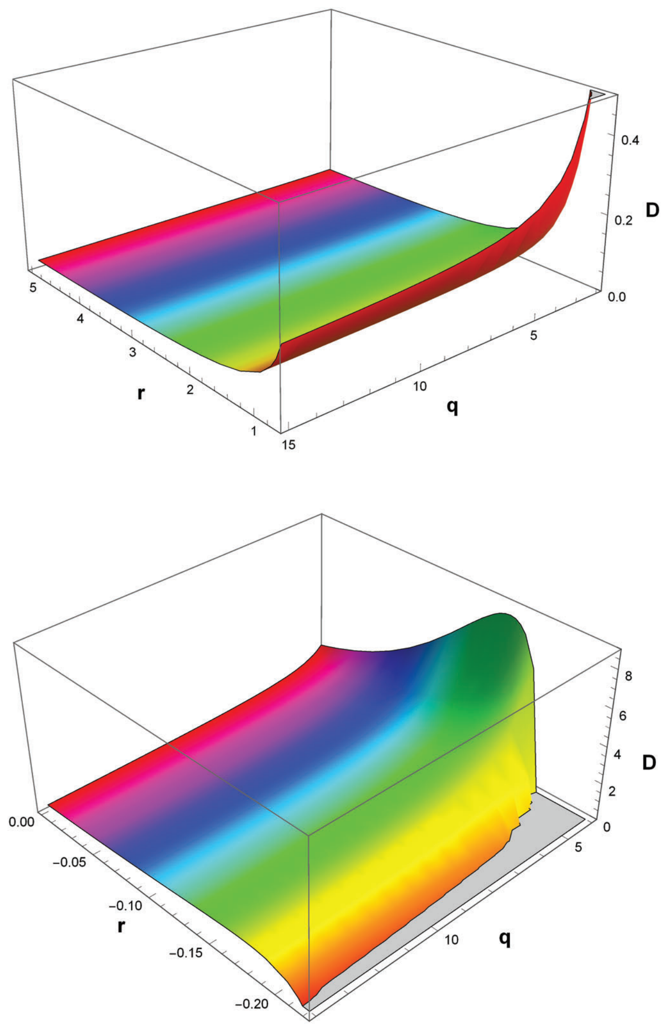
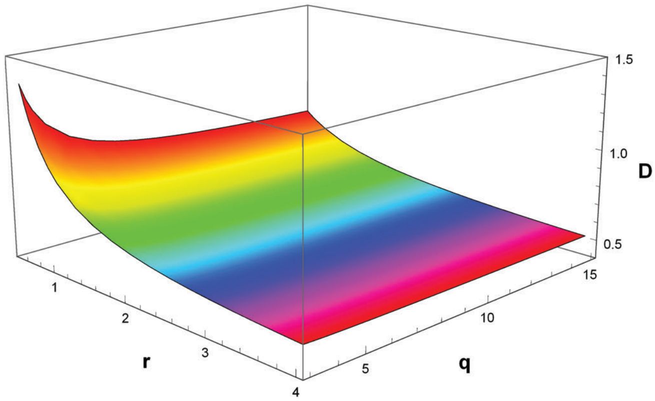


FIGURE 6 Non-linear coefficient D, appears in the solution of mKdV, is plotted for different values of q and positive r (upper panel) and negative r (lower panel)

Solitary structures for different values of q when $r = 1$, which corresponds to the flat-topped electron velocity distribution, are shown in Figure 2. Note that like in Figure 1, the amplitude of the soliton increases but width decreases as q increases, however, the amplitude of the soliton is greater in magnitude by comparison with the kappa-like distribution. Figure 3 is plotted for different values of q for the fixed value of spiky distribution, that is, $r = -0.15$. We see that the increase in the tail parameter q (or decreasing non-thermal population in the tail) mitigates the width whereas it enhances the amplitude of the mKdV soliton.

Figure 4 manifests the propagation characteristics of the solitary structures for increasing values of flat-topped distribution r when $q = 2$. Note that the amplitude of the soliton monotonically increases but the width decreases as r increases. From Figures 2



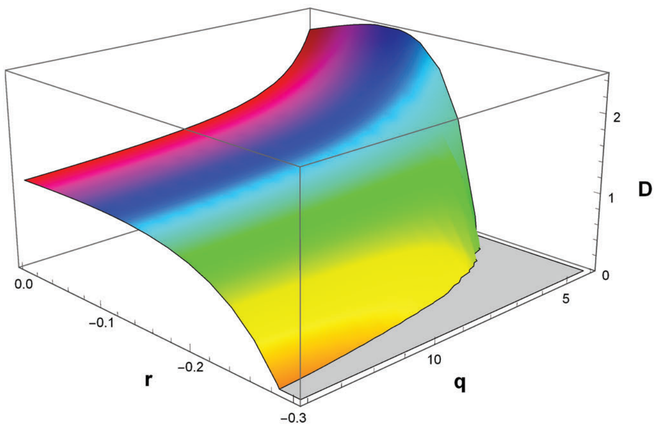


FIGURE 7 Non-linear coefficient D, appearing in the solution of KdV, is plotted for different values of q and positive r (upper panel) and negative r (lower panel)

and 3, we conclude that the tail of the distribution in the presence of flat top or spiky nature contributes less significantly to the soliton amplitude by comparison with the flatness parameter r, which essentially means that the low-energy behaviour makes a more pronounced effect than the high-energy part of the distribution function.

In Figure 5, we plot the solitary structures for different negative values of r when $q = 5$, which means that we are looking at the spiky electron distribution here. This case has yielded very interesting results. We note that as the negative value of r increases, that is, the distribution becomes spikier, the amplitude of the soliton first decreases and then increases for $r > -0.14$, but width of the soliton increases with the increase in the negative value of r. It is worth mentioning here that we cannot extend the range of negative q beyond a certain value as the coefficient of non-linearity flips sign for those values and they are not allowed for mKdV solitons as the soliton amplitude involves the square root. We shall explore this feature more in Figure 6.

To get real solution (21) for the mKdV equation, the non-linearity coefficient D appearing in the solution must be positive. In Figure 6, we plot the coefficient D for different values of r and q for which it remains positive. In Figure 6 (upper panel), coefficient D is plotted for different values of q and positive r. We can see that the non-linearity coefficient D remains positive for a range of q and positive values of r. In Figure 6 (lower panel), non-linearity coefficient D is plotted for different values of q and negative r. We can note that there is a limit on the negative value of r (approximately $r > -0.18$) beyond which the coefficient D becomes negative for different values of value q (shown as dashed area) and mKdV solution ceases to exist as it does not remain real. The upper and lower panels of Figure 7 explore the same behaviour for the non-linearity coefficient for KdV equation. Once again, it is found that the non-linearity coefficient D changes sign only for the spiky electron distribution beyond a threshold value of negative r represented by the dashed area. Note that the change of the non-linearity coefficient in the mKdV equation with the increase in negative values of r is quite different from that of the non-linearity coefficient of the KdV equation. The increase and decrease in the cubic non-linearity coefficient of the mKdV equation with the increase in the negative values of r is quite pronounced by comparison with its quadratic counterpart of the KdV equation.

Finally, we plot the soliton solutions of mKdV and KdV equations for the same parameters to see the difference in the solitons produced by cubic and quadratic non-linearities, respectively, in Figure 8. Note that the solitary structures for the mKdV equation form on a much shorter spatial scale by comparison with the solitary structures for the KdV equation. For illustrative purposes, we have used here the plasma parameters of the auroral zone, that is, $T_e = 6 - 12$ eV and $n_e = 5 - 12$ cm^{-3}.[40] Since we are dealing with the unmagnetized plasma whose spatial scale is Debye length and because we are dealing with the normalized parameters, it turns out that for the plasma parameters in the auroral zone, the spatial extent of mKdV soliton would

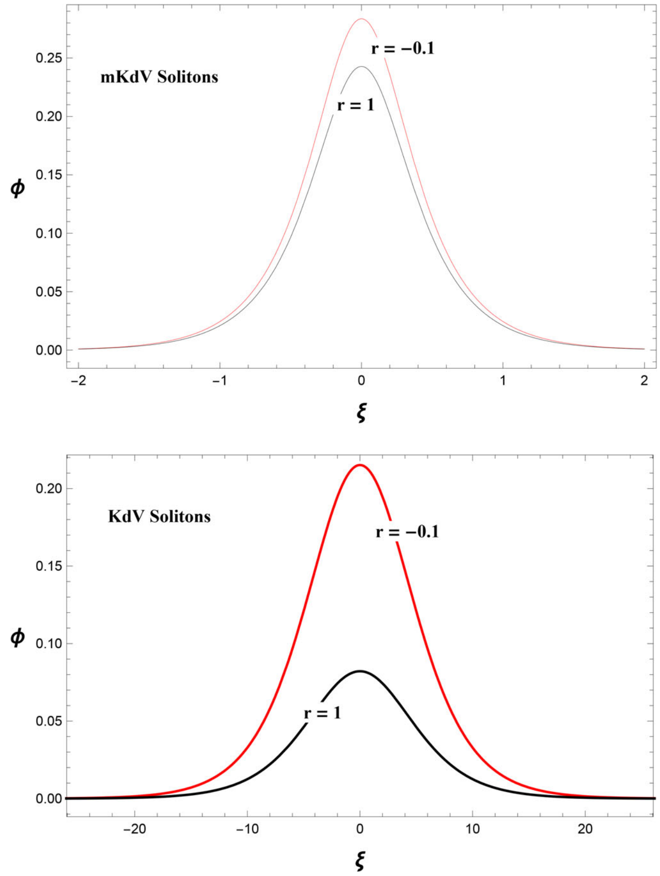


FIGURE 8 Spatial scales of KdV and mKdV solitons for different values of r when $q = 5$ and $u = 0.025$

be roughly $15 - 36$ m whereas it would be $150 - 480$ m for the KdV soliton. Note that in finding these values, we have set $q = 5$ and values of r have been chosen to be 1 and -0.1 for flat-topped and spiky distributions respectively. It is found that for mKdV soliton, the spatial extent of flat-topped and spiky electrons does not vary much, however, for the KdV solitons, it is observed that the spatial extent of the flat-topped distributed soliton is much shorter than the spiky distributed electrons. To date, no such comparison has been given to the best of our knowledge and it can be proved to be extremely beneficial to find which type of non-linearity (i.e., quadratic or cubic) dominates which regions of the auroral zone with the help of satellite data in future.

5 | CONCLUSION

In this article, we have derived the non-linear ion acoustic waves in the presence of cubic non-linearity and the electrons that follow the double spectral distribution function in order to account for the observed features of distribution functions in space plasmas. When the non-thermal features are present in the distribution function both in regions of high-phase and low-phase space densities, then the (r, q) distribution is the ultimate choice. Therefore, in this study, we have employed the (r, q) distribution to study the effect of low-energy particles on the propagation characteristics of mKdV soliton, which could not be studied otherwise by employing Maxwellian or kappa distributions. We have found that when the concentration of low-energy particles increases, that is, the flatness of the distribution increases, amplitude of the soliton increases significantly. However, with the flat top at low energies, increase in the value of q, which corresponds to decrease in the concentration of high energy particles, soliton amplitude has been found to not change appreciably. We have found that when the distribution becomes spiky at low energies, amplitude of the soliton first decreases and then increases. It has been shown that the spatial scale over which the mKdV solitons form is much shorter than the KdV solitons. We have given the estimate of the spatial extent of mKdV and KdV solitons using the parameters of the auroral zone and suggested that these estimates can, in principle, help ascertain which non-linearity is dominant in a certain region of the auroral zone. It has also been shown how the behaviour of low-energy electrons affect the spatial extent of the formation of KdV and mKdV solitons. The present investigation may be helpful to develop an in-depth understanding of the quadratic and cubic non-linearity-driven non-linear structures with reference to the non-thermal behaviour of electrons in regions of high-phase and low-phase space densities and also provide a guideline as to how to differentiate between the two in the satellite data based on the spatial scales over which they form.

ACKNOWLEDGMENTS

This research was supported by the Higher Education Commission (HEC), Pakistan Project No. 7558/Punjab/NR-PU/R&D/HEC/2017 and National Natural Science Foundation of China (Nos. 41674145).

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How to cite this article: Qureshi M, Shah K, Shi J, Masood W, Shah H. Investigation of cubic non-linearity-driven electrostatic structures in the presence of double spectral index distribution function. *Contributions to Plasma Physics*. 2019;e201900065. https://doi.org/10.1002/ctpp.201900065