

## ORIGINAL ARTICLE

# Modulation instability of lower hybrid waves leading to cusp solitons in electron–positron(hole)–ion Thomas Fermi plasma

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Following the idea of three-wave resonant interactions of lower hybrid waves, it is shown that quantum-modified lower hybrid (QLH) wave in electron–positron–ion plasma with spatial dispersion can decay into another QLH wave (where electron and positrons are activated, whereas ions remain in the background) and another ultra-low frequency quantum-modified ultra-low frequency Lower Hybrid (QULH) (where ions are mobile). Quantum effects like Bohm potential and Fermi pressure on the lower hybrid wave significantly reshaped the dispersion properties of these waves. Later, a set of non-linear Zakharov equations were derived to consider the formation of QLH wave solitons, with the non-linear contribution from the QLH waves. Furthermore, modulational instability of the lower hybrid wave solitons is investigated, and consequently, its growth rates are examined for different limiting cases. As the growth rate associated with the three-wave resonant interaction is generally smaller than the growth associated with the modulational instability, only the latter have been investigated. Soliton solutions from the set of coupled Zakharov and NLS equations in the quasi-stationary regime have been studied. Ordinary solitons are an attribute of non-linearity, whereas a cusp soliton solution featured by non-local nonlinearity has also been studied. Such an approach to lower hybrid waves and cusp solitons study in Fermi gas comprising electron positron and ions is new and important. The general results obtained in this quantum plasma theory will have widespread applicability, particularly for processes in high-energy plasma–laser interactions set for laboratory astrophysics and solid-state plasmas.

## KEYWORDS

instabilities, nonlinear, solitons, waves

## 1 | INTRODUCTION

This paper is an attempt to develop a set of Zakharov equations<sup>[1]</sup> to study modulation instability (MI) and the subsequent formation of non-linear stationary structures for an extremely high-energy density matter, particularly a Fermi gas that makes an assembly of electrons, positrons (holes), and ions. Here, unlike usual pair plasmas with the same mass and charge,<sup>[2–10]</sup> electrons and holes (positrons) have mass asymmetry, which is either due to interaction between the particles or some other non-linear phenomena emerging naturally or due to the different mobility of charge carriers. This mass asymmetry, however, opens up an interesting avenue for the plasma physicists to study the waves and instabilities on different time scales, such as low and high frequencies in comparison to gyro frequencies of the particles.<sup>[7,10,11]</sup>

Solid-state plasma or quantum (Fermi) liquid semiconductors have potential applications when the effective mass of charge carriers (electrons and holes) differs significantly from that of free electrons. Charge carriers in semiconductors such as lighter electrons and heavier positive holes can make up a degenerate system, for example, at  $n_e \geq 10^{16} - 10^{18} \text{ cm}^{-3}$ , with the effective mass of electrons  $m_e^* \approx (0.01 - 0.1)m_e$  and at temperature  $T < 10^2 \text{ K}$ .<sup>[12]</sup> However, this kind of plasma can be considered collisionless as the mean free path is usually longer than a few centimetres at low temperature, and even the particles are separated at a few angström distances. This can happen for two reasons: one is the Pauli exclusion principle restrictions, and the second reason is the screening of the Coulomb interaction between the particles<sup>[13–15]</sup>

Moreover, the creation of positrons with a milli-electron volt (MeV) of energy in laboratory conditions has emerged as a potential possibility to new avenues of antimatter research. There are a variety of other processes such as positronium production and Bose-Einstein condensates and astrophysical environments such as black holes and gamma ray bursts, which can be understood with the positron production in the lab.

The first experimental observations were that the temperatures of both electrons and positrons were different, and the temperature of positrons was found to be half of the effective electrons, as demonstrated by Chen et al.<sup>[16]</sup>

Where short ( $\sim 1 \text{ ps}$ ) and ultra-intense ( $\sim 1 \times 10^{20} \text{ W/cm}^2$ ) laser pulses were used to illuminate the gold targets of  $\sim \text{mm}$  thickness, positrons (coming out the back of gold target) up to  $2 \times 10^{10}$  per steradian were observed. The effective temperature of positrons was  $2.8 \pm 0.3 \text{ MeV}$ .

Difference in the masses between species is another source of asymmetry; the mass asymmetry can be initially given and is different from the temperature asymmetry. Such asymmetric plasma can be produced by the insertion of suitable ion beams into a trap.<sup>[10]</sup> As stated earlier, electron-hole plasma in certain semiconductors or electron-positron collider plasma produced by slightly different Lorentz factor beams are also possible examples of asymmetric plasma. Appropriate conditions for such plasma production could readily appear in dusty plasmas, as well as in astrophysical plasmas.

In support of the asymmetric plasma system, it is worth mentioning that results from the Tevatron collider at Fermi National Accelerator Laboratory, Batavia suggest that matter wins the antimatter. Experiments showed an unbalanced ratio of matter to antimatter going beyond imbalance predicted by the Standard Model, which has a 1% difference.<sup>[17]</sup>

Pair plasma exist in many astrophysical environments, such as those of neutron stars, bipolar outflows (jets), in active galactic nuclei, interior of accretion disks surrounding black holes, magnetospheres of pulsars and neutron stars, polar regions of neutron stars, the centre of the Milky Way galaxy, etc. and in labs such as laser beam-produced plasmas, non-linear quantum optics, microelectronic devices, etc.<sup>[9,18–23]</sup> Electron-positron plasmas are speculated to be highly degenerate and ultra-dense. The presence of ions other than electrons and positrons has also been predicted, which is why extensive studies have been carried out using quantum hydrodynamics (QHD).<sup>[24–30]</sup>

Thermal de-Broglie wave length ( $\lambda_B = \hbar/mv_T$ ) is a parameter to determine the quantum degeneracy effects and entails the spatial extension of the wave function of constituent particles due to quantum uncertainty and is either of the order or greater than the average inter-Fermionic distance, viz. ( $d = n_0^{-1/3}$ ), where  $n_0$  is the equilibrium number density. In such a scenario of high number densities, Fermi pressure dominates over the thermal pressure, which supports the compact objects against the gravitational burst.<sup>[31]</sup>

While treating the quantum plasma system, in the greater part of the existing literature, the equations of Schrodinger, Pauli, Klein-Gordon, and Dirac have been cast into fluidized variables through Madelung transforms and appropriately averaged to obtain the fluid equations<sup>[24,25]</sup> The idea is that the standard quantum equations of motion can be translated into equivalent equations of “classical” particles whose dynamics are determined by “quantum forces” (such as the gradient of the Bohm potential, exchange correlation effects, Fermionic pressure etc.) in addition to the external forces.<sup>[24]</sup> Later, Tsintsadze and Tsintsadze<sup>[25]</sup> developed a kinetic equation for Fermi plasma using a single Fermi particle concept by utilizing the non-relativistic Pauli equation with the aid of one-particle distribution function. The authors used the one-particle concept in spite of the large number of particles in the unit volume, and all of them have only one position and momentum ( $r, p$ ).<sup>[26]</sup>

The QHD formulation developed by Manfredi and Haas<sup>[21–23]</sup> has also been successfully implemented to predict new aspects of plasma in various systems, such as dense astrophysical objects, microelectronic devices, and in the laser-produced plasma, and received much attention in the plasma community.

The MI is a well-known mechanism for the energy localization of wave packets in a non-linear dispersive medium and can lead to an unstable situation that can potentially lead to the formation stable structures such as envelope or cusp solitons or rogue waves in plasma-like media. This happens when, for example, electromagnetic waves or light beams decay, eventually triggering the non-linear structures. Hidenori et al.<sup>[32]</sup> carried out the experiments on MI in electron plasma waves, and the observed results were precisely in agreement with the theory of Zakharov.<sup>[1]</sup> This kind of instability has potential applications in non-linear optics (lasers, self-focusing, non-linear radio waves, etc.), hydrodynamics, electromagnetics, etc.<sup>[33–41]</sup>; that is why a large amount of work has been devoted to this.

Lower hybrid waves are well known to admit non-linear structures, such as ordinary solitons and envelope solitons,<sup>[42–44]</sup> which have been observed in the Earth's magnetosphere by the FREJA satellite and have been examined with and without an extra charged species in plasma.

Liu et al. studied the non-linear theory of cylindrical lower hybrid drift-solitary waves in an inhomogeneous, magnetized plasma using a two-fluid model and reported attenuation in the wave amplitude and width of the solitary waves with the increase in the inhomogeneity in density.<sup>[38]</sup>

In this study, we investigate the MI of lower hybrid waves, associated growth rates, and stationary structures such as spiky and ordinary solitons in electron–positron (hole)–ion Thomas Fermi plasma. The peculiar spiky solitons exhibit a cusp at the crest unlike ordinary solitons.<sup>[45,46]</sup> The cusp type of soliton structures has been paid less attention in plasmas; however, an attempt was made by Ehsan et al.<sup>[37]</sup> to study the decay of lower hybrid wave into relatively lower-frequency lower hybrid waves in dusty plasmas, eventually forming the cusp type of solitons. The study of these phenomena is also very important because of the possibility of heating. Recently, coupling between fast lattice ions and electrons in piezoelectric semiconductor plasma using a semi-classical hydrodynamic model was studied, which led to the possible formation of the subsonic cusp-like solitons.<sup>[47]</sup>

This manuscript is organized in the following manner. In Section 2, the basic formulation of two types of lower hybrid waves is given, and the respective dispersion relations are obtained in Section 3. Section 4 deals with the mechanism of three-wave resonant interaction, and the subsequent derivation of the Zakharov and NLS equations is given in Section 5. In Section 6, the MIs and associated growth rates are examined. Section 7 demonstrates the one-dimensional analytical solutions of ordinary and cusp solitons. Finally, the main findings are recapitulated in Section 8.

## 2 | BASIC EQUATIONS

Considering the propagation of small longitudinal perturbations in electron–positron (hole)–ion plasmas, the relevant quantum Euler equations for the  $j$  species in quantum Fermi-Dirac plasmas<sup>[31]</sup> are:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_j \cdot \nabla\right) \mathbf{v}_j = \frac{q_j}{m_j} \left(\mathbf{E} + \frac{\mathbf{v}_j}{c} \times \mathbf{B}_o\right) + \frac{\hbar^2}{2m_j^2} \nabla \frac{1}{\sqrt{n_j}} (\nabla^2 \sqrt{n_j}) - \frac{\nabla P_{Fj}}{n_j m_j} - \frac{\nabla U_{j,xc}}{m_j} \quad (1)$$

and

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0 \quad (2)$$

$$\Delta \varphi = 4\pi e [n_e - n_p - z_i n_i], \quad (3)$$

where propagation vector or wave vector ( $\mathbf{k}$ ) is plotted along  $x$ -axis, and the magnetic force is plotted in the  $z$  direction ( $B_o \mathbf{z}$ ). For the reasons described in the introduction, the plasma will be treated as collisionless at low temperatures.

The last term of Equation (1) represents electron and positron exchange correlation potential, which is a complex function of Fermi particles density and is given as  $U_{j,xc} = \frac{0.985e^2}{\epsilon} n_j^{1/3} \left[1 + \frac{0.034}{a_{Bj} n_j^{1/3}} \ln(1 + 18.37 a_{Bj} n_j^{1/3})\right]$ ,<sup>[31]</sup> which is considered the attribute of the spin effects in dense systems. For the readers, it is useful to find that, for the degenerate plasma, these affects have been calculated comprehensibly in “Statistical Physics” book by Landau and Lifshitz,<sup>[13]</sup> while exchange correlations for the proton interaction have been presented by Tsintsadze et al.<sup>[48]</sup> As this depends on the number density, we cannot ignore it in dense plasma environments. In Equation (1),  $a_{Bj} = \epsilon \hbar^2 / m_j e^2$  is the well-known Bohr atomic radius.

Equation (1) is general and conveniently written; however, later, we will treat ions as classical particles. In Equation (1),  $\hbar = h/2\pi$ ,  $q_j$  the charge,  $m_j$  mass, and  $c$  is the velocity of light in a vacuum of the  $s$ th species. Here,  $j = i$  (ion),  $j = e$  (electron),  $j = p$  (positron or holes),  $q_e = -e$ ,  $q_p = +e$ , and  $q_i = Z_i e$ , with  $e$  being the magnitude of electronic charge and  $Z_i$  the number of charges on ions. In Equation (1),  $P_{Fj} = \frac{(3\pi^2)^{2/3} \hbar^2 n_j^{5/3}}{5m_j}$  is the pressure law for three-dimensional Fermi gas and can also be expressed in terms of Fermi energy, such as  $\frac{2}{5} \epsilon_F(n) n$ , where  $k_B$  is the Boltzmann constant,  $T_{Fs} = \frac{\epsilon_F}{K_B} = \frac{\hbar^2 (3\pi^2 n_{0j})^{2/3}}{2m_j K_B}$  is Fermi temperature, and  $n_s = n_{0j} + \delta n_j$  the total number density with equilibrium number density  $n_{0j}$  and perturbed number density  $\delta n_{1j}$  of  $j$ th particles. The ion component can be considered classical or quantum depending on the relevant parameters. However, in most of the situations, ions are considered cold fluid when describing the ion wave. In these dense quantum and semi-classical plasmas, the screened interaction potential cannot be characterized by the standard Debye–Huckel model according to the multi-particle correlations and the quantum mechanical effects, such as the Bohm potential, quantum pressure, and electron exchange terms,

as the average kinetic energy of the plasma particle in quantum plasmas is of the order of the Fermi energy. The thermal temperature of ions is low compared to the electrons and positrons and is therefore ignored.

### 3 | DISPERSION RELATION OF QUANTUM LOWER HYBRID WAVE

Here, we assume that the mass of electrons and positrons is different, which corresponds to electron–hole plasma in semiconductors. Because of the symmetry, we can assume the propagation vector to be  $k_x$  and  $k_y$  (magnetic field along  $Z$ ) or only  $k_x$ .

We linearize Equations (1)–(3) to obtain the linear and exact dispersion relation of lower hybrid waves, and for this, we use the solution of the plane wave. Below, we discuss two cases.

#### 3.1 | Case 1: Quantum-modified lower hybrid wave

In this case, we assume that  $\omega_{ce} > \omega > \omega_{cp}$  and that ions stay in the background; for this, the quasi-neutrality condition is given as:

$$\delta n_e \simeq \delta n_p \quad (4)$$

$$\frac{\delta n_e}{n_{0e}} = - \frac{\frac{e\varphi}{m_e} k_0^2}{\left[ \Omega_{ce}^2 + k^2 \left( V_{Fe}^2 + \frac{\hbar^2 k^2}{4m_e^2} \right) \right]} \quad (5)$$

and

$$\frac{\delta n_p}{n_{0p}} = \frac{\frac{e\varphi}{m_p} k_0^2}{\left[ \omega^2 - \Omega_{cp}^2 - k^2 \left( V_{Fp}^2 + \frac{\hbar^2 k^2}{4m_p^2} \right) \right]}, \quad (6)$$

where  $\Omega_{cs} = (eB_0/m_s c)$  denotes the cyclotron frequency of  $s$  species and  $V_{FE}^2 = \frac{1}{3}(3v_{Fj}^2 - \alpha_j - 2\eta_j)$  represents the combination of both Fermi velocity and exchange correlation effect. There,  $v_{Fj}^2 = \frac{2k_B T_{Fj}}{m_j}$  is the Fermi speed,  $\alpha_s = 0.985(n_j^{1/3} e^2/m_j \epsilon)$ , and  $\eta_j = 1 + (18.376n_j^{1/3}/a_B)m_j \epsilon$ .

Now considering ions in the background and using the quasi-neutrality condition  $\delta n_e = \delta n_p$ , we obtain a dispersion relation for the lower hybrid waves propagating in electron positron ion plasma  $\omega^2 \gg \omega_{cp}^2$

$$\omega^2 = \Omega_{LQ}^2 + \left( \frac{n_{0p}}{n_{0e}} \right) \left( U_{FS}^2 + \frac{\hbar^2 k^2}{4m_e m_p} \right) k^2, \quad (7)$$

where  $\Omega_{LQ} = [(n_{p0}/n_{e0})\omega_{ce}\omega_{cp}]^{1/2}$  and  $U_{FS} = [P_{Fe}^2/3m_e m_p]^{1/2}$  are lower hybrid frequency and positron sound velocities in Fermi plasma, respectively, whereas the last term represents the Madelung contribution. Deriving (7), we have assumed that the effective mass of electron is less than that of positrons (i.e.  $m_p > m_e$ ). In semiconductors, for example, we often have situations when the mass of the hole becomes much greater than the effective mass of the electrons, and so,  $T_{Fp} < T_{Fe}$ . In deriving (7),  $\omega_{ce} > \omega > \omega_{cp}$  has been taken into account. Equation (7) has a spatial dispersion term, the contribution of which comes from the mass of positrons, and this term can play an effective role in the excitation of new modes. This also shows that the electron Thomas Fermi screening length ( $\lambda = \frac{1}{3}v_{Fe}/\omega_{pe}$ ) decreases as the strength of magnetic field increases, and thus, the particles in the Debye cloud remain mostly confined [49].

#### 3.2 | Case 2: Ultra-low-frequency lower hybrid wave (QULH)

For the dispersion relation of QULH wave, ions are activated and the quasi-neutrality condition now reads as:

$$\delta n_e = \delta n_p + \delta n_i \quad (8)$$

we obtain

$$\omega_L^2 = \left[ \Omega_{\text{ULQ}}^2 + \left( \frac{n_{i0}}{n_{p0}} \right) \left( V_{\text{FS}}^2 + \frac{\hbar^2 k_L^2}{4m_i m_p} \right) k_L^2 \right], \quad (9)$$

where condition  $\omega_{ce} \gg \omega_{cp} \gg \omega_L \gg \omega_{ci}$  is satisfied, and  $\Omega_{\text{ULQ}} = \sqrt{(n_{i0}/n_{p0})\omega_{cp}\omega_{ci}}$ , and  $V_{\text{FS}}^2 = [P_{\text{Fi}}^2/3m_p m_i]^{1/2}$ . For the readers, it is interesting to note that quantum effects can also be important for the ions that are highly massive compared to electrons and positrons, for example,  $T \leq T_{\text{Fi}}$  at  $n_i \simeq 10^{22} \text{ cm}^{-3}$ ,  $m_i \simeq 10^{-24} \text{ g}$ ,  $T_{\text{Fi}} \simeq 10^0 \text{ K}$ , where  $T$  is thermal temperature.

## 4 | EXCITATION OF QULH MODE

Now using the concept of non-linear wave–wave interactions, which are also known as resonant wave–wave scattering or the decay instability, we consider the possible decay of the quantum lower hybrid wave with frequency  $\omega$  and wave vector  $k$  into two waves, a quantum-modified lower hybrid (QLH) wave having frequency  $\omega'$  and wave number  $k'$  and a QULH wave with frequency  $\omega_L$  and wave number  $k_L$ . This simple physical picture can be obtained from Equations (7) and (9), provided the energy and momentum are conserved, that is,

$$\omega - \omega' = \omega_L$$

$$k - k' = k_L, \quad (10)$$

where the components of momentum  $k$ ,  $k'$ , and  $k_L$  are directed along the  $x$ -axis; thus,  $k$  and  $k_L$  are scalars. From the above relations, we obtain

$$\omega - \omega' \simeq \left[ \left( \frac{n_{i0}}{n_{p0}} \right) \omega_{cp} \omega_{ci} \right]^{1/2} = \left[ \left( \frac{n_{p0}}{n_{e0}} \right) \frac{1}{\omega_{cp} \omega_{ce}} \right]^{1/2} \left( v_{\text{sp}}^2 + \frac{\hbar^2 k^2}{4m_e m_p} \right) k(k - k'), \quad (11)$$

here, propagation is in the  $x$  direction only. Thus, using this simple model, we have shown the possibility of the three-wave interaction, which leads to the generation of the QULH waves. Here, we do not calculate growth rates for the three-wave resonant interaction but will calculate more significant growth rates associated with the MI in Section 4.

## 5 | CONSTRUCTION OF ZAKHAROV EQUATIONS

Now, for the excitation of the QULH mode, we will solve for the low frequency density variations and include in our considerations the convective derivative term ( $\delta \mathbf{v}_p \cdot \nabla \delta \mathbf{v}_p$ ), which leads to the ponderomotive force. Thus, we obtain the following equations for the positrons

$$\frac{\partial \delta \mathbf{v}_p^{\text{ul}}}{\partial t} + \langle \delta \mathbf{v}_p \cdot \nabla \delta \mathbf{v}_p \rangle = \frac{e}{m_p} \left( \mathbf{E}^{\text{ul}} + \frac{1}{c} \delta \mathbf{v}_p^{\text{ul}} \times \mathbf{B}_o \right) - \frac{v_{\text{FP}}^2}{n_{p0}} \frac{\partial \delta n_p^{\text{ul}}}{\partial x} + \frac{\hbar^2}{4m_p^2} \frac{\partial^3 \delta n_p^{\text{ul}}}{\partial x^3}, \quad (12)$$

where the angular brackets denote the averaging over a typical lower hybrid wave period and wavelength,  $\delta \mathbf{v}_p$  is the positron velocity for the QLH waves, and  $\mathbf{E}^{\text{ul}}$  is the electric field for the ultra-low frequency field. From the ion continuity equation, we obtain

$$\frac{\partial}{\partial t} \left( \frac{\delta n_p^{\text{ul}}}{n_{0p}} \right) + (\nabla \cdot \delta \mathbf{v}_p^{\text{ul}}) = 0. \quad (13)$$

The ion dynamics are governed by the following equations of momentum and continuity:

$$\frac{\partial \delta \mathbf{v}_i^{\text{ul}}}{\partial t} = - \frac{e \nabla \varphi^{\text{ul}}}{m_i} \quad (14)$$

and

$$\frac{\partial}{\partial t} \left( \frac{\delta n_i^{\text{ul}}}{n_{0i}} \right) + (\nabla \cdot \delta \mathbf{v}_i^{\text{ul}}) = 0. \quad (15)$$

here, the ponderomotive force effect on ions compared to positrons is ignored, and a heavier mass will not cause non-linearity to appear in the ion dynamics. Using the quasi-neutrality condition  $\delta n_p^{\text{ul}} + \delta n_i^{\text{ul}} = 0$  and straightforward algebraic steps, we obtain a Zakharov-like equation<sup>[11]</sup>:

$$\left( \frac{\partial^2}{\partial t^2} + \Omega_{\text{ulh}}^2 - \frac{n_{i0}}{n_{p0}} V_{\text{FS}}^2 \frac{\partial^2}{\partial x^2} + \frac{\hbar^2}{4m_p m_i} \frac{n_{i0}}{n_{p0}} \frac{\partial^4}{\partial x^4} \right) \frac{\delta n_i}{n_{0i}} = - \frac{m_p}{2m_i} v_\phi^2 \frac{\partial^2}{\partial x^2} \left| \frac{\delta n_p}{n_{p0}} \right|^2, \quad (16)$$

where  $v_\phi = (\omega/k)$ , and the last term is the source term arising due to the ponderomotive force. Ponderomotive force comes from the fast time scale in which positrons were involved, and it helps to excite the QULH wave on a slow time scale.

Now, to obtain the second Zakharov equation, which is also known as non-linear Schrodinger equation, we start with the dispersion relation (7) and treat  $\omega$  and  $k$  as operators given by  $\left[ \omega = \omega_0 + i \frac{\partial}{\partial t} \right]$  and  $\left[ k = k_0 - i \frac{\partial}{\partial x} \right]$ <sup>[34]</sup> and obtain

$$i \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial x} - \alpha \frac{\partial^3}{\partial x^3} \right) \delta n_p + \beta \frac{\partial^2 \delta n_p}{\partial x^2} - \Delta \omega \delta n_p - \frac{\hbar^2}{8m_e m_p} \frac{\partial^4 \delta n_p}{\partial x^4} + \Omega_{\text{LQ}} \frac{\delta n_i}{n_{0p}} \delta n_p = 0, \quad (17)$$

where  $\Omega_{\text{LQ}} = (n_{p0}/n_{e0}(\omega_{ce}\omega_{cp}))^{1/2}$ ,  $v_g (= \partial\omega/\partial k$  the group velocity), and  $\alpha$ ,  $\beta$  and  $\Delta \omega$  (the non-linear frequency correction) are defined as:

$$v_g = \left( \frac{n_{e0}}{n_{p0}} \omega_{ce} \omega_{cp} \right)^{-1/2} \left( U_{\text{FS}}^2 + \frac{3k^2 \hbar^2}{2m_p m_e} \right) k \quad (18)$$

$$\alpha = \left( \frac{n_{e0}}{n_{p0}} \omega_{ce} \omega_{cp} \right)^{-1/2} \frac{\hbar^2}{2m_i m_p} k \quad (19)$$

$$\beta = \frac{1}{2} \left( \frac{n_{e0}}{n_{p0}} \omega_{ce} \omega_{cp} \right)^{-1/2} \left( U_{\text{FS}}^2 + \frac{3}{2} \frac{k^2 \hbar^2}{m_p m_e} \right) \quad (20)$$

and

$$\Delta \omega = \frac{\frac{n_{p0}}{n_{e0}} \left[ \omega_{ce} \omega_{cp} + k^2 \left( U_{\text{FS}}^2 + \frac{\hbar^2 k^4}{4m_p m_e} \right) \right] - \omega_0^2}{2\omega_0}; \quad (21)$$

Equations (16) and (17) together make a set of Zakharov-like equations, which will be used in the following sections to study the MI and stationary structures.

## 6 | MI OF QLH WAVES

Now, we investigate the MI of the QLH waves, and to analyse the amplitude modulation of the QLH wave, we use Madelung's representation<sup>[34]</sup> in the  $x$  direction only:

$$\delta n_p \sim a(x, t) e^{iS(x, t)}, \quad (22)$$

where the amplitude  $a$  and the phase  $S$  are real, and substitution of (22) into (16) and (17) gives real and imaginary parts, respectively.

$$-a_0 \left( \frac{\partial}{\partial t} + \mathbf{v}_g \cdot \frac{\partial}{\partial x} \right) \delta S + \beta \frac{\partial^2}{\partial x^2} \delta a - \gamma \frac{\partial^4}{\partial x^4} \delta a + \Omega_{\text{LQ}} a_0 \frac{\delta n_i}{2n_{0p}} = 0 \quad (23)$$

$$\frac{\partial}{\partial t} \delta a + \mathbf{v}_g \frac{\partial \delta a}{\partial x} - \alpha \frac{\partial^3 \delta a}{\partial x^3} + 2\beta a_0 \frac{\partial^2 \delta S}{\partial x^2} = 0 \quad (24)$$

and

$$\left( \frac{\partial^2}{\partial t^2} + \Omega_{\text{ULQ}}^2 - \frac{n_{i0}}{n_{p0}} V_{\text{FS}}^2 \frac{\partial^2}{\partial x^2} + \frac{n_{i0}}{n_{p0}} \frac{\hbar^2}{4m_p m_i} \frac{\partial^4}{\partial x^4} \right) \frac{\delta n_i}{n_{0i}} = - \frac{m_p}{m_i} a_0 \frac{\partial^2 \delta a}{\partial x^2}, \quad (25)$$

where  $\gamma = \hbar^2/8m_p m_e \sqrt{\omega_{ce}\omega_{cp}}$ . To this end, we linearize Equations (23)–(25) with respect to the perturbations, which are represented as  $a = a_0 + \delta a$ ,  $S = S_0 + \delta S$ , where  $a_0$  and  $S_0$  denote the equilibrium values, and  $\delta a$  and  $\delta S$  are the small perturbations.



We seek a plane wave solution proportional to  $\exp[i(\mathbf{k}_L \cdot \mathbf{r} - \omega_L t)]$ ; here,  $\mathbf{k}_L$  and  $\omega_L$  are the wave number and frequency of the modulation, respectively. Finally, we obtain the following dispersion relation for the modulation of quantum lower hybrid wave

$$[(\omega_L - k_L v_g)^2 - \alpha k_L^3 (\omega_L - k_L v_g) - \beta k_L^2 (\beta k_L^2 + \gamma k_L^4)] [\omega_L^2 - \Omega_L^2] = \left( \frac{m_e}{m_i} \right) \left( \frac{n_{0p} n_{0i}}{n_{0e}^2} \right) \left( U_{FS}^2 + \frac{3}{2} \frac{\hbar^2 k_0^2}{m_p m_e} \right) k_L^4 a_0^2, \quad (26)$$

where  $\Omega_L^2 = \left[ \Omega_{ULQ}^2 + \left( \frac{n_{i0}}{n_{p0}} \right) \left( V_{FS}^2 + \frac{\hbar^2 k_L^2}{4m_i m_p} \right) k_L^2 \right]$  From (26), we see that the diffraction term stabilizes the instability. For simplicity, we discuss three limiting cases of the dispersion relation (26).

## 6.1 | Growth rates

**Case 1.** First, we will consider the case when  $\omega_L \gg k_L v_g$ ,  $\alpha k_L^3$ ,  $\beta k_L^2 (\beta k_L^2 + \gamma k_L^4)$ , and  $\omega_L \gg \Omega_L$ ; in this case, the growth rate of instability is

$$\text{Im}\omega_L = \left( \frac{m_e}{m_i} \frac{n_{p0} n_{i0}}{n_{e0}^2} \right)^{1/4} \left( U_{FS}^2 + \frac{3}{2} \frac{\hbar^2 k_0^2}{m_p m_e} \right)^{1/4} a_0^{1/2} k_L. \quad (27)$$

**Case 2.** Now, we assume  $\omega_L - k_L v_g = \Gamma$ ,  $(\omega_L^2 - \Omega_L^2) \simeq (k_L^2 v_g^2 - \Omega_L^2)$ .  $\Omega_L^2 > (k_L v_g)^2 + \beta k_L^2 (\beta k_L^2 + \gamma k_L^4)$ ; using this in (26) gives us

$$\Gamma^2 = -(n_{p0} \omega_{ce} \omega_{cp})^{1/2} \left[ \frac{2\beta m_e n_{0i} a_0^2}{m_i n_{0e}^{3/2}} \right] \frac{k_L^4}{\Omega_L^2}. \quad (28)$$

**Case 3.** At resonance,  $\omega_L - k_L v_g = \Gamma$ ,  $\omega_L - \Omega_L = \Gamma$ ; with this, (26) becomes

$$\Gamma^3 - \alpha k_L^3 \Gamma^2 - \beta k_L^2 (\beta k_L^2 + \gamma k_L^4) \Gamma = (n_{0p} \omega_{ce} \omega_{cp})^{1/2} \frac{2\beta m_e n_{0i} k_L^4 a_0^2}{m_i n_{0e}^{3/2}}. \quad (29)$$

In this case, the growth rate is much larger. Equation (29) is a well-known cubic equation of the form  $ax^3 + bx^2 + cx + d = 0$ , which has three solutions. We have plotted the growth rate (27) for different values of amplitude of modulation and propagation vector, and it is obvious from the plot that, with an increase in the value of amplitude of modulation, the growth rate also increases.

## 7 | SOLITON SOLUTIONS

Stationary structures, like solitons that are usually formed from non-linearly propagating waves, have been rigorously investigated in plasmas and other media. Here, we shall use the standard approach to investigate solitons, but we will restrict ourselves to the consideration of stationary structures. Before we proceed further, let us assume in the second Zakharov Equation (17),  $\beta \partial^2 / \partial x^2 (\delta n_p) \gg \alpha \partial^3 / \partial x^3$  and  $\hbar^2 / 8m_e m_p \partial^4 / \partial x^4 (\delta n_p)$ , and shifting to a co-moving frame of reference  $\xi = x - v_g t$  such that the perturbations vanish at  $\xi \rightarrow \pm \infty$ , which yields

$$\frac{\partial^2 \delta n_p}{\partial \xi^2} - \frac{\Delta \omega}{\beta} \delta n_p - Q \delta n_p \frac{\partial^2 \delta n_p^2}{\partial \xi^2} = 0, \quad (30)$$

where

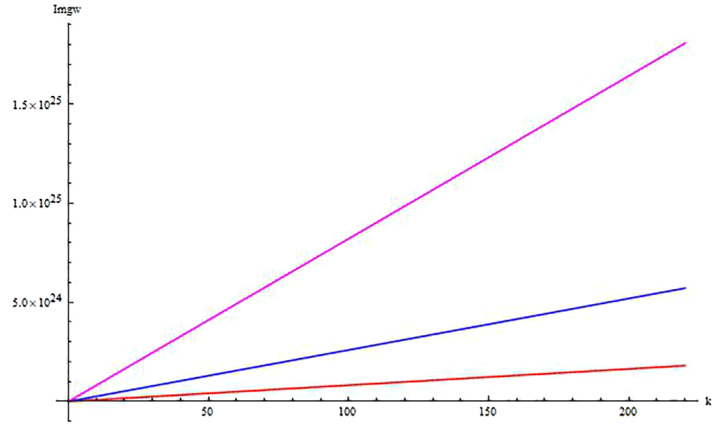
$$Q = \frac{n_{p0}}{n_{e0}} \frac{\omega_{ce} \omega_{cp}}{\left( U_{FS}^2 + \frac{3\hbar^2}{2m_p m_e} k_0^2 \right) k_0^2} \quad (31)$$

$\Delta \omega$  is the non-linear frequency shift of the solitary structure. Using (16) and (36), we shall consider two types of soliton solutions.

### 7.1 | Ordinary solitons

In the first case, we take the limit in (16):  $\partial^2 / \partial t^2 + \Omega_{ulh}^2 \ll (n_{i0} / n_{p0}) V_{FS}^2 \partial^2 / \partial x^2$  and obtain the following expression for the perturbed density:

$$\delta n_i = \left( \frac{m_p}{2n_{p0} m_i} \frac{v_\phi^2}{V_{FS}^2} \right) \langle \delta n_p^2 \rangle. \quad (32)$$



**FIGURE 1** Plot of growth rate (27) for different values of amplitude of modulation and propagation vector

Substitution of (32) into (30) and integration once yield

$$\frac{dP}{dy} = P\sqrt{1 - P^2}, \quad (33)$$

where

$$P = \left( \Omega_{LQ} \frac{m_p}{m_i} \right)^{1/2} \frac{v_\phi}{n_{p0} V_{FS}} \delta n_p \quad (34)$$

and  $y = \sqrt{2\Delta\omega/\beta\xi}$ . Equation (33) has a solution of the form

$$y = \log \left[ \frac{P}{1 + \sqrt{1 - P^2}} \right]. \quad (35)$$

which represents a standard bright soliton structure. It is important to note that we have neglected higher-order terms, which could be interesting; however, that analysis is beyond the scope of the present investigation (Figure 1).

## 7.2 | Cusp soliton

Under the limit that first and third terms of (16) compensate each other, the following expression for the perturbed ion density is obtained

$$\frac{\delta n_i}{n_{0i}} \approx -\frac{\delta n_p^2}{2n_{0i}n_{0e}k_0^2} \frac{\partial^2}{\partial x^2} \left( \frac{\delta n_p^2}{n_{0i}n_{0e}} \right) \quad (36)$$

here, we notice the interesting feature that the density perturbation is proportional to the second derivative of the amplitude of the density, which shows that the regions of higher amplitude of density correspond to highly depleted regions of local density leading to cusp solitons. Substitution of (36) into (30) gives

$$\left( \frac{dC}{dz} \right)^2 (1 - 2QC^2) = C^2, \quad (37)$$

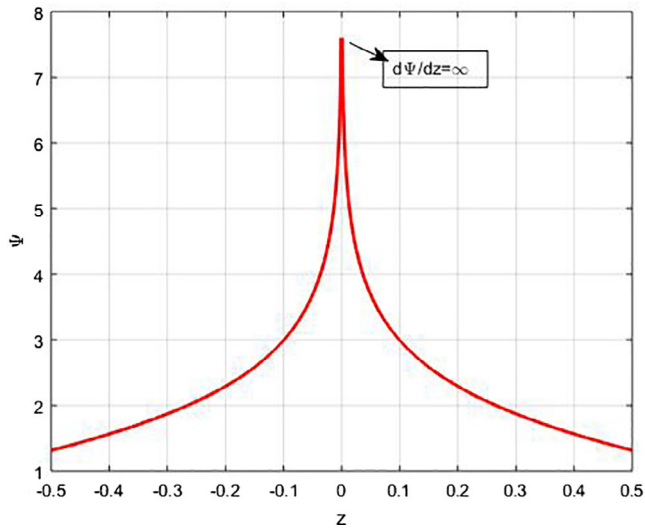
where  $C = \delta n_p/n_{p0}$  and  $z = \sqrt{\Delta\omega/\beta\xi}$ . Introducing  $\Psi = \sqrt{2QC}$  and integrating Equation (37) once, we obtain

$$\frac{d\Psi}{dz} \sqrt{1 - \Psi^2} = \Psi. \quad (38)$$

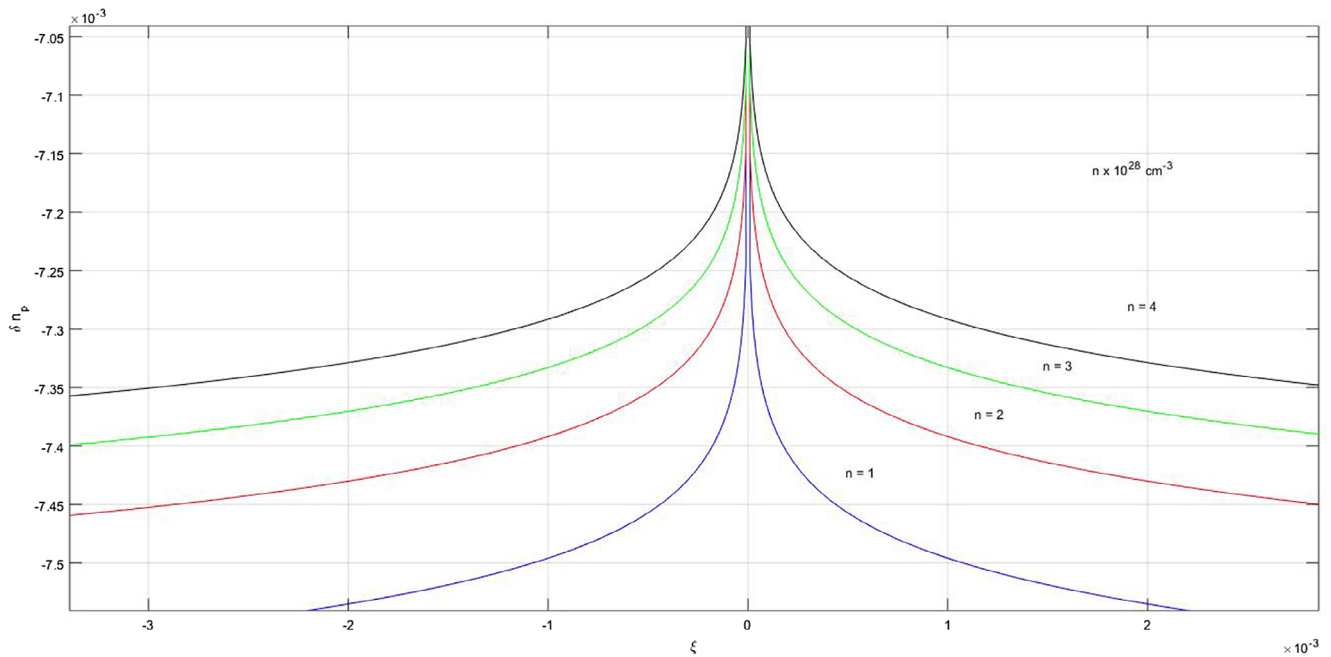
The above equation shows that, when  $\Psi$  reaches the maximum value ( $\Psi = 1$ ), the first derivative goes to infinity ( $d\Psi/dz \rightarrow \infty$ ), which shows evidence of the formation of cusp solitons. The integration of Equation (39) leads to the following solution

$$z = \sqrt{1 - \Psi^2} + \log|\Psi| - \log|1 + \sqrt{1 - \Psi^2}| \quad (39)$$





**FIGURE 2** A schematic illustration of cusp soliton solution given by (39) with general features of  $d\Psi/dz \rightarrow \infty$  at its maxima



**FIGURE 3** Cusp soliton with the variation of number density

or in terms of  $\xi$  and amplitude of density.

$$\xi = \sqrt{\frac{\beta}{\Delta\omega}} \sqrt{1 - \frac{2Q}{n_{p0}^2} \delta n_p^2} + \log \left| \frac{\sqrt{2Q}}{n_{p0}} \delta n_p \right| - \log \left| 1 + \sqrt{1 - \frac{2Q}{n_{p0}^2} \delta n_p^2} \right| \quad (40)$$

the graph (Figure 2) of which exhibits an infinite discontinues slope (cusp) at its crest and justifies the term ‘‘cusp soliton’’, but all physical quantities remain constant. This type of soliton is quite different from the well-known smooth solitons because of the infinite first derivative at its maxima.

In Figure 3, we have examined the effect of variations in the cusp with the number density for the typical plasma number density of the order of  $n_{0e} \sim 1.5 \times 10^{28} \text{ cm}^{-3}$ , with a very high ambient magnetic field  $B_0 \sim 10^{10} \text{ G}$  and using the physical constants in cgs units viz.,  $c = 3 \times 10^8 \text{ cm/s}$ ,  $m_e = 9.1 \times 10^{-28} \text{ g}$ ,  $m_i = 1.67 \times 10^{-24} \text{ g}$ , and  $\hbar = 1.057 \times 10^{-27} \text{ erg s}$ . The pattern shows that the amplitude of the density varies, and the width of the cusp changes substantially as the number density changes.

## 8 | CONCLUSIONS

During this research, a theoretical model was presented for the excitation of QULH oscillating at frequency. This model uses the decay of a relatively high-frequency QLH wave into a relatively lower frequency QLH and QMLH based on three-wave resonant interaction. MIs of QLH waves are investigated, and their growth rates are studied. In addition, one-dimensional, non-linear localized structures of bright solitons and non-linear nonlocal structures like cusp solitons are obtained. The generation of cusped solitary waves is considered by the modulation of the lower hybrid wave amplitude. We believe that such an approach to lower hybrid waves is important and demands attention. To the best of the authors' knowledge, this stationary cusp soliton solution has not been studied for the degenerate plasma system.

## ACKNOWLEDGMENT

One of us (Z.E) is grateful to the Chair, department of Physics, CUI, Prof. Sadia Manzoor for making her visit to National Center for Physics (NCP) – Islamabad possible, where the major part of this work was carried out. Hospitality from the DG and CAAD office of the NCP – Islamabad is gratefully acknowledged. Help from Dr. Yousaf Habib related to Matlab and useful comments from anonymous referee are also appreciated.

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**How to cite this article:** Ehsan Z, Tsintsadze N, Fedele R, Haque Q, Shah H. Modulation instability of lower hybrid waves leading to cusp solitons in electron–positron(hole)–ion Thomas Fermi plasma. *Contributions to Plasma Physics*. 2019;1–12. <https://doi.org/10.1002/ctpp.201800132>