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Energy transport of circularly polarized waves in bi-kappa distributed plasmas

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Abstract. The energy transport of circularly polarized waves (CPW) in bi-kappa distributed plasmas is studied using kinetic theory. Energy flux is examined by taking into account the wave-particle interaction. We investigate how the energy flux is affected by the variation of thermal speed, temperature anisotropy (the parallel and perpendicular temperatures are different with respect to the direction of ambient magnetic field, i.e., $T_\perp > T_\parallel$), index κ and the wave frequency. It is found that the CPW transport their energy rapidly over distances for smaller values of the thermal speed, the index κ and the wave frequency, whereas for low values of temperature anisotropy the waves deliver their energy slowly. Thus the above-mentioned parameters play an important role in the transport of wave energy. Possible applications of the present analysis are discussed.

1 Introduction

Understanding the interaction of electromagnetic (EM) waves with collisionless plasma is one of the important problems in plasma physics. It has been studied extensively how the wave attenuates spatially when it interacts with the conducting medium [1–7]. This analysis is helpful to understand the heating/excitation of EM waves in plasma [8] in different fields like laser plasma interaction [9], tokamaks [10,11], material processing [12] and in inductively coupled plasmas [13,14]. For instance, laser energy transmission and absorption in hot plasmas is also predicted on the basis of spatial damping of the wave [15].

Different velocity distributions have been employed to examine the propagation of waves in various plasma environments. The systems in which the particles are in thermal equilibrium are well described by the Maxwellian distribution function. However, when the particles are out of thermal equilibrium, like in collisional [16] and collisionless plasmas [17,18], kappa distribution function is more suitable. Applications of kappa distribution have been discussed by many authors in space [19–23] and laboratory plasmas [24]. Meige and Boswell [25] performed simulation to confirm the experimental findings of kappa distribution proposed by Granovski [26] and Godyak et al. [27].

From the in-situ observations the terrestrial magnetosphere and the solar wind show temperature anisotropic electron velocity distribution [16,18,28]. The temperature anisotropies are the sources of free energies that can trigger different kinetic instabilities [29–32]. The dispersion properties of the wave change significantly with the parameters: plasma beta, temperature anisotropy, index kappa and plasma to gyrofrequency ratio [33,34]. When the perpendicular temperature of electrons is larger than the parallel temperature with respect to ambient magnetic field ($A = T_\perp/T_\parallel > 1$) with the combination of plasma beta and other plasma parameters, the whistler modes destabilize. On the other hand when $A = T_\perp/T_\parallel < 1$, the firehose instability is driven [18,31]. In magnetospheric environment, two distinct particle populations exist i.e., low temperature and high temperature which determine the propagation characteristics and the instabilities of the waves, respectively [35–37]. Temperature anisotropy plays an important role in the electromagnetic fire-hose instability, electron cyclotron resonance (ECR), intense laser matter interaction and plasma processing [38–41].

The energy flux density of the electromagnetic wave is given by the Poynting vector. The divergence of Poynting flux determines how the energy changes when the wave propagates through a particular region. Poynting flux analyzer (PFX) on board the Akebono satellite [42] and the plasma wave instrument (PWI) [43] are used to measure the electric field and magnetic field components of the wave. The electric field helps to understand the plasma transport and acceleration in the magnetosphere. Earlier, the spatial attenuation of electromagnetic waves has been studied for laboratory plasma by various authors e.g., Ferrante et al. [7] and Kaganovich et al. [35,36], but not

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much attention has been given as to how the waves transport their energy over distance.

The purpose of this study is to understand how the circularly polarized waves in kappa distributed plasma transport their energy during the propagation. We discuss it by taking into account the wave particle interaction (resonant case). In the cyclotron resonance, the energy exchange occurs only when the velocity of electrons is comparable with the phase velocity of the wave.

2 Mathematical formulation

Using Vlasov-Maxwell set of equations, we get the generalized linear dispersion relation for parallel propagating right circularly polarized waves (RCPW) as

$$\omega^2 = c^2k^2 - \pi\omega\omega_{pe}^2 \int_{-\infty}^{\infty} \mathrm{d}p_\parallel \int_0^\infty p_\perp^2\, \mathrm{d}p_\perp \times \left[\frac{\frac{\partial f_0}{\partial p_\perp} - \frac{k}{m\omega}(p_\parallel \frac{\partial f_0}{\partial p_\perp} - p_\perp \frac{\partial f_0}{\partial p_\parallel})}{(\omega - kv_{_\parallel} - \omega_{ce})}\right], \quad (1)$$

where ω is the wave frequency, k is the wavenumber, ω_{ce} is the gyrofrequency of electrons, f_0 is arbitrary equilibrium distribution function and $p_{\parallel,\perp}$ is the momentum of electrons in parallel and perpendicular direction to the ambient magnetic field. We have employed here the temperature anisotropic kappa distribution function given by

$$f_0 = \frac{n_0}{\pi^{\frac{3}{2}}\theta_\perp^2\theta_\parallel} \frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \left[1 + \frac{v_\parallel^2}{\kappa\theta_\parallel^2} + \frac{v_\perp^2}{\kappa\theta_\perp^2}\right]^{-\kappa-1}, \quad (2)$$

with

$$\theta_{\perp,\parallel}^2 = \left(\frac{2\kappa-3}{\kappa}\right) v_{t_{\perp,\parallel}} \ ; \quad v_{t_{(\perp,\parallel)}} = \sqrt{\frac{T_{(\perp,\parallel)}}{m}}.$$

Using the above distribution function and after executing some algebraic steps, the dispersion relation (1) reduces to

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\omega_{ce}}{k\theta_\parallel} Z(\xi) - \frac{\theta_\perp^2}{\theta_\parallel^2}\left[1 + \xi Z(\xi)\right]\right], \quad (3)$$

where $\xi = \frac{\omega - \omega_{ce}}{k\theta_\parallel}$ and $Z(\xi)$ is the modified plasma dispersion function defined as [44–49]

$$Z(\xi) = \frac{\Gamma(\kappa)}{\sqrt{\pi}\kappa^{1/2}\Gamma(\kappa - 1/2)} \int_{-\infty}^{\infty} \frac{\mathrm{d}s}{s-\xi}\left[1 + \frac{s^2}{\kappa}\right]^{-\kappa}. \quad (4)$$

Although in literature the dispersion relation of parallel propagating electromagnetic waves has been derived and solved numerically by many authors to investigate the instability/damping of waves, in the present paper our focus is to examine the energy transfer of parallel propagating electromagnetic waves over distance which has not been reported in the literature earlier according to the best of our knowledge. This work is based on the analytical approach in which we solve equation (6) for complex wavenumber to get the imaginary wavenumber k_i and then use that k_i in the Poynting flux theorem.

The expansion of plasma dispersion function for small argument i.e., $\xi \ll 1$ is given by

$$Z(\xi) \simeq \frac{i\sqrt{\pi}\Gamma[\kappa]}{\sqrt{\kappa}\Gamma[\kappa - \frac{1}{2}]}\left(1 + \frac{\xi^2}{\kappa}\right)^{-\kappa} - \frac{\sqrt{\pi}}{\kappa\Gamma[\kappa - \frac{1}{2}]}\frac{\Gamma[\kappa + \frac{1}{2}]}{\Gamma[\frac{3}{2}]}\xi + \frac{\sqrt{\pi}}{\kappa^2\Gamma[\kappa - \frac{1}{2}]}\frac{\Gamma[\kappa + 1 + \frac{1}{2}]}{\Gamma[1 + \frac{3}{2}]}\xi^3. \quad (5)$$

We use small argument expansion of plasma dispersion function in equation (3) and obtain the dispersion relation as,

$$k^2 - \frac{\omega^2}{c^2} + \frac{\omega_p^2}{c^2}\left[1 - \frac{\omega_{ce}}{k\theta_\parallel}\frac{i\sqrt{\pi}\Gamma[\kappa]}{\sqrt{\kappa}\Gamma[\kappa - \frac{1}{2}]} - \frac{\theta_\perp^2}{\theta_\parallel^2} \times \left\{1 + \frac{i\sqrt{\pi}\Gamma[\kappa]}{\sqrt{\kappa}\Gamma[\kappa - \frac{1}{2}]}\left(\frac{\omega - \omega_{ce}}{k\theta_\parallel}\right)\right\}\right] = 0. \quad (6)$$

The reason to take small argument of the plasma dispersion function is to have strong wave-particle interaction whereas for large argument the wave-particle interaction (the pole contribution term) would be rather weak and the imaginary part of the wavenumber would be negligible.

Assuming $k_i^2 \ll k_r^2$, the real part and the imaginary part of the above dispersion relation are respectively

$$k_r^2 = \frac{\omega^2}{c^2} + \frac{\omega_p^2}{c^2}\left(\frac{\theta_\perp^2}{\theta_\parallel^2} - 1\right), \quad (7)$$

and

$$k_i = \frac{\sqrt{\pi}\Gamma[\kappa]}{\sqrt{\kappa}\Gamma[\kappa - \frac{1}{2}]}\frac{\omega_p^2}{c^2}\frac{\left[\frac{\omega_{ce}}{\theta_\parallel} + \frac{\theta_\perp^2}{\theta_\parallel^2}\left(\frac{\omega - \omega_{ce}}{\theta_\parallel}\right)\right]}{2\left[\frac{\omega^2}{c^2} + \frac{\omega_p^2}{c^2}(\frac{\theta_\perp^2}{\theta_\parallel^2} - 1)\right]}, \quad (8)$$

where we have taken $\omega > \omega_{ce}$ and $\theta_\perp^2/\theta_\parallel^2 > 1$. Using the definitions of $\theta_\perp^2$ and $\theta_\parallel^2$ we may write $A = T_\perp/T_\parallel$.

To determine how the electromagnetic wave delivers its energy to the plasma particles, we apply the Poynting flux theorem for the steady state i.e., $\nabla.\mathbf{S} = -P$ [50–53], where P and $\mathbf{S}$ are the power dissipation and Poynting vector defined as

$$P = \frac{1}{2}\mathrm{Re}(\mathbf{J}^*.\ \mathbf{E})\ , \quad (9)$$

and

$$\mathbf{S} = \frac{\mathrm{Re}}{\mu_0}(\mathbf{E}^* \times \mathbf{B}). \quad (10)$$

In the preceding equations (9) and (10), $\mathbf{J}, \mathbf{E}$ and μ_0 are the current density of electrons, perturbed electric field and magnetic permeability, respectively.

The wave polarization is taken in such a way that the ambient magnetic field $\mathbf{B}_0$ is along z-axis, the electric field and magnetic field perturbations lie in x–y plane. Current

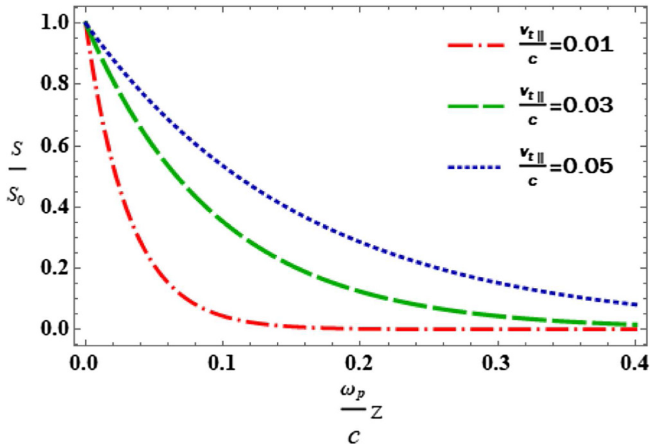

Fig. 1. Poynting flux (S/S_0) versus distance $(z\omega_p/c)$ for isotropic Maxwellian case $(A = 1, \kappa = \infty)$.

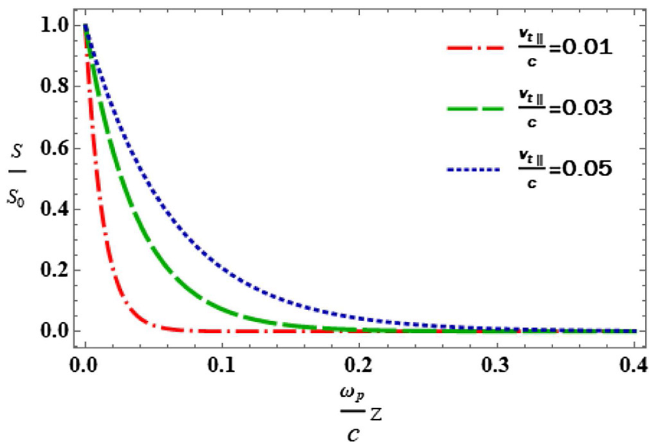

Fig. 2. Poynting flux (S/S_0) versus distance $(z\omega_p/c)$ for fixed values of index kappa $(\kappa = 2)$ and temperature anisotropy $(A = 2)$.

density is derived from the Ampere's Law. Resultantly we obtain the power dissipation as

$$P = -\frac{1}{2}\mathrm{Re}[ik\mu_0(B_xE_y - B_yE_x)]. \tag{11}$$

On simplifying the expressions of P and $\mathbf{S}$ we recast the energy flux theorem as

$$\frac{\partial S}{\partial z} = -k_i S_z \tag{12}$$

whose solution is

$$S(z) = S(0)\ \exp[-k_i\ z], \tag{13}$$

where $S(0)$ shows the energy at the point where the wave starts to travel and k_i is defined in equation (8). It is important to note how the spatial damping factor k_i appears in the energy flux theorem.

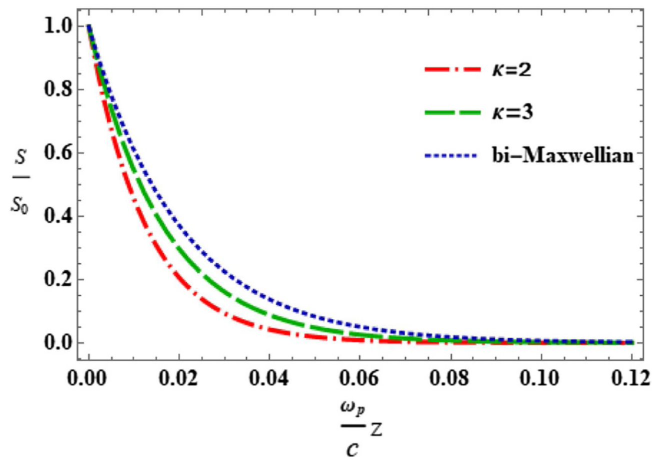

Fig. 3. Poynting flux (S/S_0) versus distance $(z\omega_p/c)$ for fixed values of temperature anisotropy $(A = 2)$ and thermal speed $(v_{t\parallel}/c = 0.01)$.

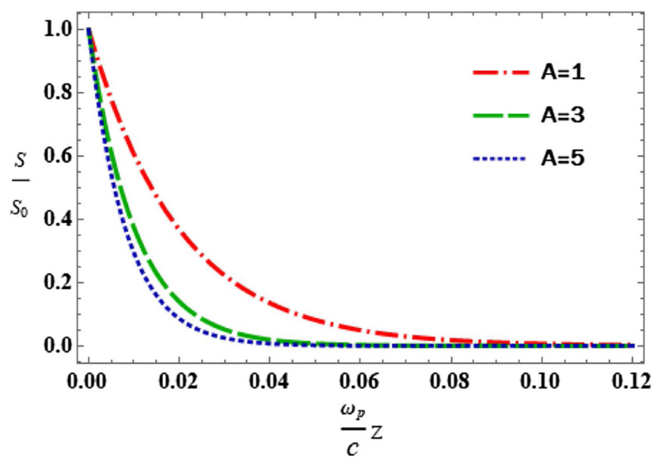

Fig. 4. Poynting flux (S/S_0) versus distance $(z\omega_p/c)$ for fixed values of index kappa $(\kappa = 2)$ and thermal speed $(v_{t\parallel}/c = 0.01)$.

3 Results and discussion

To examine the effect of temperature anisotropy and the index κ on the energy flux of RCPW, we plot equation (13). The results obtained are applicable for the plasma sheet region [54], having electron number density $n = 0.5\,\mathrm{cm}^{-3}$ and magnetic filed $B_0 = 10^{-4}$ G. Figure 1 is plotted for isotropic Maxwellian plasma, it is noted that for low values of thermal speed of electrons the waves deliver their energy rapidly at shorter distances, whereas for larger values of thermal speed the waves transport their energy rather slowly over the longer distances. Figure 2 depicts that the waves deliver their energy at shorter distances when both the temperature anisotropy and index kappa are taken into account. Figure 3 shows that for low values of index kappa (κ), the wave transports its energy rapidly. It may be due to more resonate particles for low values of index κ. In Figure 4, we plot the energy flux (S/S_0) vs. normalized distance $(z\omega_p/c)$. It is seen from the figure that the wave delivers its energy rapidly with the

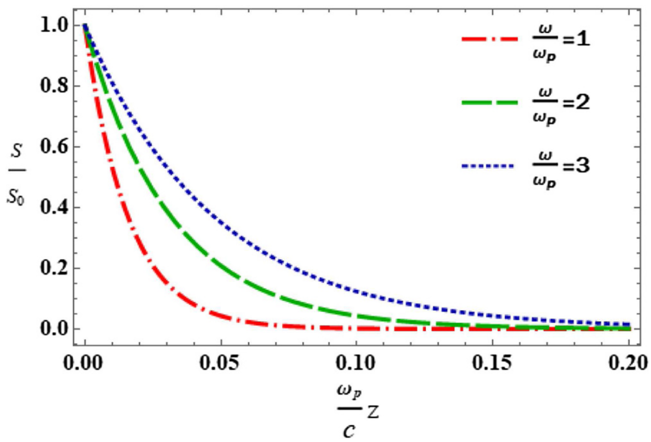

Fig. 5. Poynting flux (S/S_0) versus distance ($z\omega_p/c$) for isotropic Maxwellian case ($A = 1$, $\kappa = \infty$) and thermal speed ($v_{t\|}/c = 0.01$).

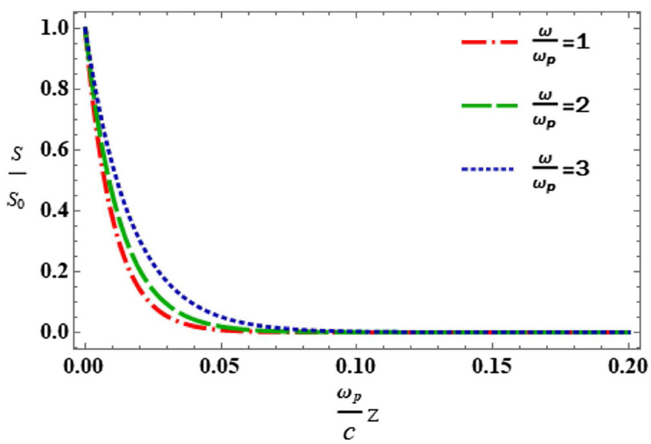

Fig. 6. Energy flux (S/S_0) versus distance ($z\omega_p/c$) for fixed values of anisotropy ($T_\perp/T_\| = 2$), index kappa ($\kappa = 2$) and thermal speed ($v_{t\|}/c = 0.01$).

increase of temperature anisotropy ($T_\perp/T_\|$). The possible explanation is the following. In the anisotropic plasma, the number of resonant particles is more and thus the wave transports its energy rapidly over distances. Figure 5 is for the isotropic Maxwellian plasma which shows that the wave transfers its energy over longer distances at higher values of wave frequency. In Figure 6, the temperature anisotropy and the index kappa are taken into account, the same trend is observed as to Figure 5 but this time the wave delivers its energy more rapidly over distances.

This approach may be helpful to explain the physical mechanism that may favour the transmission of energy during the interaction of solar wind with magnetosphere, especially during magnetosheath irregularities observed deep inside the terrestrial magnetosphere by various spacecrafts like the Radio Plasma Imager (RPI) on the Imager for Magnetopause-to-Aurora Global Exploration (IMAGE) satellite around the Earth [55–57].

In summary, we have used kinetic approach to study the energy transport for the parallel propagating right handed circularly polarized waves in bi-kappa distributed plasmas. It is found that the energy flux changes significantly over distances with the variation of the thermal speed, temperature anisotropy, index kappa and wave frequency. From Figures 1 and 2, we see that the waves transfer their energy over longer distances for larger values of the thermal speed of electrons. Figures 3 and 4 show that for large values of temperature anisotropy ($T_\perp/T_\| > 1$) and low values of index kappa, the wave would transfer its energy rapidly. In Figures 5 and 6, we observed that the waves transport their energy over longer distances at higher values of the wave frequency. It is also noted that the waves deliver their energy rapidly in kappa distributed plasma, possibly due to the more high energy particles, as compared to Maxwellian velocity distributed plasma. The results may be applicable to the laser plasma interaction, for understanding physical mechanisms involved in the material processing where the distribution is expected to be non-Maxwellian [58]. This analysis is extendable to answer similar questions of Poynting flux for the relativistic and ultra relativistic plasmas.

Author contribution statement

Tajammal H. Khokhar is the corresponding author and was involved with the analytical calculations, literature survey and preparation of the manuscript. Imran A Khan, Dr. H. A Shah and Dr. G. Murtaza contributed to the interpretation of the results. They have read and approved the final manuscript.

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