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Novel features of electromagnetic waves in an isotropic degenerate electron-ion plasma

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Abstract

Within the framework of kinetic theory, the nonlinear interaction of electromagnetic waves (EMWs) with a degenerate electron-ion plasma is studied to account for the electron quantum mechanical effects. For this purpose, a specific quantum regime is considered, for which the degenerate electron Fermi velocity is assumed to be of the order of the group velocity of EMWs. This eventually leads to the existence of a nonlinear Landau damping rate for the EMWs in the presence of electron ponderomotive force. The electron–ion density oscillations may have arisen from the nonlinear interaction of EMWs, leading to a new type of nonlinear Schrödinger equation in terms of a complex amplitude for electromagnetic pump waves. The profiles of nonlinear damping rates reveal that EMWs become less damped for increasing the quantum tunneling effects. The electrostatic response of the linear electrostatic waves is also investigated and derived from a linear dispersion for the ion-acoustic damping rate. The latter is a direct function of the electron Fermi speed and does not rely on the Bohm tunneling effect. The obtained results are numerically analyzed for two microwaves of different harmonics in the context of nonrelativistic astrophysical dense plasma environments, e.g. white dwarfs, where the electron quantum corrections cannot be ignored.

Keywords: nonlinear interaction of electromagnetic waves, kinetic nonlinear Schrödinger, degenerate electron-ion plasma

(Some figures may appear in colour only in the online journal)

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1. Introduction

The interaction of waves with a plasma medium is quite pertinent for understanding the basic plasma phenomena [1–4], e.g. the Landau damping (LD) rates can be studied to show the linear and nonlinear behavior of plasmas and indicate the wave damping without any collisions. The concept of collisionless wave damping was first theoretically presented by a Russian Physicist, Landau, in 1946 [5]. Eighteen years later, this effect was verified in laboratory experiments by Malmberg and Wharton [6]. The investigation of linear and nonlinear waves and more specifically, the analyses concerning the nonlinear propagation of high-frequency electromagnetic waves (EMWs) have been carried out for a long period of time [7–14], investigating novel properties of classical plasmas. However, in recent years, the works [15–18] have mostly been concerned with the LD of electrostatic and EMWs in the classical regime and only limited investigations have been analyzed to study the LD in quantum-dense plasmas, where quantum collective waves and quantum scales become more important and play a significant role in the behavior of plasma particles. In fact, degenerate dense plasmas not only have relevance in the laboratory, but in astrophysical environments [19, 20]. Quantum effects have often been ignored in the study of nonlinear wave–particle interactions, which may be incorporated to investigate the nonlinear LD phenomena [21–26]. Only a few investigations have been carried out with quantum settings in the context of linear theory. Specifically, the impact of degeneracy (arbitrary) of electrons has been examined with regard to the linear LD of electrostatic waves [27], showing the linear LD during an x-ray Thomson scattering experiment [28]. Later, a 1D quantum Liouville–Poisson system was employed to simulate the nonlinear LD [29], degeneracy effects and linear LD caused by particle trapping. Moreover, with utilization of numerical simulations in a 3D form, the nonlinear LD associated with the plasma waves can carry a finite orbital angular momentum, which is primarily transferred to the resonant electrons. In these studies, the plasma waves are represented in the form of Laguerre–Gaussian profiles, and significant modifications occur in the LD [30]. Direct evidence of LD in a turbulent space plasma [31] suggests that it has played a significant role in the dissipation process, in which the energy can be transferred from the electric field to the electrons. The nonlinear stage of the Langmuir wave analysis [32] has also determined that Langmuir waves can carry a finite amplitude and oscillation frequency that are larger than the damping rate (found in the linear approximation) and one does not have a damping effect, while only showing a periodic structure. However, the efforts [33–35] cannot be ignored in studying the nonlinear LD in a degenerate Fermi gas. In order to recognize the distinguishing role of quantum effects [36–40], distinct features of the EMWs need to be studied through wave–particle interactions and nonlinear LD phenomena at quantum scales.

Degenerate effects become relevant in plasmas if the electron thermal energy (k_BT_e) is comparable to or smaller than the electron Fermi energy $\epsilon_{Fe}\left[=\frac{\hbar^2}{2m_e}\left(3\pi^2 n_{e0}\right)^{2/3}\right]$, where n_{e0} (m_e) is the electron equilibrium number density (electron mass). Degenerate species involving nonlocal effects introduce new scales and coupling parameters in a plasma medium. In this context, a quantum kinetic description is to be considered to identify the linear and nonlinear LD phenomena. Various models have been adopted for this purpose in the literature. However, one of the most appropriate models has been reported [41] recently for deriving a new class of quantum kinetic equations and modeling degenerate Fermi plasmas, relying on the original framework of kinetic theory. The main objective is to identify the interaction scales where electromagnetic (EM) and electrostatic waves can be coupled in a degenerate electron–ion (EI) plasma and where the quantum nature of the plasma species cannot be ignored. A key aspect of the present analysis is to calculate the singularity present in the momentum integration. Since the phase speed of the EMWs is greater than the speed of light, it results in the suppression of the singularity in the usual Landau description and accordingly the existing treatment of singularity may lead to linear damping, which is not valid anymore. Consequently, one may experience a nonlinear LD of EMWs in degenerate plasmas.

Since the Fermi speed of the plasma species does not resonate with the phase speed of EMWs (the phase speed of EMWs is often much greater than the speed of light), the Fermi speed in degenerate plasmas may take the order of the group speed of the EMWs because the group speed is always smaller when compared to the speed of light. Hence, the nonlinear coupling of EMWs with a degenerate plasma through the wave—particle interaction leads to the generation of beat waves having frequencies of $\omega_0-\omega_0'$ and corresponding wave numbers of k_0-k_0', propagating with a group speed and resonating with a Fermi speed as $v_g\left[=\frac{\omega_0-\omega_0'}{|k_0-k_0'|}\right]\equiv v_{Fe}$. This refers to the new definition of the nonlinear LD, where v_{Fe} stands for the electron Fermi speed, as was defined in [24, 42] for classical plasmas. Thus, degenerate electrons with an electron Fermi speed of the order of phase speed of EMWs are the most favorable candidates to exchange their energy via the wave–particle interaction and may lead to the nonlinear LD of EMWs. On the other hand, if the assumption such as $v_g\equiv v_{Fe}$ does not remain valid, then the only linear LD for electrostatic waves occurs, where the wave phase velocity matches the thermal velocity of plasma species. Consequently, the LD of the transverse EMWs cannot be investigated.

When high-frequency EMWs interact nonlinearly with a plasma medium, the distribution of plasma particles is significantly affected, resulting in density oscillations or longitudinal waves that propagate along the electric field direction. Recently, Zhu *et al* [43] studied the dispersive properties of the electron waves and challenged the traditional conditions attributed to low-temperature and high-density plasmas. They obtained an expression for the LD rate by taking into account the electron waves with quantum correction due to the Bohm potential at normal temperatures and high densities. They noticed that the LD rate reduces in the presence of quantum effects. In the present model, a study of density oscillations is carried out that gives rise to local and nonlocal nonlinearities and accounts for the nonlinear LD of transverse

EMWs. In our analysis, the density oscillations in a degenerate EI plasma are calculated by using the Vlasov equation for degenerate plasma particles. A new type of kinetic nonlinear Schrödinger equation (KNLSE) is obtained for fermions by making use of Maxwell equations. It is shown that quantum mechanical effects significantly modify the wave frequency and group velocity and affect the consequent energy transfer rate of the modulated EMWs. The nonlinear LD rate is computed in the presence of electron Bohm potential and electron degeneracy effects. Two different frequency regimes are considered to modify the real linear frequencies of electrostatic waves with quantum Bohm potential and reveal the linear damping of electrostatic waves to be unaffected by the Bohm tunneling effect.

2. Basic formulation

In order to study the nonlinear interaction of EMWs with a quantum degenerate collisionless unmagnetized plasma, we consider the well-known Vlasov equations for the jth plasma species (j equals e for electrons and i for ions) and account for the electron quantum Bohm potential and nonrelativistic ponderomotive force effects. The EM wave is assumed to propagate along the z-axis, whereas the degenerate species are modeled with the Fermi–Dirac distribution. This specifically gives rise to the quantum statistical effects [41] of the degenerate plasma species. Thus, the Vlasov equations along with the charge-neutrality condition for a degenerate EI plasma can be governed by the following:

$$\left(\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}\right)\langle f_{e1}\rangle + \left(e\frac{\partial\Phi}{\partial z}-\frac{\partial\Phi_{pe}}{\partial z}+\frac{\hbar^2}{4m_en_{e0}}\frac{\partial}{\partial z}\nabla^2 n_{e1}\right)\frac{\partial\langle f_{e0}\rangle}{\partial p_z}=0, \quad (1)$$

$$\left(\frac{\partial}{\partial t}+v_z\frac{\partial}{\partial z}\right)\langle f_{i1}\rangle + \left(-e\frac{\partial\Phi}{\partial z}+\frac{\hbar^2}{4m_in_{i0}}\frac{\partial}{\partial z}\nabla^2 n_{i1}\right)\frac{\partial\langle f_{i0}\rangle}{\partial p_z}=0, \quad (2)$$

and

$$n_{e1}=n_{i1}, \quad (3)$$

where $f_{e1}(f_{i1})$ is the perturbed electron (ion) distribution function with electron (ion) Fermi–Dirac equilibrium distribution function $f_{e0}(f_{i0})$. The angular bracket denotes the averaging over the spatio-scale and the temporal period of the EMWs [44]. Here, the potentials have been assumed to be the function of slow time $\frac{1}{\omega_0}$ and space $\frac{2\pi}{k_0}$ scales in equations (1) and (2). The ponderomotive force of the electrons in the nonrelativistic limit can be defined as [45] $F_{ze}=-\partial_z\Phi_{pe}$ with $\Phi_{pe}=\frac{e^2}{2m_ec^2}|A_{\perp 0}|^2$, where e is the electronic charge, $A_{\perp 0}$ and c are the amplitude of the high-frequency EM pump waves and speed of light in vacuum, respectively. Note that equation (2) is a simple equation modified with quantum correction, which provides information about quantum particles.

Solving equations (1) and (2) by using Zakharov's approximation [44], i.e. by making use of the linearization theory and Fourier transformation $f_{e1,i1}(k,\omega,v)\sim\int f_{e1,i1}(r,v,t)\exp[i(k\cdot r-\omega t)]drdt$, we immediately obtain the oscillating functions, respectively, as,

$$f_{e1}(k,\omega)=\left\{e\Phi(k,\omega)-\Phi_{pe}(k,\omega)-\frac{\hbar^2k^2}{4m_e}\frac{n_{e1}(k,\omega)}{n_{e0}}\right\}\times\frac{k}{\omega-kv_z}\frac{\partial f_{e0}(\varepsilon)}{\partial p_z}, \quad (4)$$

and

$$f_{i1}(k,\omega)=-\left\{e\Phi(k,\omega)+\frac{\hbar^2k^2}{4m_i}\frac{n_{i1}(k,\omega)}{n_{i0}}\right\}\frac{k}{\omega-kv_z}\frac{\partial f_{i0}(\varepsilon)}{\partial p_z}. \quad (5)$$

It is important to mention here that only the ponderomotive force of the electrons is taken into account for nonlinear coupling and the corresponding force for ions neglected due to their large mass. In addition, supposing that degenerate electrons and ions both follow the well-known Fermi–Dirac distributions, which can be expressed in terms of the Heaviside function (or step function) as,

$$f_{e0,i0}(\varepsilon)=\frac{1}{\exp[\varepsilon-\varepsilon_{Fe,i}]/k_BT_{e,i}+1}\equiv\Theta(\varepsilon_{Fe,i}-\varepsilon), \quad (6)$$

where $\varepsilon_{Fe}=m_ev_{Fe}^2/2\ \left[\varepsilon_{Fi}=m_iv_{Fi}^2/2\right]$ being the electron (ion) Fermi energy, is much larger compared to the thermal energy [41] of the plasma particles. The conversion of the derivative of the step function into the delta function can easily be performed through the relation,

$$\partial_\varepsilon\Theta=-\delta(\varepsilon_{Fe,i}-\varepsilon)=-\delta\left(\frac{p_{Fe,i}^2}{2m}-\frac{p^2}{2m}\right). \quad (7)$$

Taking into account equations (6) and (7) and keeping in mind the properties of the Dirac delta function, we may write,

$$\mathbf{k}\cdot\frac{\partial f_{oe}}{\partial\mathbf{P}}=-\frac{\mathbf{k}\cdot\mathbf{P}}{p_{Fe,i}}\delta(p_{Fe,i}-p). \quad (8)$$

Next, with the introduction of equation (8) into equations (4) and (5) and integrating over the velocity, in a spherical polar coordinate system, we may eventually obtain,

$$n_{e1}(k,\omega)=\frac{3n_{e0}}{m_ev_{Fe}^2F_e}\{e\Phi(k,\omega)-\Phi_{pe}(k,\omega)\}\times\left(1-\frac{\omega}{2kv_{Fe}}\ln\frac{\omega+kv_{Fe}}{\omega-kv_{Fe}}\right), \quad (9)$$

and

$$n_{i1}(k,\omega)=-\frac{3n_{0i}}{m_iv_{Fi}^2F_i}e\Phi(k,\omega)\left(1-\frac{\omega}{2kv_{Fi}}\ln\frac{\omega+kv_{Fi}}{\omega-kv_{Fi}}\right), \quad (10)$$

with

$$F_{e,i} = 1 + \frac{3\hbar^2 k^2}{4m_{e,i}^2 v_{Fe,i}^2}\left(1 - \frac{\omega}{2kv_{Fe,i}}\ln\frac{\omega + kv_{Fe,i}}{\omega - kv_{Fe,i}}\right),$$

where $v_{Fe,i} = 2T_{Fe,i}/m_{e,i}$ and $T_{Fe,i}$ are the EI Fermi temperatures. Equations (9) and (10) show the perturbed EI number densities incorporating quantum effects and can be used to investigate the nonlinear LD of transverse EMWs in a degenerate EI plasma.

Next, to study the nonlinear LD rate of the EM waves in a degenerate EI plasma, we need to first expand the density perturbations in equations (9) and (10) in the following frequency regimes:

$$kv_{Fi} \ll \omega \ll kv_{Fe}, \tag{11}$$

obtaining, respectively, the relations,

$$n_{e1}(k,\omega) = \frac{3n_{e0}}{m_e v_{Fe}^2\{1 + H_e^2(1 + iA_{Fe})\}} \times \{e\Phi(k,\omega) - \Phi_{pe}(k,\omega)\}(1 + iA_{Fe}), \tag{12}$$

and

$$n_{i1}(k,\omega) = \frac{n_{i0}}{m_i}\frac{k^2}{\omega^2}e\Phi, \tag{13}$$

where $H_e\left(= \frac{\sqrt{3}\hbar k}{2m_e v_{Fe}}\right)$ represents the quantum effects caused by the quantum Bohm potential and $A_{Fe}\left(= \frac{\pi\omega}{2kv_{Fe}}\right)$ the damping term involving the degenerate electrons, so that $A_{Fe}^2 \ll 1$. In deriving equations (12) and (13), we have used expansions of the logarithmic arguments for $x \ll 1$ and $x \gg 1$, respectively, and utilized $\ln[-1] = -i\pi$ [42] and $\ln\left(\frac{1+x}{1-x}\right) = 2\left(x + \frac{x^3}{3} + .\right)$ in equations (9) and (10). It should also be noted that the Bohm tunneling effect of ions is neglected compared to the tunneling effect of electrons. Thus, by substituting equations (12) and (13) into equation (3), we eventually arrive at,

$$e\Phi(k,\omega) = \frac{T_{Fe}\left\{1 + H_e^2(1 + iA_{Fe})\right\}}{3n_{e0}(1 + iA_{Fe})}n_{i1}(k,\omega) + \Phi_{pe}(k,\omega). \tag{14}$$

In addition, by substituting equation (14) into equation (13), we produce,

$$(\omega^2 + i\omega^2 A_{fe} - k^2 v_s^2)\frac{n_{e1}(k,\omega)}{n_{e0}} = \frac{k^2}{m_i}(1 + iA_{fe})\Phi_{pe}(k,\omega). \tag{15}$$

The ion–sound speed is now modified by the electron tunneling effect and can be expressed in the form $v_s = c_s(1 + H_e^2)^{1/2}$, where $c_s = (T_{Fe}/m_i)^{1/2}$ is the usual Fermi speed for degenerate species and $n_{e0} \simeq n_{i0}$.

Back substitution of the quantities, such as $-i\omega = \partial_t$, $\omega^2 = -\partial_t^2$, and $k^2 = -\partial_x^2$ yields the following relation:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}v_s^2\right)\frac{n_{e1}}{n_{e0}} - \frac{1}{2}\frac{1}{v_{Fe}}\frac{\partial^2}{\partial t^2}\wp\int_{-\infty}^{\infty}\frac{dz'}{z' - z}\wp\int_{-\infty}^{z'}dz''\frac{\partial}{\partial t}\frac{n_{e1}}{n_{e0}} = \frac{1}{m_i}\frac{\partial^2\Phi_{pe}}{\partial z^2} - \frac{1}{2m_i}\frac{\partial^2}{\partial z^2}\frac{1}{v_{Fe}}\wp\int_{-\infty}^{\infty}\frac{dz'}{z' - z}\wp\int_{-\infty}^{z'}dz''\frac{\partial\Phi_{pe}}{\partial t}. \tag{16}$$

Here, the formula $\frac{|k|}{k} = \frac{1}{i\pi}\wp\int_{-\infty}^{\infty}\frac{dz}{z}e^{ikz}$ is utilized in obtaining equation (16), showing the symbol $\wp$ as the principal value of the integral. Equation (16) is the generalization of Zakharov's set of equations [44], where the third term (in the LHS) and the last term (in the RHS) govern the nonlinear LD phenomena. Rewriting equation (16) by introducing a new moving frame as,

$$\xi = z - v_g t \text{ with } \frac{\partial}{\partial t} \ll v_g\frac{\partial}{\partial\xi},\ \Phi_{pe}(z,t) = \Phi_{pe}(\xi) \text{ and } n_{e1}(z,t) = n_{e1}(\xi), \tag{17}$$

we obtain

$$\frac{n_{e1}(\xi)}{n_{e0}} + \beta\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\frac{n_{e1}(\xi)}{n_{e0}} = \frac{1}{m_i(v_g^2 - v_s^2)}\Phi_{pe}(\xi) + \frac{1}{m_i}\frac{\beta}{v_g^2}\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\Phi_{pe}(\xi), \tag{18}$$

where v_g is the group speed and $\beta = \frac{1}{2}\frac{1}{v_g^2 - v_s^2}\frac{v_g^3}{v_{Fe}}$. Multiplying both sides of equation (18) by an operator $\left(1 - \beta\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\right)$ and using the following Poincare–Bertrand formula [44]:

$$\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\wp\int_{-\infty}^{\infty}\frac{d\xi'' F(\xi'')}{\xi' - \xi} = (i\pi)^2 F(z), \tag{19}$$

we produce a new expression in the following form:

$$\frac{n_{e1}(\xi)}{n_{e0}} = \frac{1}{1 + \beta^2\pi^2}\left[\frac{1}{m_i(v_g^2 - v_s^2)}\left(1 - \beta\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\right)\Phi_{pe}(\xi) + \frac{\pi^2}{m_i}\frac{\beta^2}{v_g^2}\Phi_{pe}(\xi) + \frac{1}{m_i}\frac{\beta}{v_g^2}\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\Phi_{pe}(\xi)\right]. \tag{20}$$

2.1. Subsonic case

Next, we discuss the electron density oscillations in supersonic and subsonic regimes. First, in the subsonic regime, i.e. $v_s \gg v_g$, i.e. $\beta\left(= -\frac{v_g^3}{v_{Fe}v_s^2}\right) \ll 1$, so the second and third terms in the RHS of equation (20) may be neglected. As a result, the electron density oscillations reduce to,

$$\frac{n_{e1}}{n_{e0}} = \left[-\frac{1}{m_i v_s^2}\Phi_{pe}(\xi) - \frac{1}{m_i}\frac{v_g}{2v_{Fe}v_s^2}\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}\Phi_{pe}(\xi)\right]. \tag{21}$$

2.2. Supersonic case

In the supersonic regime $v_g \gg v_s$ and $\beta^2\pi^2 \gg 1$, equation (20) reduces to the form,

$$n_{e1} = \frac{n_{e0}}{m_i v_g^2}\Phi_{pe}(\xi) = \frac{n_{e0}}{m_i v_g^2}\frac{e^2|A_{\perp 0}|^2}{2m_e c^2}. \tag{22}$$

Note that in the supersonic regime, the nonlocal nonlinear term appears and nonlinear LD disappears. It is also clear from the above equation that the density fluctuations of Fermi electrons become a function of the amplitude of EM waves and are an inverse function of the group velocity of the pump EM wave.

3. KNLSE for fermions

It is well-known [46] that mathematical representation of the EM wave packet having the complex amplitude $A_\perp$ is the Schrödinger equation, where the scaled Planck constant ($\hbar$) and potential energy are interchanged by the propagation number $k(=2\pi/\lambda)$ and refractive index, $\eta(=ck/\omega)$, respectively. The Schrödinger equation becomes nonlinear due to the fact that the refractive index is a function of the wave amplitude. Hence, the possible competition between the nonlinearity and diffraction terms may lead to a wide spectrum of effects arising during the nonlinear interaction of EM waves with the plasma medium. We need to solve the Maxwell equations in this context to obtain the nonlinear Schrödinger equation in terms of an EM wave packet having a complex amplitude $A_\perp$, as

$$-\nabla^2\mathbf{A}_\perp + \frac{1}{c^2}\partial_t^2\mathbf{A}_\perp = \frac{4\pi e\mathbf{J}}{c}, \tag{23}$$

where J is the plasma current density in the presence of a circularly polarized EM wave pulse. Since in our consideration, we have assumed the amplitude of the EM wave to be a slowly varying function of space and time coordinates, we have to substitute $A_\perp(x,t) = A_{\perp 0}(x,t)\exp i(k_0 z - \omega_0 t)$ in the above equation to obtain the NLSE for a degenerate plasma medium as,

$$i\left(\frac{\partial}{\partial t} + v_g\frac{\partial}{\partial z}\right)A_{\perp 0}(x,t) + \frac{c^2\nabla_z^2}{2\omega_0}A_{\perp 0}(x,t) + \delta^2 A_{\perp 0}(x,t) - \frac{\omega_{pe}^2}{2\omega_0}\frac{n_{e1}}{n_{e0}}A_{\perp 0}(x,t) = 0, \tag{24}$$

where $v_g = (k_0c^2/\omega_0)$ represents the group velocity of the EM wave, $A_{\perp 0}(x,t)$ is the time- and space-dependent amplitude of the EM wave, $\delta = \left[\left(\omega_0^2 - c^2k_0^2 - \omega_{pe}^2\right)/2\omega_0\right]^{1/2}$ is the nonlinear correction shift of frequency of the EM wave with electron plasma frequency $\omega_{pe} = \left(4\pi n_{e0}e^2/m_e\right)^{1/2}$. It is pertinent to mention that as the collisionless degenerate EI plasma under study is isotropic, the contribution of the electron current density is much larger in comparison to the ion–current densities in equation (24) [44]. Substituting the expression of n_{e1} into the Schrödinger equation (24), while taking into account equation (17), we obtain a new NLSE containing both local and nonlocal nonlinear LD terms, as

$$\left\{2i\omega_0\frac{\partial}{\partial t} + c^2\frac{\partial^2}{\partial\xi^2} + \delta^2 + \frac{\omega_{pe}^2}{2m_i v_s^2}\frac{e^2}{m_e c^2}\left(|A_{\perp 0}|^2 + \frac{1}{2}\frac{v_g}{v_{Fe}}\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi' - \xi}|A_{\perp 0}|^2\right)\right\}A_{\perp 0}(\xi,t) = 0. \tag{25}$$

The last two terms on the LHS of the above equation appear due to the nonlinear interaction of EMWs with the plasma under consideration such that; the third term is the local nonlinear term, whereas the last term is the nonlocal nonlinear term governing the nonlinear LD phenomena. To derive the nonlinear LD rate, we substitute $A_{\perp 0} = b_0 + b_1\ \exp[i(k\xi - \omega t)] + c.c$ with b_0 being the constant. Using the identities $\wp\int_{-\infty}^{\infty}\frac{d\xi'}{\xi'-\xi} = 0$ and $\wp\int_{-\infty}^{\infty}\frac{d\xi'}{(\xi'-\xi)}\exp ik(\xi' - \xi) = i\pi$, we finally obtain the following result:

$$\mathrm{Im}\,\omega = -\frac{\pi}{4}\frac{\omega_{pe}^2}{\omega_0}\frac{1}{m_i v_s^2}\frac{e^2|b_0|^2}{m_e c^2}\frac{v_g}{v_{Fe}}. \tag{26}$$

This describes the nonlinear LD of the transverse EM waves in a degenerate EI plasma, which is significantly affected by the Bohm quantum correction via the modified sound speed v_s. The nonlinear LD rate is found to be the direct function of the amplitude of the EM wave.

4. Linear electrostatic waves

In order to study the linear properties of electrostatic waves in a degenerate EI plasma, we need to first assume that EM waves are absent and ignore the nonlinear ponderomotive force in equation (1). By taking into account the collisionless damping due to wave–particle interaction, we insert the expressions n_{e1} and n_{i1} into the charge-neutrality equation (3) to obtain,

$$1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2F_e}\left(1 - \frac{\omega}{2kv_{Fe}}\ln\frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}\right) + \frac{3\omega_{pi}^2}{k^2v_{Fi}^2F_i}\left(1 - \frac{\omega}{2kv_{Fi}}\ln\frac{\omega + kv_{Fi}}{\omega - kv_{Fi}}\right) = 0. \tag{27}$$

For high-frequency quantum electron waves, we consider the electrons following the frequency regime $\omega \gg kv_{Fe}$ in the background of static ions. In this case, the series expansion $\ln(\frac{1+x}{1-x}) = 2(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots)$ is to be used in equation (27), which eventually yields,

$$\omega = \left(\omega_{pe}^2 + \frac{3}{5}k^2v_{Fe}^2 + \frac{\hbar^2k^4}{4m_e^2}\right)^{1/2}. \tag{28}$$

This clearly indicates that high-frequency degenerate electron oscillations propagate in a collisionless plasma without delivering any energy to the medium, so there is no LD in this case.

Next, we consider the intermediate frequency regime, i.e. $kv_{Fi} \ll \omega \ll kv_{Fe}$ to study the ion-acoustic (IA) waves and their linear damping rate. Hence, in this case, equation (27) can be reduced to,

$$1+\frac{3(1+iA_{Fe})}{k^2\lambda_{Fe}^2\left\{1+\frac{3\hbar^2k^2}{4m_e^2v_{Fe}^2}(1+iA_{Fe})\right\}}-\frac{\omega_{pi}^2}{\omega^2}=0. \quad (29)$$

Using $\omega=\omega_r+i\omega_i$ in equation (29), where $|\omega_i| \ll |\omega_r|$, one can decompose the real and imaginary parts of the frequency, as

$$\omega_r^2=\frac{1}{3}k^2c_s^2(1+H_e^2), \quad (30)$$

and

$$\omega_i=-\frac{\pi}{12}\frac{m_e kv_{Fe}}{m_i}, \quad (31)$$

where $c_s\,[=\omega_{pi}\lambda_{Fe}]$ is ion-sound speed with ion-plasma oscillation frequency ω_{pi} and electron Fermi length λ_{Fe}. Note that equation (30) is derived in the long-wavelength limit, representing the linear dispersion relation of IA waves. However, equation (31) indicates the linear damping of the IA waves as a direct function of electron Fermi speed only, so the degenerate electrons play a crucial role in absorbing the oscillations, where the Bohm tunneling effect does not contribute to the linear damping of IA waves in degenerate EI plasmas.

5. Numerical analyses

In order to illustrate our findings for the nonlinear LD rate caused by the interaction between transverse EMWs with a degenerate EI plasma at quantum scales, we solve equations (22) and (26) numerically for nonlinear and linear damping responses. We also choose typical quantum plasma parameters in the atmosphere of white dwarfs, where the electron density is $n_{e0}=(10^{24}\text{–}10^{26})$ cm^{-3} [47, 48] along with other physical constants in CGS units, e.g. $c=2.997\times10^{10}$ cm s^{-1}, $m_e=9.109\times10^{-28}g$, $e=4.8\times10^{-10}\,stat$coloumb, $\hbar=1.05\times10^{-27}$ cm^2 gs^{-1} and $k_B=1.3807\times10^{-16}$ cm^2 gs^{-2} K^{-1}. In order to satisfy the conditions for nonlinear LD damping of EM waves, i.e. $v_g[=\frac{\omega_0-\omega_0'}{k_0-k_0'}]\sim v_{Fe}$, we need to consider EM waves having different harmonics, e.g. microwaves of typical frequencies of the order of magnitude, $\omega_0=40\times10^6$ Hz and $\omega_0'=30\times10^6$ Hz with corresponding wavelengths $\lambda=2000$ cm and $\lambda'=5000$ cm, respectively. As a result, these frequency ranges lead to group speed of the order of $v_g\sim5.30\times10^8$ cm s^{-1}. We also take into account the specific value of electron number density as $n_{e0}=4\times10^{24}$ cm^{-3} to find out the electron Fermi length $\lambda_{Fe}\sim5.036\times10^{-9}$ cm, electron plasma frequency $\omega_{pe}\sim1.128\times10^{17}$ s^{-1} and electron Fermi speed $v_{Fe}=5.682\times10^8$ cm s^{-1}, which is comparable to v_g of the EM waves. In a subsonic regime, the group velocity is always assumed to be much smaller than the sound speed, i.e. $v_g\ll v_s$ and the nonlinear LD rate in this regime can be studied by normalizing

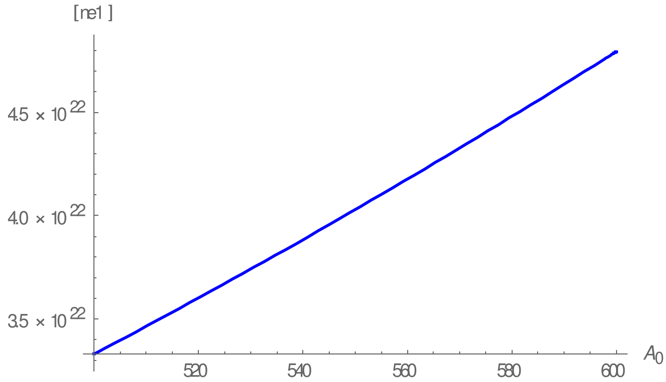

Figure 1. Normalized density fluctuations of degenerate electrons $\left(\frac{n_{e1}}{n_{0e}}\right)$ are plotted against the amplitude (A_0) of EMWs (as described by equation (22)) at a fixed value of group velocity $v_g=7.96\times10^7$ cm s^{-1}.

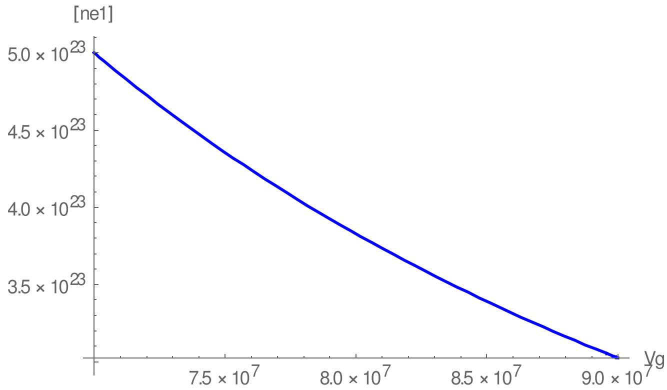

Figure 2. Normalized density fluctuations of degenerate electrons $\left(\frac{n_{e1}}{n_{0e}}\right)$ are plotted against the group velocity (v_g) of EMWs (as described by equation (22)) at a fixed value of $A_0=1703$ cm.

equation (26) with scaled parameters as $\tilde{c}=c/v_{Fe}$, $\tilde{k}=k\lambda_{Fe}$, $\tilde{k}_0=k_0\lambda_{Fe}$ and $\tilde{\omega}_0=\omega_0/\omega_{pe}$. The quantum tunneling parameter then yields $\tilde{H}_e=(\frac{\sqrt{3}}{2}\frac{\hbar\omega_{pe}}{m_e v_{Fe}^2}\tilde{k})\equiv0.176$ for electron density $n_{e0}=4\times10^{24}$ cm^{-3} and normalized wave number $\tilde{k}=0.503$. Thus, the nonlinear LD rate for microwaves in a degenerate quantum plasma turns out to be $\mathrm{Im}\tilde{\omega}=-6.637$ for normalized values of acoustic speed $\tilde{v}_s\left[=\tilde{c}_s(1+\tilde{H}_e^2)^{1/2}\right]\sim0.023$ and group velocity $\tilde{v}_g(=\tilde{k}_0\tilde{c}^2/\tilde{\omega}_0)\sim0.016$.

Equation (22) is plotted in figure 1 to display the density oscillations of degenerate electrons in a supersonic regime, as a direct function of amplitude (A_0) of the EMWs, whereas the electron density fluctuations are an inverse function of the group velocity of EMWs, see figure 2. Figure 3 represents how the normalized nonlinear LD rate ($\mathrm{Im}\tilde{\omega}$) of microwaves (of different harmonics) varies with the amplitude (b_0) of the EMWs in a subsonic regime, as shown in equation (26), as a function of Bohm potential that corresponds to the specific range of electron density concentration $(3-5)\times10^{24}$ cm^{-3}. It is evident from figure 3 that the nonlinear LD rate of microwaves increases as the Madelung term decreases. In

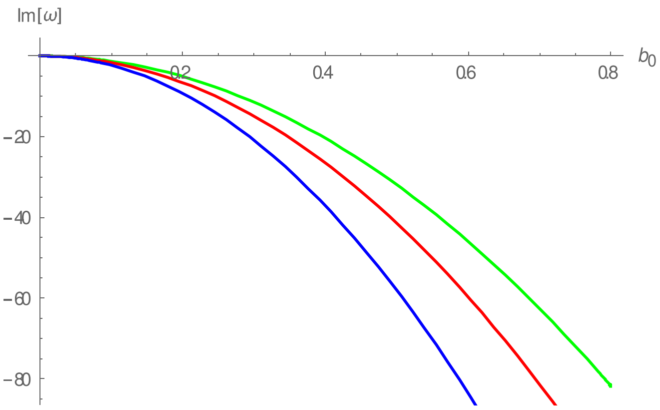

Figure 3. Normalized NLD rate of microwave $\mathrm{Im}\,\tilde{\omega}(=\omega/\omega_{pe})$ is plotted as a function of amplitude (b_0) (as described by equation (26)) for different values of Bohm tunneling potential: $H_e \sim 0.183$ (green curve), 0.176 (red curve) and 0.168 (blue curve).

other words, as we move towards the laboratory plasma parameters, the Madelung term will start playing a significant role in enhancing the nonlinear LD rate of the EMWs. In spite of the fact that the diffraction (Madelung) term is usually less than the pressure term in the momentum equation, in our present consideration this term plays a crucial role in enhancing the nonlinear LD rate of microwaves.

6. Conclusion

We have presented the nonlinear interaction of EM waves in a degenerate EI plasma by using the kinetic treatment of quantum species. In such a plasma, the group velocity (v_g) of the EM wave is approximately equal to the electron Fermi velocity v_{Fe}. For nonlinear LD rate of the EM waves, the properties of the step function and Fermi–Dirac distribution are utilized to derive the perturbed densities of the degenerate electrons and ions in the presence of electron ponderomotive force. It is noted that in a supersonic regime, the kinetic NLS equation includes only a local cubic nonlinearity and the nonlinear LD term disappears. Furthermore, the density oscillations of the Fermi electrons become a function of amplitude of the EM wave, while its inverse relation is shown with the group velocity of the EM waves in a supersonic regime. For considering the nonlinear LD in a subsonic regime, we have also obtained a KNLSEk, which involves both local and nonlocal nonlinear terms, where the latter is responsible for the nonlinear LD rate. Note that in the subsonic regime, the EM wave damps via the nonlinear LD rate in such a way that the Bohm potential plays a crucial role in the nonlinear LD rate of the EM wave. Next, the electrostatic response of the linear waves is studied accounting for the Bohm tunneling and Fermi statistical effects. The linear damping rate for the IA waves reflects that degenerate electrons play a key part in absorbing the oscillations, but the Bohm tunneling effect does not contribute to the linear damping in degenerate EI plasma. Numerical graphs support our analytical results and may prove useful for understanding the nonlinear damping of the EM waves in dense astrophysical plasmas, such as white dwarfs, active galactic nuclei, neutron stars, etc.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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